Subtle and surprising aspects of Angular Momentum
At null infinity

(Emphasizing compact binary coalescences)

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ORGANIZATION

1. Introduction & setting the stage
   (BMS; Newman & Penrose; AA; AA & Streubel, Winicour; ....
   Reviews: AA, Centenary volume, Bierri & Yau (eds) 20
   AA, Campiglia & Laddha (GRG 2018))

2. Compact Binary Coalescences
   - Contrast with source-free solutions
   - The black hole kick
   - Supertranslation ambiguity in Angular Momentum
   - Notion of past & future stationarity as $u \to \pm\infty$
   - Surprising consequences
   - Theory & observations

3. Summary & Discussion
   - Recent proposal to remove supertranslation ambiguity
   - Surprise in Maxwell & YM theories
   - $\Lambda > 0$

1. Introduction: Setting the stage.

- Space time $(\hat{M}, \hat{g}_{ab})$ that is asymptotically flat at null ∞.

  (M, g_{ab}) : conformal completion.

  $\mathscr{I}^+$: Future boundary : $S^2 \times \mathbb{R}$, null, generators $\propto n^a$ equipped with $(q_{ab}, n^b) \approx (\omega^2 q_{ab}, \omega^{-1} n^b)$ such that q_{ab} : $0++$, $\mathcal{L}_n q_{ab} = 0$, $q_{ab} n^b = 0$, $\mathcal{L}_n \omega = 0$.

  This is the universal structure of $\mathscr{I}$

- Subgroup of Diff$(\mathscr{I}^+)$ that preserves this universal structure : The BMS group $\mathcal{B}$. This provides an intrinsic characterization of $\mathcal{B}$, w/o ref to space-time interior.

$$\mathcal{B} = \mathcal{S} \ltimes \mathcal{L} \qquad : \text{semi-direct product}$$

$\mathcal{S}$: ∞-dim, Abelian, normal subgroup of $\mathcal{B}$

$\mathcal{L} = \mathcal{B}/\mathcal{S}$: the 6-dim Lorentz subgroup.

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- Structure of $\mathcal{B}$ similar to that of $\boxdot$, with $\mathcal{S}$ replacing the 4-d group of translations of $\boxdot$

  $\mathcal{B}$ does admit a unique Abelian 4-d normal subgroup of translations $\mathcal{T} \subset \mathcal{S}$. $\Rightarrow$ well-defined 4-momentum But while $\boxdot$ admits a 4-parameter family of Lorentz subgroups $\mathcal{L}$, $\mathcal{B}$ admits an infinite parameter family, related by supertranslations

$\Rightarrow$ supertranslation ambiguity in the notion of angular momentum.

- Supertranslation generators $\xi^a = f n^a$, with $\mathcal{L}_n f = 0$.

  f : conformal weight $+1$: $(q_{ab}, n^a) \to (\omega^2 q_{ab}, \omega^{-1} n^a) \Rightarrow f \to \omega f$

  f Not just a function!

  can select a canonical 4-d subspace of translations $\mathcal{T}$. But, no natural notion of $\perp$ space of "pure supertranslations." The $Y_{\ell m}$ decomposition varies as we change (q_{ab}, n^a) (Chen, Wang², Yau)

1.B Radiative Modes in full GR.

$*$ Invariant characterization:

(M, g_{ab}); completion; $\nabla_a g_{bc} = 0$; ∇ induces D on $\mathscr{I}^+$

s.t. $\quad D_a q_{bc} = 0 \quad \& \quad D_a n^b = 0$

conformal freedom in g_{ab} $\longrightarrow$ Equivalence classes $\{D\}$

$$(\{D'_a\} - \{D_a\})\, K_b = \sigma_{ab}\,(n^c K_c); \qquad \underbrace{\sigma_{ab} n^b = 0 \quad q^{ab}\sigma_{ab} = 0}_{TT:\ 2\ \text{DOFs per point of } \mathscr{I}^+}$$

$*$ "Non-trivial part" of curvature of $\{D\}$ is neatly captured in the asymptotic Weyl curvature: ${}^*K^{ac} = \lim \Omega^{-1}\, {}^*K^{abcd} n_b n_d \Longleftrightarrow \left\{ \begin{matrix} \Psi_4^0, \Psi_3^0 \\ \mathrm{Im}\, \Psi_2^0 \end{matrix} \right.$.

