Abstracts

Speaker: Romina Arroyo
Title: The prescribed Ricci curvature problem for naturally reductive metrics on compact Lie groups
Abstract: One of the most important challenges of Riemannian geometry is to understand the Ricci curvature tensor. An interesting open problem related with it is to find a Riemannian metric whose Ricci curvature is prescribed, that is, a Riemannian metric g and a real number $c > 0$ satisfying

$$\mathrm{Ric}(g) = cT,$$

for some fixed symmetric $(0,2)$-tensor field T on a manifold M, where $\mathrm{Ric}(g)$ denotes the Ricci curvature of g.

The aim of this talk is to discuss this problem within the class of naturally reductive metrics when M is a compact simple Lie group, and present recently obtained results in this setting.

This talk is based on work in progress with Artem Pulemotov (The University of Queensland) and Wolfgang Ziller (University of Pennsylvania).

Speaker: Florica Cirstea
Title: Existence of singular solutions to elliptic equations with critical Hardy–Sobolev growth
Abstract: We discuss the existence of positive singular solutions with sharp asymptotic profiles for elliptic equations with critical Hardy–Sobolev growth:

$$-\Delta u = \frac{u^{2^*(s)-1}}{|x|^s} - \mu u^q \quad \text{in } B_R(0) \setminus \{0\}.$$

Here, $B_R(0)$ is the open ball of radius R centred at 0 in $\mathbb{R}^n$ $(n \geq 3)$, $\mu > 0$, $q > 1$, $s \in (0, 2)$ and $2^*(s) := 2(n-s)/(n-2)$. The case $s = 0$ and $\mu = 0$ has played an important role in the study of the Yamabe problem. As a novelty, we show that the perturbation term $-\mu u^q$ gives rise to two novel singular profiles when compared with the non-perturbed case. Besides the existence of solutions with $\liminf_{|x|\to 0} |x|^{(n-2)/2} u(x) = 0$ and $\limsup_{|x|\to 0} |x|^{(n-2)/2} u(x) \in (0, \infty)$ if $q \in (2^* - 2, 2^* - 1)$, we prove that when $q \in (2^*(s) - 1, 2^* - 1)$, there are infinitely many solutions satisfying $\lim_{|x|\to 0} |x|^{s/(q-2^*(s)+1)} u(x) = \mu^{-1/(q-2^*(s)+1)}$.

The results are based on joint work with Frédéric Robert (University of Lorraine) and Jérôme Vétois (McGill University).

Speaker: Jaigyoung Choe
Title: Some minimal submanifolds generalizing the Clifford torus
Abstract: The Clifford torus is the simplest nontotally geodesic minimal surface in $\mathbb{S}^3$. It is a product surface, it is helicoidal, and it is a solution of a PDE obtained by separation of variables. We will show that there are more minimal submanifolds with these properties in $\mathbb{S}^n$ and in $\mathbb{R}^4$.

Speaker: Xianzhe Dai
Title: The Agmon estimate for eigenfunctions of Witten equation
Abstract: Witten deformation is introduced in a seminal paper by Witten almost forty years ago and has led to major applications, mainly for compact manifolds. The resulting Witten equation differs from the Laplacian by mainly a potential term. The Agmon estimate is introduced also forty years ago in Agmon's influential study of eigenfunctions of Schrodinger operators. It is an

exponential decay estimate for the eigenfunctions in terms of the so-called Agmon distance. We establish an Agmon estimate for eigenfunctions of the Witten equation under certain tameness conditions. This enables us to deduce some consequences for Witten deformation on noncompact manifolds. The work is joint with my student Junrong Yan.

Speaker: Serena Dipierro
Title: A free boundary problem driven by the biharmonic operator
Abstract: In this talk we describe a minimization problem involving the biharmonic operator, and we discuss some results obtained in collaboration with Aram Karakhanyan and Enrico Valdinoci about the free boundary condition, the regularity of the solutions and that of their free boundary, and a monotonicity formula.

Speaker: Takashi Kagaya
Title: Existence of non-convex traveling waves for a surface diffusion flow with contact angle condition
Abstract: We consider an evolving plane curve with two endpoints, which can move freely on the x-axis with generating constant contact angles. For the evolution of this plane curve governed by surface diffusion, we discuss the existence, the uniqueness and the convexity of traveling waves. The main results show the uniqueness and the convexity can be lost in depending on the conditions of the contact angles. The talk is based on a joint work with Yoshihito Kohsaka (Kobe University).

