BMS, MODULAR INVARIANCE & MINKWOSKIAN HOLOGRAPHY

Arjun Bagchi, IIT Kanpur.

"Flat Asymptotia", Okinawa , Japan.

March 16, 2021.

WHY FLATSPACE HOLOGRAPHY?

- 20+ years and 20000+ citations of the Maldacena correspondence.
- Holography has changed the way we look at quantum gravity and indeed a lot of theoretical physics.
- Holography is primarily understood for AdS spacetimes.
- Our universe is clearly not AdS.
- For many practical applications (e.g. astrophysics), the universe is well approximated by flat spacetimes.
- Need to understand flat holography to build towards a hologram of the real world.
- Also need to understand holography beyond AdS in order to attempt understanding quantum gravity in general.

OUTLINE OF THE REST OF THE TALK

- Holography for Flat spacetimes.

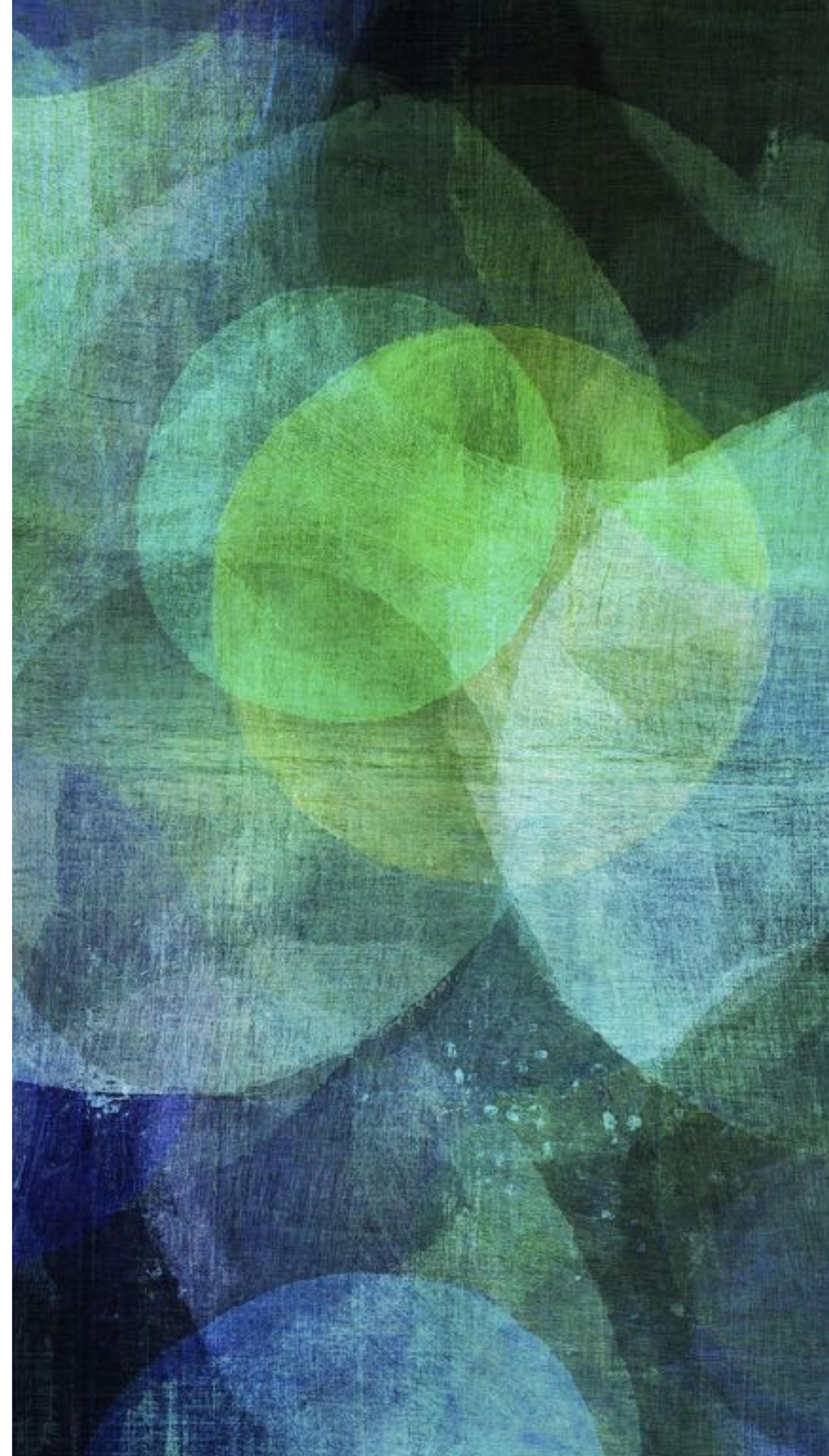
FLAT COSMOLOGY & ENTROPY

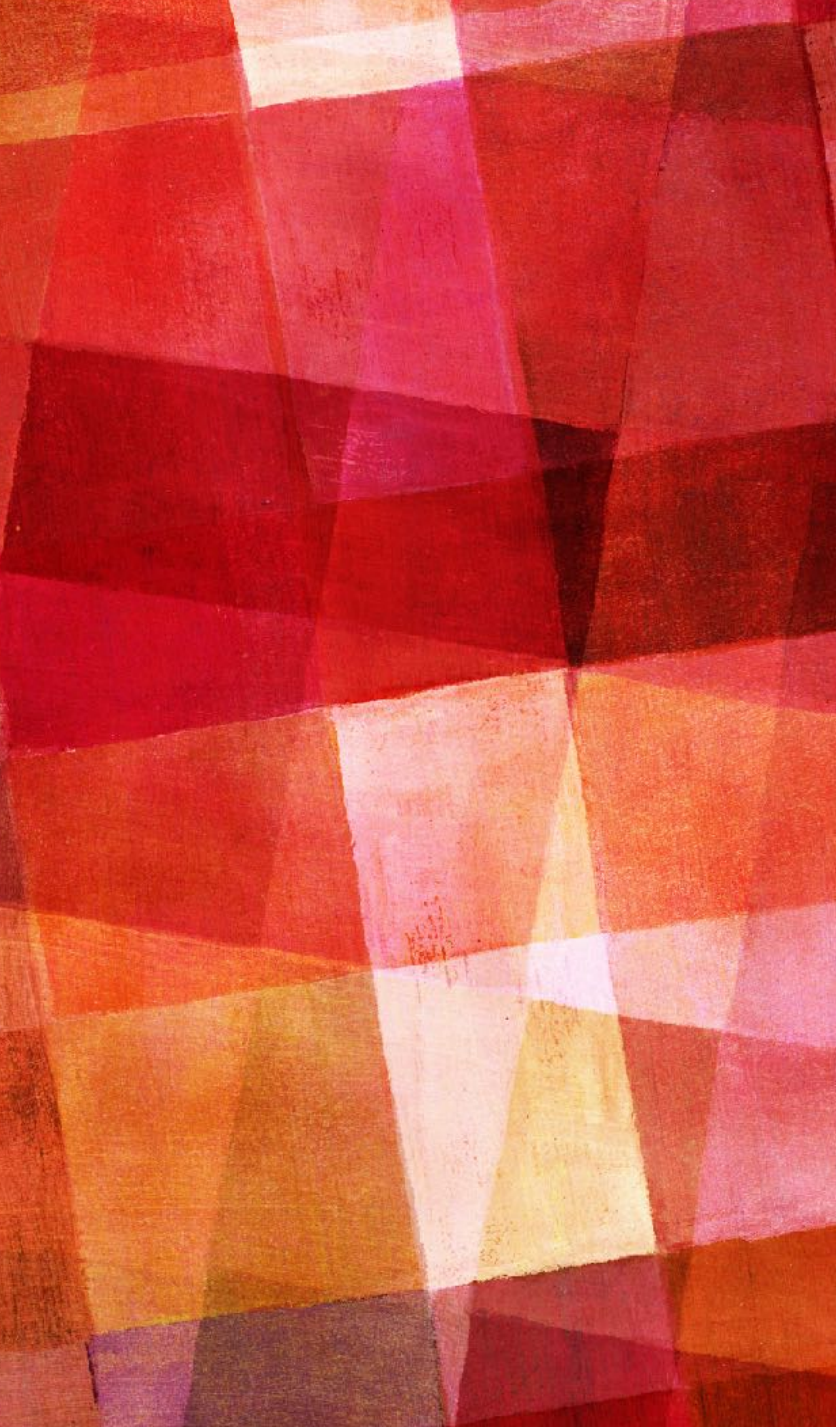
- BMS and modular invariance.
- BMS Cardy formula.
- Flat cosmologies and Cardy counting.

