Flat space holographic c-functions

Daniel Grumiller

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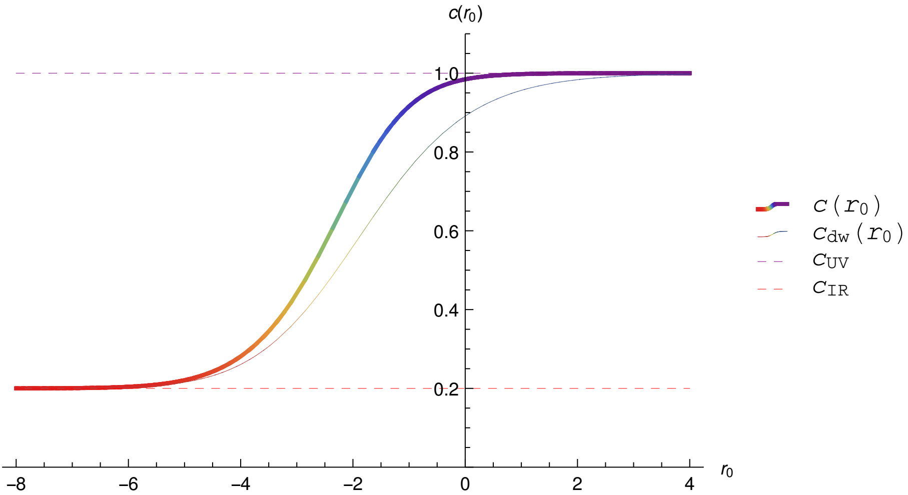
Flat Asymptopia Workshop
OIST, March 2021

FWF

with Max Riegler



$c(r_0)$
1.0
0.8
0.6
0.4
0.2
–8
–6
–4
–2
0
2
4
r_0
$c(r_0)$
$c_{\rm dw}(r_0)$
$c_{\rm UV}$
$c_{\rm IR}$

Outline

Flat space holography based on BMS_3

Holographic c-functions in $\mathrm{AdS}_3/\mathrm{CFT}_2$

BMS_3 entanglement entropy

Flat space domain walls and BMS_3 c-functions

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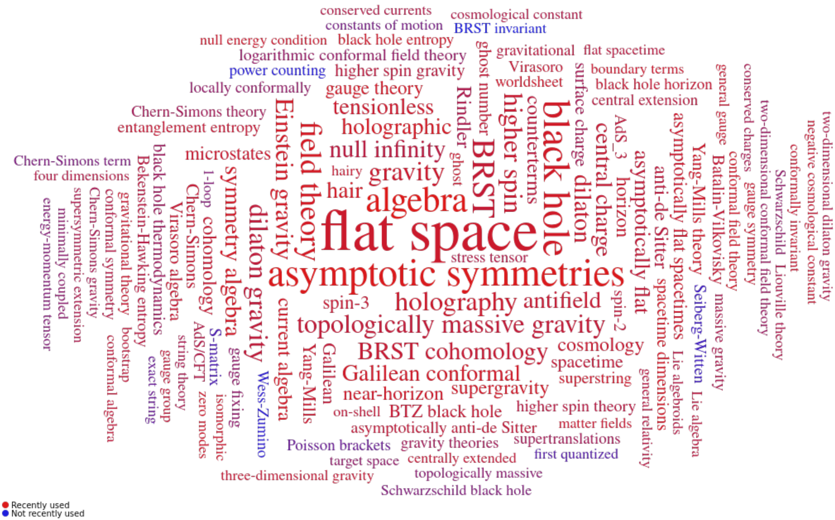
Motivation and overview



word cloud generared using today's speakers

Motivation and overview

See previous talk by Arjun Bagchi!

BMS_3

Asymptotic symmetries in asymptotically flat space for 3d Einstein gravity:
Ashtekar, Bicak, Schmidt '96; Barnich, Compère '06

$$[L_n,\, L_m] = (n-m)\, L_{n+m}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\, n(n^2-1)\, \delta_{n+m,\,0}$$
$$[M_n,\, M_m] = 0$$

reminder: some holographic checks based on BMS_3 symmetries

- cardyology Bagchi, Detournay, Fareghbal, Simón; Barnich '12
- entanglement entropy Bagchi, Basu, DG, Riegler '14
- stress-energy correlators Bagchi, DG, Merbis '15
- ... see Arjun's talk! ...