Speaker: Yoshihito Kohsaka
Title: Stability of the steady states for evolving surfaces by surface diffusion
Abstract: Stability of the steady states of the surface diffusion equation will be studied. The surface diffusion equation was first derived by Mullins to model the motion of interfaces in the case that the motion of interfaces is governed purely by mass diffusion within the interfaces. Also, this equation has a variational structure that the area of the surface decreases whereas the volume of the region enclosed by the surface is preserved. This provides the constant mean curvature surfaces (CMC surfaces) as steady states for this equation. In this talk, an axisymmetric case will be discussed.

Speaker: Kwok Kun Kwong
Title: Quantitative comparison theorems in geometry
Abstract: The classical volume comparison states that under a lower bound on the Ricci curvature, the volume of the geodesic ball is bounded from above by that of the ball with the same radius in the model space. On the other hand, counterexamples show that the assumption on the Ricci curvature cannot be weakened to a lower bound on the scalar curvature, which is the average of the Ricci curvature. In this talk, I will show that a lower bound on a weighted volume integral of the Ricci curvature is sufficient to ensure volume comparison. If time allows, I will also show an integral version of the Laplacian comparison theorem, a quantitative Gunther's theorem, a Kahlerian analogue and some other extensions.

Speaker: Ramiro Lafuente
Title: On the signature of the Ricci curvature on nilmanifolds

Abstract: In this talk, I will report on a very recent project in collaboration with Romina Arroyo, in which we completely describe all possible signatures for the Ricci curvature of left invariant metrics on an arbitrary nilpotent Lie group. The key idea in the proof is to establish a connection between zeroes of the Ricci curvature and closed orbits in a certain representation space for the general linear group.

Speaker: Ki-Ahm Lee
Title: Evolution of Level Sets in Curvature Flows
Abstract: In this talk, we are going to discuss how to develop some regularity theory on the lower dimensional objects in Nonlinear Partial Differential Equations. Specially, we consider various quantities related with lower or upper bound of geometric quantities of level sets of convex functions satisfying Gauss Curvature Flows and then optimal decay rate of each curvature near the flat spot. In addition, we will discuss higher regularity of the solution in weighted space related to decay rates.

Speaker: Jiakun Liu
Title: Regularity of free boundary in optimal transport
Abstract: In this talk, we introduce some recent regularity results of free boundary in optimal transportation with the quadratic cost. Particularly in dimension two, when the densities f, g are C^α and the domains Ω, Ω^* are C^2, uniformly convex, by adopting our recent results on boundary regularity of Monge-Ampere equations, we prove that the free boundary is $C^{2,\alpha}$. This is a joint work with Shibing Chen and Xu-Jia Wang.

Speaker: Luca Lombardini
Title: Nonlocal Plateau problem with obstacles
Abstract: The fractional perimeter and the associated minimizing sets, whose boundaries are the so called nonlocal minimal surfaces, were introduced by Caffarelli, Roquejoffre & Savin in 2010, and have since then attracted a lot of interest. In this talk I will present some recent results concerning subgraphs having finite fractional perimeter. We will define a fractional and nonlocal counterpart of the classical area functional and we will then study its minimizers, namely the nonlocal minimal graphs. In particular, we will present existence and uniqueness results, eventually in presence of (discontinuous) obstacles. Finally, we will focus on the so called highly nonlocal regimes, showing some "stickiness" phenomena, which are typically nonlocal.

Speaker: Takeyuki Nagasawa
Title: Asymptotic analysis for non-local curvature flows for plane curves with general rotation number
Abstract: Several non-local curvature flows for plane curve with general rotation number are discussed. The flows include the area-preserving one and the length-preserving one. We have a relatively good understanding of these flows for curve with the rotation number. In particular, when the initial curve is strictly convex, the flow converges to a circle as time to infinity. Even if the initial curve is not strictly convex, a global solution, if exists, converges to a circle. Here we deal with curves with general rotation number, and show not only a similar result for global solutions but also a blow-up criteria, estimates of the blow-up time and blow-up rate from below. We use a geometric quantity which has never been considered before. This is joint work with Kohei Nakamura.

Speaker: Norbert Pozar
Title: Viscosity approach to the crystalline mean curvature flow
Abstract: In this talk I will give an overview of the notion of viscosity solutions for the crystalline mean curvature flow in an arbitrary dimension, introduced recently in joint work with Yoshikazu Giga from the University of Tokyo. This problem serves as a model of crystal growth but it also has applications in image processing and related fields. Its level set formulation leads to a nonlocal, very singular parabolic equation with non-smooth, faceted solutions to which the standard viscosity theory does not apply. We introduce a reduced class of faceted test functions and show that they are sufficient to establish the comparison principle as well as an existence result for a rather general class of problems with the crystalline mean curvature.