$\{D\}$ knows about "radiative aspects" of grav. field. In particular ${}^*K^{ab} = 0 \implies N_{ab} = 0$ (Bondi news vanishes). But $\{D\}$ has no knowledge of the "Coulombic aspects" $\underbrace{\mathrm{Re}\,\Psi_2^0}_{\text{"Mass"}}$, $\underbrace{\Psi_1^0}_{\text{"Angular Momentum"}}$

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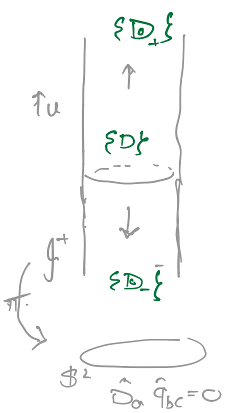
"Classical vacua $\{\mathbb{D}\}$"



$*$ $\{\mathbb{D}\}$ classical vacuum if it has trivial curvature i.e. $\mathbb{D}$ is fully determined by $\hat{D}$ on the base space $\mathbb{S}^2$

$\mathbb{D}$: No 'dynamical' information : $\Psi_4^0, \Psi_3^0, \mathrm{Im}\,\Psi_2^0 = 0$ on $\mathscr{I}$
$\Rightarrow N_{ab} = 0$

$*$ Each $\{\mathbb{D}\}$ is preserved precisely by a $\boxdot$ subgroup of the BMS group. $\mathcal{N}/\mathcal{T}$ acts simply and transitively on $\mathcal{V} = \{\{\mathbb{D}\}\}$ space of vacua.

$\Rightarrow$ $\mathcal{B}$ reduces to $\boxdot$ in absence of grav. radiation.

$*$ Each $\{\mathbb{D}\} \longrightarrow \{\mathbb{D}\}_\pm$ as $u \to \pm\infty$.

So, in any space-time, we can select two Poincaré subgroups $\boxdot_\pm$ of $\mathcal{B}$, but generically $\boxdot_- \neq \boxdot_+$. $\{\mathbb{D}\}_+ - \{\mathbb{D}\}_- = [\sigma_{ab}]_{u=-\infty}^{u=\infty}$ is an observable of GR: Total gravitional memory.

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1.C Radiative Phase space, BMS Fluxes & Charges.

* $\Gamma_{Rad} \ni \{ \{D\}$ on $\mathscr{I}^+$, $\rightarrow \{D_\pm\}$ at $(i^\pm, i^0)$ $\}$: Affine space.

Covariant phase space of GR $\longrightarrow$ symplectic structure on Γ_{Rad}

$$\Omega\big|_{\{D\}}(\delta_1, \delta_2) = \frac{1}{R} \int_{\mathscr{I}^+} (\sigma^{(1)}_{ab}\, \mathcal{L}_n\, \sigma^{(2)ab} - 1 \leftrightarrow 2)\, d^3\mathscr{I} \qquad \delta\{D\} \equiv \sigma_{ab}$$

* BMS action on $\mathscr{I}^+$: Preserves Ω $\longrightarrow$ Hamiltonians : Fluxes

$$\xi^a \in \mathfrak{b} \longrightarrow F_\xi[\Delta\mathscr{I}] \qquad \text{Linear in } \xi^a.$$

$$F_\xi[\Delta\mathscr{I}] = -\frac{1}{2R} \int_{\Delta\mathscr{I}} N^{ab} \left[(\mathcal{L}_\xi D_a - D_a \mathcal{L}_\xi)\, \ell_b + 2\, \ell_{(a} D_{b)} k \right] d^3\mathscr{I}.$$

N_{ab} : News ; $n^a \ell_a = -1$; $\mathcal{L}_\xi q_{ab} = 2k\, q_{ab}$ ($=0$ for supertranslations & Rotations)

$F_\xi[\Delta\mathscr{I}]$ Has all desired invariances

* Fluxes can be 'integrated' to obtain 2-sphere charges:

$$Q_\xi[C_1] - Q_\xi[C_2] = F_\xi[\Delta\mathscr{I}] \qquad (\partial(\Delta\mathscr{I}) = C_1 \cup C_2)$$

Ψ_2^0, Ψ_1^0 appear in $Q_\xi[C]$ through Bianchi identities

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   - Supertranslation ambiguity in Angular Momentum
   - Notion of past & future stationarity as $u \to \pm\infty$
   - Surprising consequences • Theory & observations

   (Mainly, AA, De Lorenzo, Khera (GRG 2020, PRD 2020) + Krishnan (PRD 2021); Mitman, Iozzo, Khera et al PRD (2021) Khera, AA & Krishnan (in preparation). Also: Compere et al, Elhashash & Nichols)

3. Summary & Discussion
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2.A : Source-free Solutions Vs CBCs

* Non-linear stability of Minkowski space-time : All analyses to date use source free soln with $\hat{M} = \mathbb{R}^4$:
  - Incoming radiation at $\mathscr{I}^-$ essential.
    Only soln with no incoming radiation : Minkowski space

- Christodoulou-Klainnormann initial conditions { Ψ_2^0 : spherically symmetric at i^o and zero at i^+
  $\Rightarrow$ supermomentum flux across $\mathscr{I}^\pm = 0$
  "ordinary" or "Linear" memory vanishes.