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- L_n: superrotations (diff(S^1))

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- L_n: superrotations (diff(S^1))
- M_n: supertranslations
- $c_M = 3/G$: BMS_3-central charge

Note: c_M dimensionful $\Rightarrow$ dimensionless ratios still meaningful

$$\frac{c_M}{h_M}\,, \qquad \frac{c_M^{\text{UV}}}{c_M^{\text{IR}}}\,, \qquad c_M \times \text{length}$$

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This talk: bulk theory = Einstein gravity + scalar field

$$I_{\text{bulk}}[g_{\mu\nu},\, \phi] = \frac{1}{16\pi G} \int \text{d}^3x \sqrt{-g}\, \Big(R - \frac{1}{2}\,(\partial\phi)^2 - V(\phi)\Big)$$

FWF Der Wissenschaftsfonds.

7-point function of the stress-tensor from 3D holography.

Wout Merbis

Institute for Theoretical Physics, Vienna University of Technology
Wiedner Hauptstrasse 8-10/136 A-1040 Wien, AUSTRIA

$$\langle T(z_1)T(z_2)T(z_3)T(z_4)T(z_5)T(z_6)T(z_7)\rangle =$$

Based on work presented in "Stress tensor correlators in three-dimensional gravity" with A. Bagchi and D. Grumiller. Special thanks to Friedrich Schöller for Mathematica wizardry

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Zamolodchikov c-function

Consider RG-flow from UV to IR in QFT$_2$

fineprint: the Euclidean QFT$_2$ is assumed to be renormalizable, reflection-positive, translation- and rotation-invariant

Zamolodchikov c-function

Consider RG-flow from UV to IR in QFT$_2$

- coupling constants g^i have β-functions

$$\dot{g}^i = \beta^i(g)$$

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$$\dot{g}^i = \beta^i(g)$$

- there exists a function $c(g)$ with the properties
  1. Monotonicity

$$\dot{c}(g) := \beta^i(g)\,\frac{\partial c(g)}{\partial g^i} \leq 0$$

Equivalently: $c(g)$ non-increasing under dilatations!

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  2. Saturation at fixed points

$$\dot{c}(g_*) = 0 \qquad \leftrightarrow \qquad \beta^i(g_*) = 0$$

Note: at fixed points enhancement to CFT$_2$ Virasoro symmetries

$$[L_n,\, L_m] = (n-m)\,L_{n+m} + \frac{c_{\text{Vir}}(g_*)}{12}\,n(n^2-1)\,\delta_{n+m,\,0}$$

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$$c(g_*) = c_{\text{Vir}}(g_*)$$

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- Consequence:

$$c_{\mathrm{Vir}}^{\mathrm{UV}} > c_{\mathrm{Vir}}^{\mathrm{IR}} \qquad \Rightarrow \qquad \text{more dof in UV}$$

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- Explicit construction of a c-function using stress tensor and its 2-point correlators Zamolodchikov '86

Domain wall c-function in $\mathrm{AdS}_3/\mathrm{CFT}_2$ (Freedman, Gubser, Pilch, Warner '99)

Two interesting alternative c-functions in holography context

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Two interesting alternative c-functions in holography context

domain walls in AdS$_3$ (set AdS radius to unity)

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + e^{2A(\rho)}\big(-\mathrm{d}t^2 + \mathrm{d}x^2\big) \qquad \lim_{\rho\to\infty} A(\rho) = \rho + \dots$$

holographic model for RG flow (UV: $\rho \to \infty$)

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- $\rho = \mathrm{const.}$ slices: Poincaré invariant

$$\mathrm{KVs}: \qquad \partial_t \qquad \partial_x \qquad x\partial_t + t\partial_x$$

conformal KVs: infinitely many (CFT_2 symmetries)

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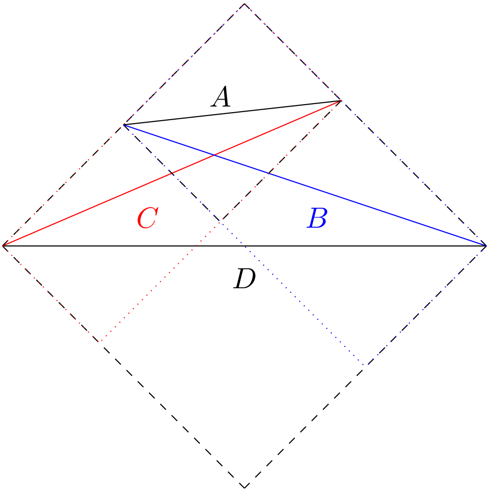
- holographic domain wall c-function

$$c_{\text{dw}}(\rho) = \frac{c^{\text{UV}}}{A'(\rho)}$$

  monotonicity implied by reality of bulk scalar field [see later!]