Speaker: Artem Pulemotov
Title: The prescribed Ricci curvature problem for homogeneous metrics in low dimensions
Abstract: We discuss the problem of finding a homogeneous Riemannian metric with given Ricci curvature on a family of compact low-dimensional spaces. Joint work with Mark Gould (The University of Queensland) and Wolfgang Ziller (The University of Pennsylvania).

Speaker: Phil Schrader
Title: Curve shortening by Sobolev gradient flow
Abstract: The classical curve shortening flow evolves planar curves in the (negative) normal direction proportional to curvature. This direction is the gradient of length with respect to a parametrization invariant Riemannian metric of class L^2 on the space of immersed curves. This space being infinite dimensional, not all Riemannian metrics are equivalent, and so we might expect different behaviour for the gradient flows associated with other metrics. I will present some results from investigating the gradient flow associated with a Sobolev H^1 metric on the space of curves (joint work with G. Wheeler and V.M. Wheeler).

Speaker: Felix Schulze
Title: On the regularity of Ricci flows coming out of metric spaces
Abstract: We consider smooth, not necessarily complete, Ricci flows, $(M, g(t))_{t\in(0,T)}$ with $\mathrm{Ric}(g(t)) \geq -1$ and $|\mathrm{Rm}(g(t))| \leq \frac{c}{t}$ for all $t \in (0,T)$ coming out of metric spaces (M, d_0) in the sense that $(M, d(g(t)), x_0) \to (M, d_0, x_0)$ as $t \to 0$ in the pointed Gromov-Hausdorff sense. In the case that $B_{g(t)}(x_0, 1) \Subset M$ for all $t \in (0,T)$ and d_0 is generated by a smooth Riemannian metric in distance coordinates, we show using Ricci-harmonic map heat flow, that there is a corresponding smooth solution $\tilde{g}(t)_{t\in(0,T)}$ to the δ-Ricci-DeTurck flow on an Euclidean ball $B_r(p_0) \subset \mathbb{R}^n$, which can be extended to a smooth solution defined for $t \in [0,T)$. We further show, that this implies that the original solution g can be extended to a smooth solution on $B_{d_0}(x_0, \frac{r}{2})$ for $t \in [0,T)$, in view of the method of Hamilton. This is joint work with Alix Deruelle and Miles Simon.

Speaker: Weimin Sheng
Title: An Anisotropic shrinking flow and L_p Minkowski problem
Abstract: In this talk, I will introduce my recent work with Caihong Yi on studying anisotropic shrinking flows and the application on L_p Minkowski problem. We consider a shrinking flow of smooth, closed, uniformly convex hypersurfaces in Euclidean $\mathbb{R}^{n+1}$ with speed $f\, u^{\alpha}\sigma_n^{-\beta}$, where u is the support function of the hypersurface, α and β are two real numbers, and $\beta > 0$, σ_n is

the n-th symmetric polynomial of the principle curvature radii of the hypersurface. We show that the flow exists an unique smooth solution for all time and converges smoothly after normalisation to a smooth solution of the equation $f\, u^{\alpha-1}\sigma_n^{-\beta} = c$ provided the initial hypersurface is origin-symmetric and f is a smooth positive even function on $\mathbb{S}^n$ for some cases of α and β. In the case $\alpha \geq 1 + n\beta, \beta > 0$, we prove that the flow converges smoothly after normalisation to a unique smooth solution of $f\, u^{\alpha-1}\sigma_n^{-\beta} = c$ without any constraint on the initial hypersuface and the function f. When $\beta = 1$, our argument provides a uniform proof to the existence of the solutions to the L_p Minkowski problem $u^{1-p}\sigma_n = \phi$ for $p \in (-n-1, +\infty)$ where ϕ is a smooth positive function on $\mathbb{S}^n$.

Speaker: Neil Trudinger
Title: Classical solvability of generated Jacobian equations in geometric optics
Abstract: The notion of generated prescribed Jacobian equation was recently introduced by us as a framework to extend the theory of Monge-Ampere type equations in optimal transportation to those in near field geometric optics. In this talk we will present recent work on the classical solvability of the second boundary value problem, which in optics models the illumination of targets with prescribed intensity by reflection or refraction.