- Chrusciel-Delay Initial conds { same, but Ψ_1^0 & hence Ang. Mom. well-defined

* Contrast with CBC of NS & BH to a Kerr BH
  - No incoming radiation (at $\mathscr{I}^-$: $\{D\} = \{D_-\}$)
    Generically : ① supermomentum flux across $\mathscr{I}^+ \neq 0$; and
  - ② $\{D_+\} \neq \{D_-\}$ $\Rightarrow$ $\boxed{\ }$ reduction at i^o & i^+ distinct.
    $\Rightarrow$ Lorentz groups defining initial $\vec{J}_{i^o}$ & $\vec{S}_{i^+}$ distinct
    conceptually meaningless to consider $(\vec{J}_{i^o} - \vec{S}_{i^+})$.

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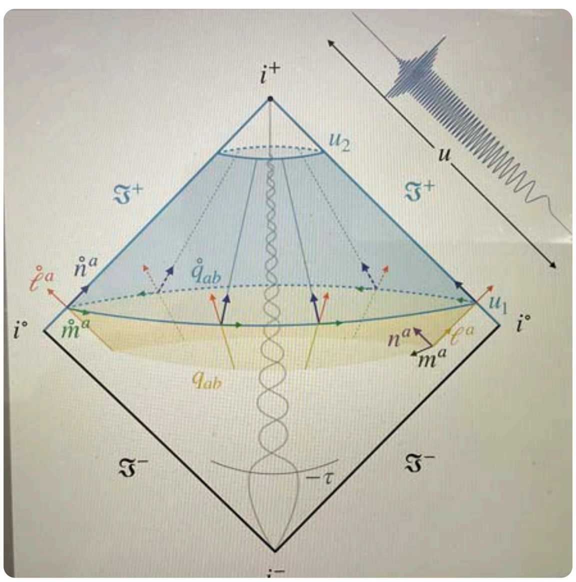
2B : Compact Binary coalescence : Setting



① Final BH kick determined by the flux of 3-momentum radiated across $\mathscr{I}$

$$F_{Y_{\ell m}}[\mathscr{I}] = \int_{\mathscr{I}^+} N^{ab} (\mathcal{L}_{(Y_{\ell m}) n} D_a - D_a \mathcal{L}_{(Y_{\ell m}) n}) \ell_b \, d^3\mathscr{I}$$

$$= \int_{\mathscr{I}^+} Y_{\ell m}(\theta, \varphi) \, N^{ab} N_{ab} \, d^3\mathscr{I}$$

$\neq 0$ Generically

② Future/Past rest frames yield

$\mathring{q}^+_{ab} = \omega^2 \mathring{q}^-_{ab}$ $\quad$ $\mathring{n}^a_+ = \omega^{-1} \mathring{n}^a_-$

unit 2-sphere metrics

$$\omega = \frac{1}{\gamma(1 - \vec{v}\cdot\hat{x})} \qquad \gamma = \frac{1}{\sqrt{1-v^2}}$$

③ $\vec{J}_-$ refers to SO(3) subgroups of $\boxed{\ }_-$ determined by past restframe

$\vec{S}_+$ refers to SO(3) subgroups of $\boxed{\ }_+$ determined by future restframe

These $SO(3)_\pm \subset B$ are related by a supertranslation & a boost.

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2.C Angular Momentum: supertranslation ambiguity

* Special Relativity : choice of origin $\vec{O} \leftrightarrow$ Lorentz subgroup $\mathcal{L}$

Displacement $\vec{O} \to \vec{O} + \vec{d}$ $\Rightarrow$ $M_{ab} \to M_{ab} + P_{[a} d_{b]}$

Boost momentum can be removed. $J^a = \epsilon^{abcd} M_{cd} (P_b/M) = \vec{J}$

$\vec{J}$: Refers to SO(3) subgroups in the rest frame where $\vec{P}=0$

*GR: i^o : $\vec{J}_-$: Past rest frame , $\vec{P}_- = 0$ selects $(\mathring{q}_{ab}, \mathring{n}^a)_-$