Casini–Huerta (CH) c-function (CH '06)



$$AD = BC \;\Rightarrow\; C = \lambda A,\; D = \lambda B,\; \lambda > 1$$

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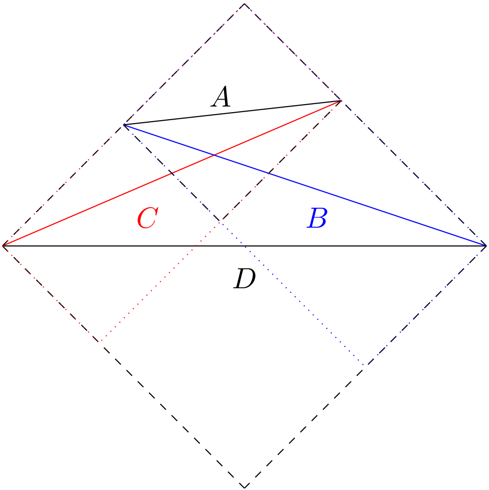

- SSA of EE

$$S(C) + S(B) \geq S(A) + S(D)$$

Lieb, Ruskai '73; Kiefer '59

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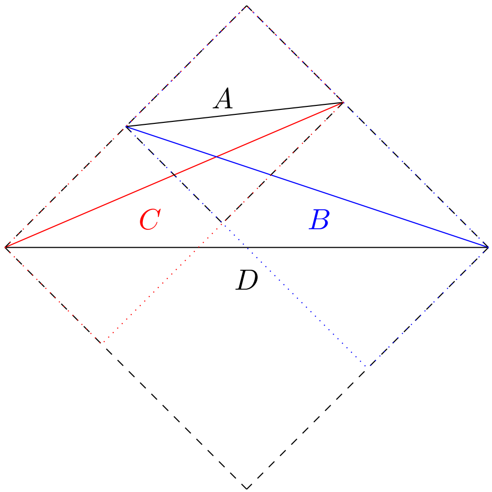

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$$S(B) - S(A) \geq S(\lambda B) - S(\lambda A)$$

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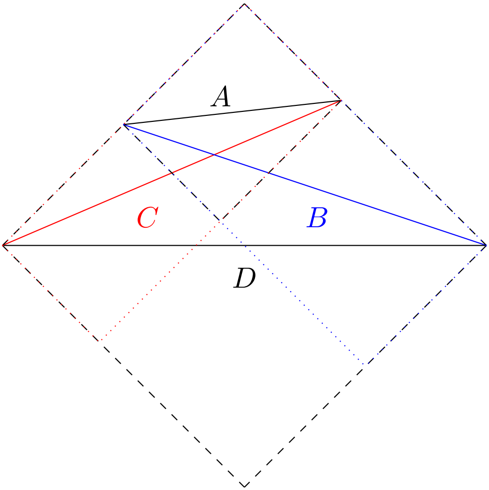

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- Differential instead difference

$$c_{\mathrm{CH}}(L) = \# \, L \, S'(L)$$

non-increasing under dilatations!

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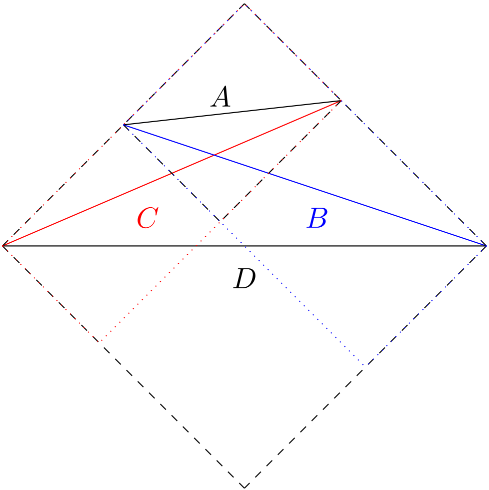

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- Differential instead difference

$$c_{\rm CH}(L) = \#\, L\, S'(L)$$

non-increasing under dilatations!