Speaker: Dong-Ho Tsai
Title: On a nonlocal curvature flow arising from the Hele-Shaw problem
Abstract: We consider long time behavior of a given smooth convex embedded closed curve $\gamma_0 \subset \mathbb{R}^2$ evolving according to a nonlocal curvature flow, which arises in a Hele-Shaw problem and has a prescribed rate of change in its enclosed area $A(t)$, i.e. $dA/dt = -\beta$, where $\beta \in (-\infty, \infty)$ is given. Specifically, when the enclosed area expands at any fixed rate, i.e. $\beta \in (-\infty, 0)$, or decreases at a fixed rate $\beta \in (0, 2\pi)$, one has the round circle as the unique asymptotic shape of the evolving curves; while for a sufficiently large rate of area decrease, one can have n-fold symmetric curves (which look like regular polygons with smooth corners) as extinction shapes (self-similar solutions).

Speaker: Enrico Valdinoci
Title: Nonlocal minimal graphs in the plane are generically sticky
Abstract: We discuss some recent boundary regularity results for nonlocal minimal surfaces in the plane. In particular, we show that nonlocal minimal graphs in the plane exhibit generically stickiness effects and boundary discontinuities. More precisely, if a nonlocal minimal graph in a slab is continuous up to the boundary, then arbitrarily small perturbations of the far-away data necessarily produce boundary discontinuities. Hence, either a nonlocal minimal graph is discontinuous at the boundary, or a small perturbation of the prescribed conditions produces boundary discontinuities. The proof relies on a sliding method combined with a fine boundary regularity analysis, based on a discontinuity/smoothness alternative. Namely, we establish that nonlocal minimal graphs are either discontinuous at the boundary or their derivative is Hlder continuous up to the boundary. In this spirit, we prove that the boundary regularity of nonlocal minimal graphs in the plane "jumps" from discontinuous to differentiable, with no intermediate possibilities allowed. In particular, we deduce that the nonlocal curvature equation is always satisfied up to the boundary. As a byproduct of our analysis, one describes the "switch" between the regime of continuous (and hence differentiable) nonlocal minimal graphs to that of discontinuous (and hence with differentiable inverse) ones. These results have been obtained in collaboration with Serena Dipierro and Ovidiu Savin.

Speaker: Xu-Jia Wang
Title: Convex hypersurfaces of prescribed curvatures
Abstract: There are several well-known problems on convex hypersurfaces of prescribed curvatures. The well known ones include the Minkowski problem, the Aleksandrov problem, the Christoffel problem, and their dual problems. In this talk, I will report some recent developments on the existence, regularity, and uniqueness of solutions on these problems, including curvature flows to the dual Minkowski problem and solving the Christoffel problem by the fundamental solution of the Laplacian.

Speaker: Guofang Wei
Title: $L^{n/2}$-curvature pinching results
Abstract: Using Ricci flow we obtain an $L^{n/2}$-curvature pinching result for Yamabe metrics which generalizes the work of Hebey–Vaugon and Gursky. We also obtain an $L^{n/2}$ version of Gromov's almost flat manifolds theorem. This is joined with Eric Chen and Rugang Ye.

Speaker: Valentina Wheeler
Title: Counterexamples to graph preservation under mean curvature flow
Abstract: Mean curvature flow is the steepest descent gradient flow for the area functional, making it a difficult flow to obtain a long time existence result for geometric objects that flow without an additional restriction. One can see this in a "simple" way by using convex objects as barrier. The celebrated result of Huisken showing that convex bodies under mean curvature flow will shrink out of existence in finite time allows us to prove the extinction of any other object contained inside(or lose regularity before). One property of the solution that facilitates the long time existence proof is that of being graphical. A mean curvature flow solution that is graphical is equivalent to a second order quasilinear strictly parabolic partial differential equation for which one can employ parabolic methods to obtain long time existence. Under graphicality Ecker and Huisken proved in '89 that entire solutions of mean curvature flow exist for all times. For boundary value problems one has to consider further arguments to manage maintaining initial graphicality up to and including the boundary. In this talk we show that for hypersurfaces moving by mean curvature flow with free boundary, preservation of graphicality holds only in very special circumstances. That is we prove that for any non-cylindrical smooth support hypersurface there exist smooth mean curvature flows with graphical initial data and free boundary on this hypersurface that become non-graphical in finite time. This is joint work with Ben Andrews (ANU).