But SO(3) subgroups now have a supertranslation freedom:

$u = $ const cuts, $R^a_{(k)}$ tangential select one SO(3)

Freedom : $u \to u + f(\theta, \varphi)$

Can eliminate by demanding : shear $\sigma_{ab} = \mathrm{TF}\, D_a D_b u \to 0$ as $u \to -\infty$

Selects 4-parameter family of SO(3) as in special relativity

Precisely the family in the $\mathfrak{B}_- \subset \mathfrak{B}$ selected by $\{\mathcal{D}\}_-$.

$$\vec{J}^-_{(i)} = -\frac{2}{R} \lim_{u_0 \to -\infty} \oint_{u_0} \mathrm{Im}(\Psi_1^o \bar{\eth} \beta_{(i)})\, d^2\mathring{V} \quad ; \quad R^a_{(i)} = \epsilon^{ab} \partial_b \beta_{(i)}.$$

$$\vec{P}^-_{(i)} = 0 \quad \& \quad E^- = -\frac{2}{R} \lim_{u_0 \to -\infty} \oint_{u_0} (\mathrm{Re}\, \Psi_2^o)\, d^2\mathring{V}_o.$$

Thus, one can eliminate supertranslation ambiguity as $u \to -\infty$

obtain P_a^- and J^a_- : Total 4-mom & Ang. mom.

* Distant Future $u \to \infty$ (i^+) :

Generically, BH kick $\Rightarrow$ Future rest frame different

$$(\mathring{q}'_{ab}, \mathring{n}'^a) = (\omega^2 \mathring{q}_{ab}, \omega^{-1} \mathring{n}^a) \quad ; \quad \omega = \frac{1}{\gamma(1 - \vec{v} \cdot \hat{x})} \quad ; \quad \gamma = \frac{1}{\sqrt{1-v^2}}$$

Again introduce

u' : $n'^a D_a u' = 1$ st $\sigma'_{ab} \to 0$ as $u' \to +\infty$, ie $\{\mathcal{D}\} \to \{\mathcal{D}\}_+$

Yields 4-parameter family of SO(3) subgroups $\subset \mathfrak{B}_+$

$$\vec{J}^+_{(i)} = \vec{S}_{(i)} = -\frac{2}{R} \lim_{u_0' \to +\infty} \oint_{u_0'} \mathrm{Im}\, \Psi_1^o\, \bar{\eth}' \beta'_{(i)}\, d^2\mathring{V} \qquad R'^a_{(i)} = \epsilon'^{ab} \partial_b \beta'_{(i)}$$

(in the future rest frame: $\vec{P}^+_{(i)} = 0$; $E^+ = -\frac{2}{R} \lim_{u_0' \to \infty} \oint_{u_0'} (\mathrm{Re}\, \Psi_2^{o\prime})\, d^2\mathring{V}'$)

But SO(3) subgroup $\subset \mathfrak{B}_+$ $\mathfrak{B}_-$ These SO(3) subgroup defining $\vec{S}_{(i)}$ relat to ne SO(3) subgroups defining $\vec{J}^-_{(i)}$ by a boost (as in special relativity) and a supertranslation!

This is the first subtlety.

$\vec{J}^-_{(i)} - \vec{S}_{(i)} \neq$ Flux of some SO(3) angular momentum

It also contains supermomentum flux.

More explicitly : Let us work with the past rest-frame $(\mathring{q}_{ab}, \mathring{n}^a)$.
suppose the kick is in x-direction : $\vec{v}\cdot\hat{x} = v \sin\theta\cos\phi$
& the asymptotically shear-free cuts are related by $u' = u + S(\theta,\varphi)$

Then :
$$R'^{a}_{(1)} = R^{a}_{(1)} + s_{(1)}\,\mathring{n}^a$$
$$R'^{a}_{(2)} = \gamma\left(R^{a}_{(2)} + v\,K^{a}_{(3)} + g_2\,\mathring{n}^a\right)$$
(similarly for $R'^{a}_{(3)}$)

$s_{(i)} = -\mathcal{L}_{R_{(i)}} S(\theta,\varphi)$

$\overset{*}{s}_{(i)} = -\mathcal{L}_{K_{(i)}} S(\theta,\varphi)$

$\mathcal{L}_{K_{(i)}} \mathring{q}_{ab} = 2 R_{(i)} \mathring{q}_{ab}$

Blue terms from special relativity, black terms new.