- Fix normalization

$$c_{\rm CH}(L) = 3\, L\, S'(L)$$

Properties of CH c-function

- Monotonicity

$$c'_{\mathrm{CH}}(L) = 3L\,S''(L) + 3S'(L) \leq 0$$

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- Fixed point values

$$\lim_{L\to 0} c_{\text{CH}}(L) = c^{\text{UV}} \qquad\qquad \lim_{L\to\infty} c_{\text{CH}}(L) = c^{\text{IR}}$$

Note: at fixed points we have

$$S(L\to 0) = \frac{c^{\text{UV}}}{3}\,\ln L \quad\Rightarrow\quad c_{\text{CH}}(L\to 0) = \lim_{L\to 0}\big(3L\,S'(L)\big) \to c^{\text{UV}}$$

$$S(L\to \infty) = \frac{c^{\text{IR}}}{3}\,\ln L \quad\Rightarrow\quad c_{\text{CH}}(L\to \infty) = \lim_{L\to \infty}\big(3L\,S'(L)\big) \to c^{\text{IR}}$$

Holzhey, Larsen, Wilczek '94; Cardy, Calabrese '06

$\Rightarrow$ is indeed a c-function different from previous ones!

Properties of CH c-function

- Monotonicity

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- Monotonicity of CH c-function

$$\frac{c'_{\text{CH}}(L)}{3L} = S''(L) - \frac{S'(L)}{L} + \frac{6}{c_{\text{CH}}(L)}\,(S'(L))^2 \leq 0$$

implies ground state QNEC [see next slide]

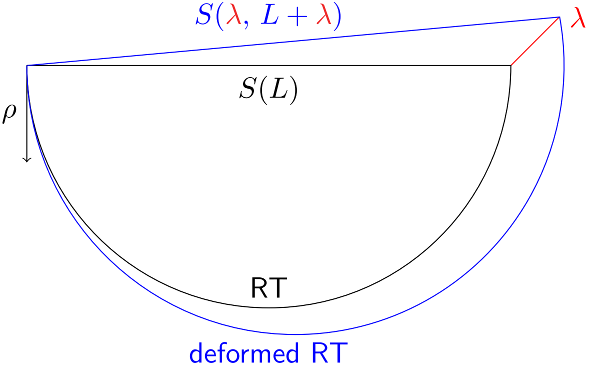
$$S''(L) - \frac{S'(L)}{L} + \frac{6}{c_{\text{UV}}}\,(S'(L))^2 \leq 0$$

Ecker, DG, Soltanpanahi, Stanzer '20

QNEC in 2d

Quantum Null Energy Condition (QNEC) in 2d:

$$2\pi \langle T_{\pm\pm} \rangle \geq \frac{\mathrm{d}^2 S}{\mathrm{d}\lambda^2}\bigg|_{\lambda=0} + \frac{6}{c^{\mathrm{UV}}} \left(\frac{\mathrm{d}S}{\mathrm{d}\lambda} \right)^2 \bigg|_{\lambda=0}$$



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- both sides of QNEC transform with Schwarzian derivative Wall '11

  under bulk diffeos (or boundary conformal trafos):

$$\delta_\xi S = \underbrace{\xi S' - \frac{c}{12}\,\xi'}_{\text{anomalous scalar}}$$

  implies

$$\delta_\xi Q = \underbrace{\xi Q' + 2\xi' Q - \frac{c}{12}\,\xi'''}_{\text{infinitesimal Schwarzian}}$$

  with $Q := S'' + \frac{6}{c}\,(S')^2$

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- QNEC saturates for states dual to vacuum Einstein solutions
  Khandker, Kundu, Li '18; Ecker, DG, Sheikh-Jabbari, Stanzer, van der Schee '20