BMS STRUCTURE CONSTANTS

- Torus one-point functions.
- BMS structure constants from 1-pt fn.
- Bulk calculations

- Conclusions.





RECIPE FOR HOLOGRAPHY

- Start with a gravitational theory in a certain spacetime.
- Compute its asymptotic symmetries.
- Declare the asymptotic symmetry group to be the symmetry of the dual field theory, which lives on the boundary of the spacetime.
- Many examples:
  - Brown-Henneaux construction in AdS_3.
  - Higher spin holography in AdS3.
- Lessons for asymptotically flat spacetime?

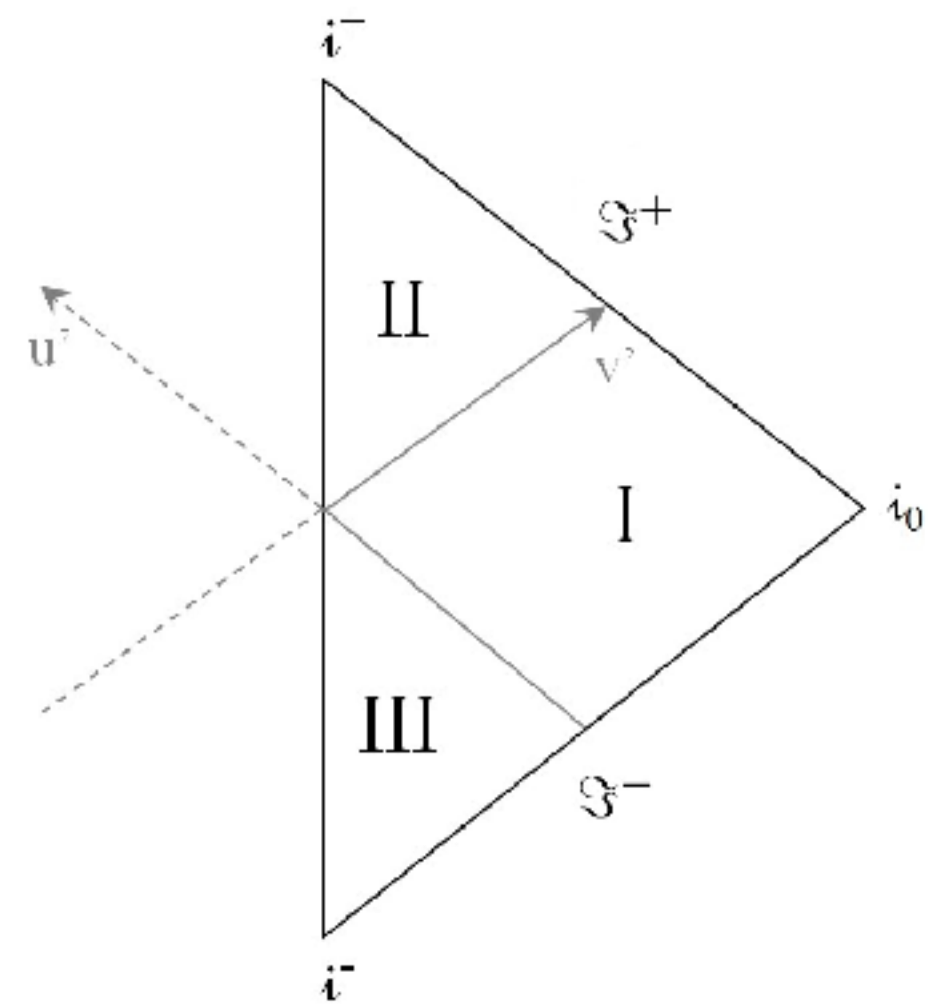
FLAT SPACE AND BMS SYMMETRIES

- Asymptotic symmetries of flat space at null infinity is given by the Bondi-Metzner-Sachs (BMS) group.
- In 3 and 4 dimensions, the BMS group is infinite dimensional.
- We shall concentrate on asymptotic symmetries of 3d flat spacetimes and hence the BMS_3 algebra.