$g_{(2)} = s_{(2)} + v R_{(3)} S + v \overset{*}{S}_{(3)}$

consequently: supermomenta feature in the expression of flux of angular momentum carried by gravitional waves :

$$\vec{S}_{(1)} - \vec{J}^{(-)}_{(1)} = F_{R_{(1)}} + F_{s_{(1)}} \; ; \qquad \gamma^{-1}\vec{S}^{(2)} - \vec{J}^{(2)}_{-} = F_{R_{(2)}} + v\,F_{K_{(3)}} + F_{g_{(2)}}.$$
(similarly for the 3rd component)

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2.D : Asymptotic stationarity as $u \to \pm\infty$

In the CBC, final state is kerr, expectation: Asymptotically as $u \to +\infty$ along $\mathscr{I}^+$, fields approach those in kerr spacetime. Borne out in numerics.

As $u \to -\infty$, it is assumed that the progenitors are so far from one another that there is stationarity. (In fact much stronger assumptions made in the wave form community.)

Let us make much weaker (but still non-trivial) assumptions:

1. Bondi news $N = N_{ab}\,\bar{m}^a \bar{m}^b = -\dot{\bar{\sigma}}^{\circ} \to 0$ as $\frac{1}{|u|^{1+\epsilon}}$ as $u \to \pm\infty$ (std assumption)
$\Rightarrow \dot{\Psi}^{\circ}_4, \dot{\Psi}^{\circ}_3, \dot{\Psi}^{\circ}_2 \sim$ News & its derivatives $\Rightarrow \Psi^{\circ}_4, \Psi^{\circ}_3, \Psi^{\circ}_2$ become asymptotically stationary (in every conformal frame (q_{ab}, n^a)).

2. $\Psi^{\circ}_1 \to 0$ in the past rest frame $(\mathring{q}_{ab}, \mathring{n}^a)$ as $u \to -\infty$ and in the future rest frame $(\mathring{q}'_{ab}, \mathring{n}'^a)$ as $u \to +\infty$. If this were to fail, Ψ°_1 would diverge in the limit.

Even this weak notion of asymptotic stationarity has interesting & unforeseen consequences.

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Consequences

- $\dot{\Psi}_1^\circ \to 0$ as $u \to -\infty$ in the past rest frame $(\mathring{q}_{ab}, \mathring{n}^b)$
  $\Rightarrow$ In this conformal frame, $\lim_{u\to-\infty} \Psi_2^\circ = -GM_{i^0}$ (spherically symm)

- $\dot{\Psi}_1^\circ \to 0$ as $u \to +\infty$ in the future rest frame $(\mathring{q}'_{ab} = \omega^2 \mathring{q}_{ab}, \ \mathring{n}'^b = \omega^{-1}\mathring{n}^b)$
  $\Rightarrow$ In this conformal frame $\lim_{u\to+\infty} \Psi_2^{\circ\prime} = -GM_{i^+}$ (spherically symm)

$$\underbrace{\Psi_2^\circ\Big|_{i^+}}_{\text{Future mass aspect in the past rest frame}} = \omega^3 \ \Psi_2^{\circ\prime} = \frac{-GM_{i^+}}{\gamma^3 (1 - v\sin\theta\cos\varphi)^3} \quad (\text{not spherical})$$

$$\approx -GM_{i^+}\left[1 + (3\sin\theta\cos\varphi)\underset{\text{kick/Recoil velocity}}{v} - \tfrac{3}{2} - 6\sin^2\theta\cos^2\varphi\, v^2 + \cdots\right]$$

Illustrates why the requirement $\dot{\Psi}_1^\circ \to 0$ cannot hold without specification of a conformal (or rest) frame.

- In the past frame we are working with, $\oint \Psi_2^\circ f\, d^2\mathring{V} = 0$ unless f is constant $\Rightarrow$ All supermomentum charges vanish @ i^0 except energy $E = -\frac{1}{4\pi G}\oint \Psi_2^\circ d^2\mathring{V} = M_{i^0}$.

2.E Subtlety and Surprise

- Recall: Difference between initial & final angular momenta:

$$\vec{S}_{(1)} - \vec{J}_{(1)}^{(-)} = F_{R_{(1)}} + F_{S_{(1)}} \ ; \qquad \gamma^{-1}\vec{S}^{(2)} - \vec{J}_-^{(2)} = F_{R_{(2)}} + v\, F_{K_{(3)}} + F_{g_{(2)}}.$$

(similarly for the 3rd component)

Fluxes of supermomenta that feature are non-zero if & only if the corresponding final supermomentum charges @ i^+ are non-zero
(because $\Psi_2^\circ\big|_{i^0} = -GM_{i^0}$ and $\oint S_{(i)}\, d^2\mathring{V} = 0 = \oint g_{(i)}\, d^2\mathring{V}$.)