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- for boost invariant ground states:

$$0 \geq \frac{\mathrm{d}^2 S}{\mathrm{d}\lambda^2}\Big|_{\lambda=0} + \frac{6}{c^{\text{UV}}}\left(\frac{\mathrm{d}S}{\mathrm{d}\lambda}\right)^2\Big|_{\lambda=0} = S''(L) - \frac{S'(L)}{L} + \frac{6}{c^{\text{UV}}}\,(S')^2$$

since $S(\lambda,\, L+\lambda) = S(0, \sqrt{(L+\lambda)^2 - \lambda^2})$

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  since $S(\lambda,\, L+\lambda) = S(0, \sqrt{(L+\lambda)^2 - \lambda^2})$
  Ecker, DG, Sheikh-Jabbari, Stanzer '20
- ground state QNEC necessary for monotonicity of CH c-function

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EE in BMS_3 invariant QFTs (Bagchi, Basu, DG, Riegler '14)

Using tricks similar to CFT_2 (Renyi entropy, twist operators, uniformization maps, universality of 2- and 3-point functions, ...) we find

$$S = \frac{c_L}{6} \ln \Delta x + \frac{c_M}{6} \frac{\Delta u}{\Delta x}$$

resembles EE for CFTs with gravitational anomaly

$$S = \frac{c_L + c_R}{6} \ln \Delta x + \frac{c_L - c_R}{6} \underbrace{\kappa}_{\text{boost}}$$

Castro, Detournay, Iqbal, Perlmutter '14

and EE for warped CFTs

$$S = C_0 \ln \Delta x + C_1 \Delta u$$

Castro, Hofman, Iqbal '15

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Remarks:

- neglected above cut-off dependence (more on this on next slide!)

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- for vanishing c_M: recover expected result for chiral half of CFT_2

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- for vanishing c_L:

$$S = \frac{c_M}{6}\,\frac{\Delta u}{\Delta x}$$

  non-zero only if $\Delta u \neq 0$ (makes sense from Carrollian perspective)

EE in BMS_3 invariant QFTs (Bagchi, Basu, DG, Riegler '14)

Using tricks similar to CFT_2 (Renyi entropy, twist operators, uniformization maps, universality of 2- and 3-point functions, ...) we find

$$S = \frac{c_L}{6}\,\ln\Delta x + \frac{c_M}{6}\,\frac{\Delta u}{\Delta x}$$

Remarks:

- neglected above cut-off dependence (more on this on next slide!)
- result only true for state analogous to vacuum on plane
- for vanishing c_M: recover expected result for chiral half of CFT_2
- for vanishing c_L:

$$S = \frac{c_M}{6}\,\frac{\Delta u}{\Delta x}$$

  non-zero only if $\Delta u \neq 0$ (makes sense from Carrollian perspective)

Focus on $c_L = 0$ and $c_M \neq 0$

Cut-off from boost invariance

planar vacuum in CFT_2 boost invariant, but EE without cut-off is not

$$S = \frac{c}{3}\,\ln\Delta x \to \tilde{S} = \frac{c}{3}\,\ln\Delta\tilde{x} \neq S$$

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boost invariance restored by multiplicative cut-off term

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apply this now to BMS_3 EE; Carrollian boosts:

$$\Delta x \to \Delta x \qquad\qquad \Delta u \to \Delta u + \gamma\Delta x$$

thus, BMS_3 EE without cut-off not boost invariant:

$$S = \frac{c_M}{6}\,\frac{\Delta u}{\Delta x} \to \tilde{S} = \frac{c_M}{6}\left(\frac{\Delta u}{\Delta x} + \gamma\right)$$

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boost invariance restored by additive cut-off term $\epsilon_u \to \epsilon_u + \gamma\epsilon_x$

$$S = \frac{c_M}{6}\left(\frac{\Delta u}{\Delta x/\epsilon_x} - \epsilon_u\right) \to \tilde{S} = S$$

note: from dimensional arguments also multiplicative cut-off term ϵ_x

Uniformization of BMS_3 EE (DG, Parekh, Riegler '19)

- in AdS_3/CFT_2: simple uniformization of (holographic) EE for all states dual to vacuum solutions ('Bañados geometries')

$$S = \frac{c}{6}\,\ln\frac{\ell^+(x_1^+,\,x_2^+)\ell^-(x_1^-,\,x_2^-)}{\varepsilon^2}$$

with $\ell = \psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)$ and $\psi'' = \mathcal{L}\psi$ (Hill's eq.)

