# MINI SYMPOSIUM

# Computational Problems in Low-dimensional Topology

# March 12–14, 2018 OIST Conference Center

Organizers: Tirasan Khandhawit, Dale Koenig, Nick Owad, Robert Tang, Anastasiia Tsvietkova (OIST) This workshop is supported by OIST

# **Schedule**

# Monday, March 12

**Transportation** 

 $\frac{\text{From}}{} \Rightarrow \frac{\text{To}}{} \frac{\text{Pickup Time}}{}$ 

Hotel Moon Beach ⇒ OIST Auditorium 9:40

10:00-10:50 Jane Gilman (Rutgers University, Newark)

Computability Models: Algebraic, Topological and Geometric

11:10-12:00 Gaven Martin (Massey University)

Random Discrete Groups

12:00-13:20 Lunch

### 13:20-14:10 Henry Segerman (Oklahoma State University)

A survey of veering triangulations and computation

14:30-15:20 Hyungryul Baik (Korea Advanced Institute of Science and Technology)

Asymptotic translation length in curve complexes

15:40-16:30 Hidetoshi Masai (Tohoku University)

Fibered commensurability of random mapping classes

16:40-17:00 Quick tour and group photo on a bridge

17:00-Dinner (at OIST Restaurant (Floor B, Center Bldg.))

# **Transportation**

<u>From</u>  $\Rightarrow$  <u>To</u> <u>Pickup Time</u> OIST Auditorium  $\Rightarrow$  Hotel Moon Beach 19:00

# Tuesday, March 13

# **Transportation**

 $\frac{\text{From}}{} \Rightarrow \frac{\text{To}}{} \frac{\text{Pickup Time}}{}$ 

 $\overline{\text{Hotel Moon Beach}} \Rightarrow \overline{\text{OIST Auditorium}} = 8:40$ 

# 9:00-9:50 Luisa Paoluzzi (Université d'Aix-Marseille)

Computational challenges of character varieties

# 10:10-11:00 Christian Zickert (University of Maryland)

Ptolemy varieties for higher dimensional manifolds

# 11:20-12:10 Craig Hodgson (University of Melbourne)

Counting genus 2 surfaces in 3-manifolds

12:20 Boarding a bus for lunch and excursion

13:00- Lunch at Restaurant "PEACE" in Southeast Botanical Gardens

14:30- EXCURSION/free time

17:30 – Dinner (at Nakadomari Restaurant)

# **Transportation**

<u>From</u>	$\Rightarrow$	<u>To</u>	<u>Pickup Time</u>
(For everybody)			
OIST Auditorium	$\Rightarrow$	Restaurant (lunch)	12:20
(For those who choose <u>no</u>	<u>t</u> to tou	r the gardens)	
Restaurant	$\Rightarrow$	<b>Hotel Moon Beach</b>	14:30
Hotel Moon Beach	$\Rightarrow$	Restaurant (dinner)	17:15
(For those who choose to	tour the	e gardens)	
Botanical Garden	$\Rightarrow$	Restaurant (dinner)	16:45
		, ,	
(For everybody)			
Restaurant	⇒	Hotel Moon Beach	20:00
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# Wednesday, March 14

# **Transportation**

 $\frac{\text{From}}{} \Rightarrow \frac{\text{To}}{} \frac{\text{Pickup Time}}{}$ 

Hotel Moon Beach ⇒ OIST Auditorium 8:40

## 9:00-9:50 Kazuhiro Ichihara (Nihon University)

Exceptional surgeries on knots and links

# 10:10-11:00 Alan Reid (Rice University)

Congruence link complements: A 3-dimensional Rademacher Conjecture

## 11:20-12:10 Ser Peow Tan (National University of Singapore)

Hyperbolic jigsaws and families of pseudomodular surfaces.

12:10-13:10 Lunch

## 13:10-14:00 Carlo Petronio (UNIVERSITA' DI PISA)

On the Hurwitz existence problem

# 14:20-15:10 Dan Romik (University of California, Davis)

Computational approaches to the moving sofa problem

## **Transportation**

 $\frac{\text{From}}{} \Rightarrow \frac{\text{To}}{} \frac{\text{Pickup Time}}{}$ 

OIST Auditorium ⇒ Hotel Moon Beach 15:30

17:30 – Dinner (at Tsubaki) (5 minutes walk from Moon Beach Hotel)

(Last modified: Mar. 5)

# **Abstract**

## **Hyungryul Baik**

Asymptotic translation length in curve complexes

We study the asymptotic translation length on curve complexes of the pseudo-Anosov surface homeomorphisms. We first show that the minimal asymptotic translation length of Torelli groups and pure braid groups are asymptotically  $1/\chi(S)$  where  $\chi(S)$  is the Euler characteristic of the surface. If the time permits, we also discuss the asymptotic translation length of pseudo-Anosov monodromies of primitive elements in Thurston's fibered cone. This talk represents joint work with Hyunshik Shin and Chenxi Wu.

### Jane Gilman

Computability Models: Algebraic, Topological and Geometric

In this talk we survey various computational models for problems, procedures and algorithms in Low Dimensional Topology including BSS machines, bit-computability, symbolic computation, G-Fenchel matrix computability, and extended bit-computability (EBC), a model currently under development by Tsvietkova and Gilman. We apply these to Riley's procedure and discreteness algorithms as in Gilman (1997) and also in the more recent work of Kapovich (2016) and the Series-Tan-Yamashita Diagonal Slice Algorithm (2014). We contrast algebraic models with topological models and geometric models. We present the recently completed algorithm for adapted bases. Part of the talk is joint work with Tsvietkova.