When are these final supermomenta non-zero?

- special case ①: Grav. memory vanishes, i.e. $\begin{cases} \eth^2 D\xi_+ = \eth^2 D\xi_- \\ \text{or, } \boxed{\ }_+ = \boxed{\cdot}_- \\ \text{as in special relativity.} \end{cases}$
  Then $s(\theta,\varphi) = 0 \Rightarrow S_{(i)} = 0 = g_{(i)}$.
  $\Rightarrow$ supermomentum terms vanish; just as one expects.

- special case ②: Grav memory $\neq 0 \Rightarrow s(\theta,\varphi)$ nontrivial ST.
  But kick/recoil velocity v vanishes. $\Rightarrow (\mathring{q}_{ab}, \mathring{n}^a) = (\mathring{q}'_{ab}, \mathring{n}'^a)$
  Fixed conf frame but $[SO(3)]_\pm$ related by a supertranslation.

  However, $v = 0 \Rightarrow \Psi_2^{\circ\prime} = -GM_{i^+}$ in the past conf. frame
  $\Rightarrow$ supermomentum terms vanish! 2nd surprise/subtlety.
  ok to compare apples with oranges in practice!

- Thus, in CBCs, because of "asympotic stationarity as $u \to \pm\infty$" the supertranslation ambiguity in angular momentum shows up only when: Grav Memory $\neq 0$ AND kick velocity $\neq 0$.

  But this is the generic case.
  Nonetheless, the supermomentum flux is small for small v:

$$\underbrace{\mathcal{F}_{(f)} = P_{(f)}}_{S_{(i)} \text{ or } g_{(i)}} \Big|_{i^+} = + \frac{1}{4\pi G} \oint_{u=-\infty} d^2\mathring{V} \; f(\theta,\varphi) \; G M_{i^+} \underbrace{\left[\cancel{1} + (\beta \sin\theta \cos\varphi) v + \dots \right]}_{\Psi_2^{\circ} |_{u=\infty}}$$

- In NR simulations of CBCs $v \sim 10^{2}$-10^{3} kms/s. Therefore the supertranslation ambiguity in the notion of angular momentum is not likely to be relevant for current LIGO detectors. But interesting for 3-g detectors that are being planned. In this respect, similar to gravitational memory which has not been definitively measured.

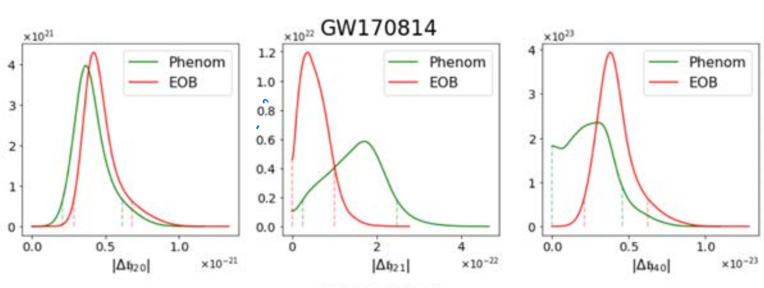
- But memory and supermomentum and angular momentum balance laws I discussed are already being used as diagnostic tools to test the "goodness" of model waveforms currently used (Phenom, EOB, surrogate) & as pointers for further improvements.

2.F Theory & observations: illustrative Examples.

supermomentum balance law + CBC with a kick velocity $\vec{v}$:

$$\underbrace{\left[\eth^2 \sigma^{\circ} \right]_{u=-\infty}^{u=\infty}}_{\sim \text{ Memory}} = \frac{G M_{i^+}}{\gamma^3 (1 - \vec{v} \cdot \hat{x})^3} - G M_{i^{\circ}} + \int_{-\infty}^{\infty} du \, |\dot{\sigma}^{\circ}|^2 \qquad (2\sigma^{\circ} = \mathring{h}_+ + i \mathring{h}_\times)$$

LIGO/Virgo provide posterior probability distributions for symbols in blue $\longrightarrow$ PPDs for memory.