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- for BMS_3 invariant QFTs: analogous statement (set cut-off $\varepsilon_x = 1$)

$$S = \frac{c_L}{6}\,\ln\ell_x + \frac{c_M}{6}\left(\frac{\ell_u}{\ell_x} - \varepsilon_u\right)$$

where ℓ_u and ℓ_x are bilinears in analogue of solutions to Hill's eq.

$$\mu'' = \mathcal{M}\,\mu \qquad \nu'' = \mathcal{M}\,\nu + \mathcal{N}\,\mu \qquad \text{with } \partial_u\mathcal{N} = \partial_x\mathcal{M}$$

Note: when $\mathcal{M}, \mathcal{N} = const.$ this includes all flat space cosmologies

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- example: global flat space, $\mathcal{M} = -\frac{1}{4}$, $\mathcal{N} = 0$

$$S = \frac{c_L}{6}\,\ln\big(2\sin(\Delta x/2)\big) + \frac{c_M}{6}\left(\frac{\Delta u}{2}\,\cot(\Delta x/2) - \varepsilon_u\right)$$

recovers indeed result for EE obtained in Bagchi, Basu, DG, Riegler '14

Flat space holographic EE

Different options for flat space holographic EE:

- use CS formulation and attach Wilson lines at endpoints of interval
  Basu, Riegler '15
    - pro: Wilson line natural in CS formulation; generalizable to higher spins
    - con: geometric meaning unclear; need suitable bc's at endpoints of Wilson line; cut-off dependence not obvious

Flat space holographic EE

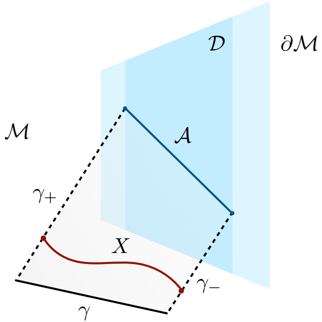
Different options for flat space holographic EE:

- use CS formulation and attach Wilson lines at endpoints of interval Basu, Riegler '15
- Rindler method Jiang, Song, Wen '17
  - pro: BMS analogue of Casini, Huerta, Myers '11; geometric interpretation in terms of geodesics (see next point)
  - con: not clear how to generalize to states dual to non-vacuum solutions

Flat space holographic EE

Different options for flat space holographic EE:

- use CS formulation and attach Wilson lines at endpoints of interval Basu, Riegler '15
- Rindler method Jiang, Song, Wen '17
- swing geodesics Hijano, Rabideau '17; Apolo, Jiang, Song, Zhong '20
  - pro: geometric interpretation in terms of geodesics; also applicable to other situations (e.g. warped CFT)
  - con: not clear how it works for domain walls; extremality?



swing construction decoded:

- $\mathcal{A}$: entangling region in FT
- $\gamma_\pm$: null geodesics ('ropes')
- X: generic spacelike geodesic
- γ: extremal spacelike geodesic ('bench')

picture from 2007.10740

Details of holographic EE for simplest case using swings

We ignore cut-off issues on this slide

Consider the locally flat metric

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + r^2\,\mathrm{d}x^2$$

- boundary interval: fixed $\Delta u > 0$ and $\Delta x > 0$

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$$\Delta s^2 = \Delta x^2\, r_0 r_1 - 2(r_1 - r_0)\,\Delta u$$

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- geodesic not extremal, but rather saddlepoint
- get same result for HEE if we change r_1 to arbitrary values!

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- entanglement inequalities?

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  SSA saturates for Poincaré vacuum for fixed slope $\Delta u_i/\Delta x_i = \gamma$

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> Consider non-vacuum states on gravity side to study holographic c-function and address (though not resolve) some of the issues above

Outline

Flat space domain walls

$$\mathrm{d}s^2 = -e^{A(r)}\,2\,\mathrm{d}u\,\mathrm{d}r + e^{2A(r)}\;\mathrm{d}x^2$$

as solutions of Einstein–Klein–Gordon bulk theory

$$I_{\text{bulk}}[g_{\mu\nu},\,\phi] = \frac{1}{16\pi G}\int \mathrm{d}^3x\sqrt{-g}\,\Big(R - \frac{1}{2}\,(\partial\phi)^2 - V(\phi)\Big)$$

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- $r = r_0 = const.$ slices have degenerate metric $0\ \mathrm{d}u^2 + 0\ \mathrm{d}u\,\mathrm{d}x + e^{2A(r_0)}\,\mathrm{d}x^2$ whose conformal KVs form BMS_3

$$\text{KVs:} \qquad \partial_u \qquad\qquad \partial_x \qquad\qquad x\,\partial_u + \underbrace{f(r)}_{f'(r)=e^{-A(r)}}\ \partial_x$$

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- can translate all AdS_3 domain walls into flat space domain walls!