### **Craig David Hodgson**

Counting genus 2 surfaces in 3-manifolds

The 3D-index of a hyperbolic knot complement M is a powerful invariant introduced by the physicists Dimofte, Gaiotto and Gukov, giving a collection of formal power series in q with integer coefficients, indexed by a pair of integers. When both integers are zero, the constant term is 1 and we show that the coefficient of q can be expressed in terms of the numbers of genus 2 normal and almost normal surfaces in a suitable ideal triangulation of M. Further, this coefficient also has a purely topological description, in terms of the numbers of isotopy classes of genus 2 Heegaard surfaces and genus 2 incompressible surfaces in M. We will give examples illustrating these results, and sketch an approach to the proof, which also gives algorithms for counting isotopy classes of the above genus 2 surfaces. (This is based on joint work with Stavros Garoufalidis, Hyam Rubinstein, Henry Segerman, Neil Hoffman, and Nathan Dunfield.)

#### Kazuhiro Ichihara

Exceptional surgeries on knots and links

I will give a survey of my results on exceptional Dehn surgeries on knots and links. It will includes classification theorems of exceptional surgeries on alternating knots, on Montesinos knots, and on 2-bridge links.

Part of contents are based on joint works with In Dae Jong and Hidetoshi Masai.

#### **Gaven Martin**

Random Discrete Groups

This talk seeks to motivate and explore what it means to choose a random subgroup of SL(2,C). Then we seek to identify a geometrically natural probability distribution from which we can make meaningful calculations and discuss the probability that a finitely generated subgroup is discrete – particularly in the case of two-generator subgroups. An interesting special case concerns groups generated by two parabolics where precise answers are possible. Fixing the genus of a punctured surface then allows us to think about random conformal structures. The easiest case is the punctured torus where there is a very natural probability distribution given by looking at random ideal quadrilaterals and connecting this to a Gaussian like distribution on the moduli space allows us to compute the expected value of geometric invariants, like shortest geodesic and the distance to the origin in the Teichmüller metric.

## Hidetoshi Masai

Fibered commensurability of random mapping classes

We consider random walks on the mapping class group and discuss their fibered commensurability, or symmetry. We explain that such random mapping classes are minimal in their commensurability class. If time permits, we also discuss some applications of the minimality to the random mapping tori.

#### Luisa Paoluzzi

Computational challenges of character varieties

I will discuss some joint work with J. Porti (some still in progress) providing insight on the character varieties of knots. One of our main concerns is to detect ramification phenomena that may appear over fields of positive characteristic. I will present some results in this direction, focusing on the computational aspects that arise during the study.

## Carlo Petronio

On the Hurwitz existence problem

To a branched cover between closed surfaces one can associate a combinatorial datum, given by the genera and orientability of the source and target surfaces, the total degree, the number of branching points, and the partitions of the total degree given by the local degrees at the preimages of the branching points. This datum must satisfy some necessary conditions, notably the Riemann-Hurwitz formula, and some other ones in the non-orientable case. An old and still unsettled problem asks for which combinatorial data these necessary conditions are actually sufficient for the existence of a corresponding branched cover. I will report on recent and less recent progress on this problem.

#### **Alan Reid**

Congruence link complements: A 3-dimensional Rademacher Conjecture

The Rademacher Conjecture in dimension 2, stated that there are only finitely congruence surfaces of genus zero that are finite covers of the modular orbifold. This was answered in the 1970's by Denin. A 3-dimensional analogue of the Rademacher Conjecture asks whether there are only finitely many congruence link complements. This talk will describe some recent progress, including a complete enumeration of all the principal congruence link complements in  $S^3$ , thereby answering a question of Thurston. The ideas in the proof involve a mix of theory and computation.

### **Dan Romik**

Computational approaches to the moving sofa problem

The moving sofa problem is a well-known open problem in geometry. It asks for the planar shape of largest area that can be moved around a right-angled corner in a two-dimensional hallway of width 1. In this talk I will survey the known results about the problem, which has a surprisingly rich structure, and explain several ways in which its study is informed by computational methods. In particular, I will discuss recent results derived in joint work with Yoav Kallus, in which we developed and implemented an algorithm to prove new upper bounds for the area of a moving sofa shape, improving a 1968 result by Hammersley.

# **Henry Segerman**

A survey of veering triangulations and computation

I will give an overview of veering triangulations, a combinatorial tool introduced by Agol that describes manifolds with pseudo-Anosov bundle or flow structures. I'll discuss the ways in which computers have been used to generate examples and counterexamples, visualizations and statistical data for veering triangulations.

### **Ser Peow Tan**

Hyperbolic jigsaws and families of pseudomodular surfaces.

It is well known that the set of cusps of the modular group PSL(2,Z) is the set of rationals including infinity. In general, determining the set of cusps of a given Fuchsian group is a difficult question and not many families of examples are known where the set of cusps is completely determined. A pseudomodular group is a Fuchsian group which is not commensurable with the modular group but which has cusp set all of the rationals including infinity. The corresponding surface is called a pseudomodular surface. Long and Reid constructed a few examples of pseudomodular groups which belong to different commensurability classes and asked if there are infinitely many commensurability classes of pseudomodular groups. We give an affirmative answer to this question raised by Long and Reid. We do this by introducing a general construction of surfaces whose fundamental domains are obtained by gluing together marked ideal triangular tiles, which we call hyperbolic jigsaw surfaces. In the case of jigsaw surfaces made up of the two simplest tiles, we show that there is a pseudo-euclidean algorithm associated to the groups which brings every rational to infinity, so that all such surfaces are pseudomodular. This is joint work with Beicheng Lou and Anh Duc Vo.

### **Christian Zickert**

Ptolemy varieties for higher dimensional manifolds

We discuss how the theory of shape and Ptolemy coordinates on representation varieties extends to higher dimensional manifolds.