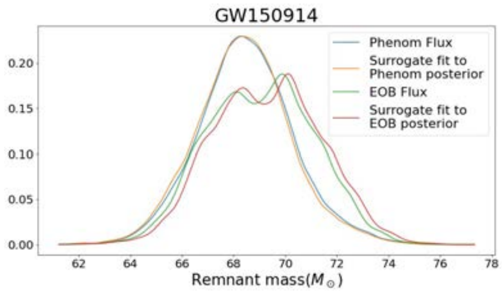


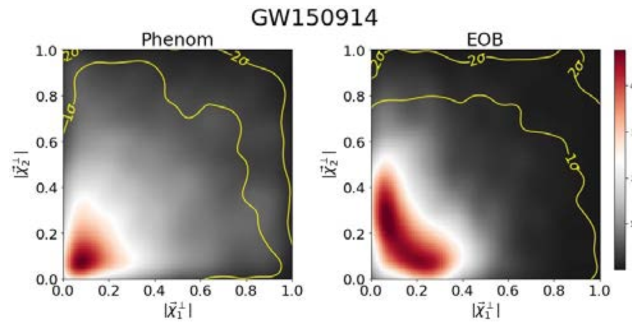
- AA, De Lorenzo, Khera GRG (2020)
- Khera, Krishnan, AA De Lorenzo (2021)

viewpoint we introduced:

Gravitational memory is an inferred observable (like M_{i^+}, $\vec{S}_{i^+}$) can be used to improve waveform models. Example: GW170814 SEOBNRv3 & IMRPhenomPv2 comparison $\longrightarrow$ Need for improvement of higher modes.

Second Example : Same two models examined with another inferred observable M_{rem} calculated with the balance law.
First LIGO Event : Analysis from 2015-16.





We found a bi-modal Hump in SEOBNRv3 and traced to the way precession (spin components $\perp$ to $\vec{L}$) was included in the model. Again: A diagnostic tool for improving the model. Angular momentum considerations.

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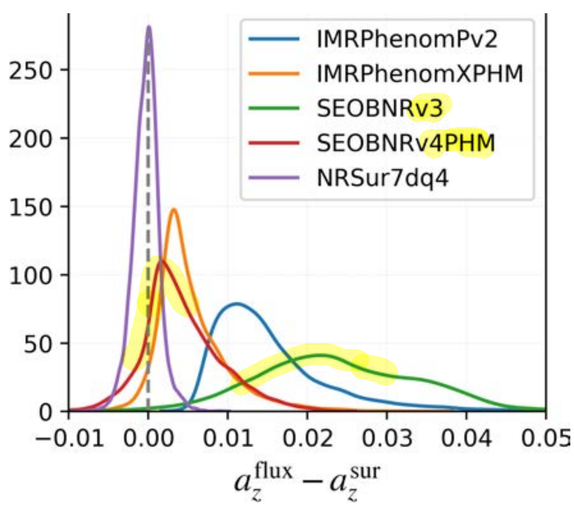
3rd Example : Use of Angular momentum balance laws



Recent improvements in waveform models :

IMRPhenomPv2 ⟶ IMRPhenomXPHM
SEOBNRv3 ⟶ SEOBNRv4P

Diagnostic tool : Balance law for the z-component of angular momentum.

Clearly brings out the extent of improvement.

Even the NR result is instructive : Brings out synergy !

Khera, AA, Krishnan (in preparation)

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(Chen, Wang², Yau (2021 arXiv); AA & Bonga (CQG 2021)
AA, Bonga & Kesavan (PRL 2016); AA (RoPP 2018)).

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3.A Summary

- For every BMS vector field $\xi^a \in \mathfrak{b}$, we have F_ξ, $Q^{(i^0)}_\xi$, $Q^{(i^+)}_\xi$ with the balance law: $Q^{(i^+)}_{(\xi)} - Q^{(i^0)}_{(\xi)} = F_\xi$ (and its local versions).
  $\xi \to Q_{(\xi)}$ and $\xi \to F_{(\xi)}$: linear in ξ^a.

- Supertranslation ambiguity: we do have preferred Poincaré subgroups $\boxdot_+$ & $\boxdot_-$ in any space-time. But (gravitational memory $\neq 0$) $\Longleftrightarrow$ $\boxdot_+ \neq \boxdot_-$; two are related by a supertranslation.
  If there is a non-zero kick, the asymptotic rest-frames also distinct $(\mathring{q}_{ab}, \hat{n}^a)_+ \neq (\mathring{q}_{ab}, \hat{n}^a)_+ = (\mathring{q}'_{ab}, \hat{n}'^a)$

  $\Rightarrow$ Rotations $R^a_{(i)}$ (defining $\vec{J}_-$) and $R'^a_{(i)}$ (defining $\vec{S}_+$) are related by a boost and a supertranslation.

- Hence $\vec{S}_{i^+} - \vec{J}_{i^0}$ = Flux of rotational angular momentum + Flux of boost angular momentum + Flux of supermom.