Domain wall c-function

Is holographic domain wall function

$$c_{\mathrm{dw}}(r) = \frac{c_M^{\mathrm{UV}}}{A'(r)}$$

c-function for Carrollian QFTs with BMS_3 invariant UV & IR fixed points?

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- all desired properties of flat space holographic c-function

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Take flat space domain wall related to AdS domain wall with super-potential $W(\phi) = -2 - \frac{1}{4}\,\phi^2 - \frac{\alpha}{8}\,\phi^4$ (mass $m^2 = -\frac{3}{4}$ above BF)

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- can read off BMS$_3$ central charges at fixed points

$$c_M^{\text{IR}} = \frac{c_M^{\text{UV}}}{1 - \frac{1}{16\alpha}} < c_M^{\text{UV}}$$

more info in plot of flat space domain wall c-function

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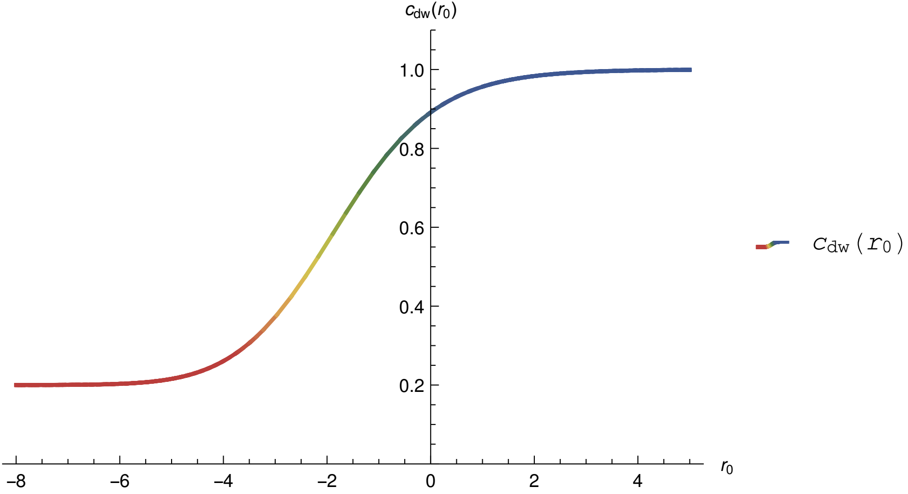

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$$\frac{1}{6\Delta x}\, c_M(\Delta u,\, \Delta x) = \frac{\partial S}{\partial \Delta u}$$

Casini–Huerta-like c-function?

Try to exploit $\text{AdS}_3/\text{CFT}_2$ logic in reverse

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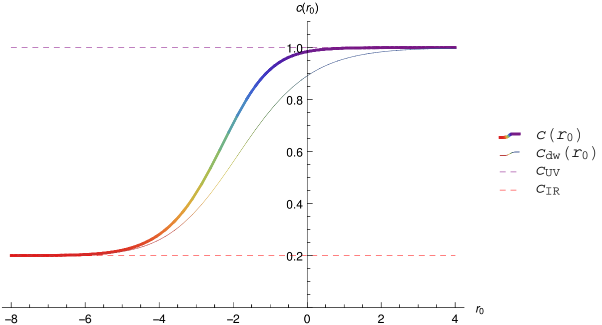

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- insertion into our proposal yields

$$c_M(r_0) = \frac{c_M^{\text{UV}}}{A'(r_0) + \frac{A''(r_0)}{A'(r_0)}}$$

But wait...

...is this really always a c-function for any flat space domain wall flowing between two fixed points?

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- no! only if the scalar field mass in the associated AdS_3 domain wall is between $[-\frac{3}{4}, 0]$

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Thanks for your attention!