  If $\boxdot_- = \boxdot_+$ (memory $= 0$) : No supermomentum term.
  Surprise : If $\boxdot_- \neq \boxdot_+$, but $V_{kick} = 0$: supermomentum term turns out to vanish because of asymptotic stationarity.
  (But this assumption is not compelling; should keep eyes open for circumstances where it is violated).

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- Because the effect vanishes for $v_{kick}=0$, and expected BH kicks are 'small', supertranslation term too small for the sensitivity of the present LIGO/Virgo detectors. But likely to be non-negligible for 3G detectors. But the detailed balance law is already useful as a diagnostic check on waveform models as they are consequences of exact GR.

3B : Discussion.

(i) Recent proposal to eliminate the supertranslation ambiguity in general (i.e. without reference to asymptotic stationarity).

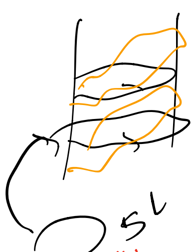
CW²Y propose a new formula for $\tilde{Q}_R$ for R: Rotation generators in $\mathcal{B}_i$ starting from quasi-local expressions. Work in a fixed rest frame ("No kick for CBC").

Result: $\tilde{Q}_R(i^+) - \tilde{Q}_R(i^0) =: \tilde{F}_R$ has the property

$$\tilde{F}_R = \tilde{F}_{R+(\mathcal{L}_R f)} \quad \text{if} \quad f_s = \underbrace{\sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} f_{\ell m} Y_{\ell m}(\theta,\varphi)}_{\text{"pure supertranslation"}}$$

Linearity $\Rightarrow$ $\tilde{F}_{(\mathcal{L}_R f_s)} = 0 \quad \forall f_s$

Tension: ① Standard supermomentum flux will not vanish for all supertranslations. Meaning of $\tilde{F}_{\mathcal{L}_R f_s}$?



② A pure supertranslation in one conformal frame (q_{ab}, n^a) has $\ell'=0,1$ components in another. So there will be a nontrivial constraint on the new energy momentum flux as well.

—o—o—

(ii) Surprise in Maxwell & YM theory: Angular momentum

Broadbrush-stroke summary (skipping important finer points)

- Phase space of radiative modes : A_a on $\mathscr{I}$, $A_a n^a = 0$ (can work in Mink space but not essential)

$\underbrace{A_a n^a = 0}$: $A_a m^a$: Radiative Modes

$$\Omega(\delta_1 A, \delta_2 A) = \int_{\mathscr{I}} [\delta_1 A \wedge \delta_2 {}^*F - 1 \leftrightarrow 2]$$

$\downarrow$ Determine Φ_2^0, $\operatorname{Im}\Phi_1^0$

[·] Action preserves Ω; Hamiltonians H_ξ

ξ : Translation : $H_\xi = \int_{\mathscr{I}} T_{ab}\, \xi^a\, \epsilon^b{}_{mnp} = \int_{\mathscr{I}} T_{ab}\, \xi^a n^b\, du\, d^2S$

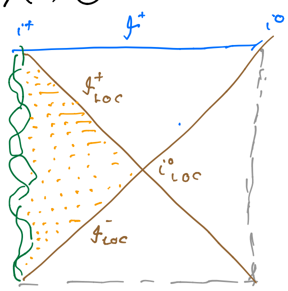
But for rotations $H_\xi = \int_{\mathscr{I}} T_{ab}\, \xi^a n^b\, du\, d^2S$ only if total charge $=0$

In general: The RHS has 'coulombic information' ($\operatorname{Re}\Phi_1^0$) that is not captured by radiative modes & hence by H_ξ

To recover it, one has to extend the radiative phase space appropriately.

surprise : A priori one would have thought that angular momentum carried by electromagnetic waves across $\mathscr{I}^+$ shouldn't care about total charge in space-time. But it does!

(iii) $\Lambda > 0$ case : Binary star system to illustrate the idea (can also consider CBC : details more complicated)



viewpoint :

① Focus on the part of space-time that the binary can influence

② Impose no incoming radiation cond^n on $H^-(i^0)$ (Require it to be a weakly isolated horizon)

③ Drag symmetries from $\mathscr{I}^-_{Loc}$ to $\mathscr{I}^+_{Loc}$ via i^0_{Loc}, Define fluxes of energy & angular momentum across $\mathscr{I}^+_{Loc}$

④ For linear gravity, Einstein's quadrupole formula has been extended in the PN, Post deSitter approximation