$$[L_n, L_m] = (n-m)L_{m+n} + \frac{c_L}{12}\delta_{n+m,0}(n^3-n)$$

$$[L_n, M_m] = (n-m)M_{m+n} + \frac{c_M}{12}\delta_{n+m,0}(n^3-n)$$

$$[M_n, M_m] = 0.$$



Penrose Diagram of Minkowski spacetime

- M's: supertranslations. Angle dependent translations along the null direction.
- L's: superrotations. Diffeos of the circle at infinity.
- For Einstein gravity, $c_L = 0, \quad c_M = \frac{3}{G}$

Barnich and Compere 2006

FROM ADS TO FLATSPACE

- Can obtain flat space by taking the radius of AdS to infinity.
- How do we see this at the level of the symmetry algebra?
- Start with 2 copies of Virasoro algebra that form asymptotic symmetries of AdS3.

$$[\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{L}_{n+m} + \frac{c}{12}\delta_{n+m,0}(n^3 - n).$$
$$[\bar{\mathcal{L}}_n, \bar{\mathcal{L}}_m] = (n-m)\bar{\mathcal{L}}_{n+m} + \frac{\bar{c}}{12}\delta_{n+m,0}(n^3 - n).$$
$$[\mathcal{L}_n, \bar{\mathcal{L}}_m] = 0$$

- The central terms of the left and right copies: $c = \bar{c} = \dfrac{3\ell}{2G}$
- We take the following limit:

$$\boxed{L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)} \quad \text{where } \epsilon = \frac{1}{\ell} \to 0.$$

- Easy to see that this contracts 2 copies of Virasoro algebra to BMS3 algebra.
- The central terms $c_L = c - \bar{c} = 0$ and $c_M = \epsilon(c + \bar{c}) = \dfrac{3}{G}$
- This agrees with the intrinsic analysis.

ROAD TO MINKOWSKIAN HOLOGRAPHY

- The field theory dual to Minkowski spacetimes should inherit the asymptotic symmetries of flat spacetimes.
- For 3D Minkowski spacetimes, the dual theory should be a 2D field theory living on the null boundary of flatspace and it should have BMS3 as its underlying symmetry algebra.

A Bagchi 2010

- We would have two separate tools to study these field theories.
  - The intrinsic way: use only symmetries of BMS.
  - The limiting way: use the singular limit from 2d CFTs.
- We will be attempting to understand aspects of flatspace from a field theory which lives on $\mathcal{I}_+$. This will give only a partial understanding of Minkowskian holography.
- For the entire picture, one needs to patch two field theories living on the two null boundaries.

OTHER POSSIBLE ROADS TO FLAT HOLOGRAPHY

- *BFSS Matrix model* : Banks, Fishler, Shenker, Susskind '96.

- *Flat Holography and Celestial CFTs:*
  Strominger et al. and a lot of other people recently.
  Following up on BMS/CFT proposal of Barnich and Troessaert '10 and
  AdS/dS slicing proposal of De Boer, Soludukhin '03.

  Main difference: we take into account the null direction on the boundary.
  See Banerjee, Ghosh, Gonzo (2020) for a possible reconciliation between two approaches.

- *AdS/Ricci-flat correspondence*: Caldarelli, Camps, Gouteraux, Skenderis '12-'13.

We will not discuss these here.

FLAT HOLOGRAPHY: SOME CHECKS OF PROPOSAL

- Asymptotic density of states from the field theory and the bulk
  [AB, Detournay, Fareghbal, Simon 2012; Barnich 2012.]

- Multipoint correlation functions of the EM tensor in the boundary and bulk.
  (*) Novel phase transitions from zero-point functions. [AB-Detournay-Grumiller-Simon'13]
  (*) Matching of higher point correlations [AB, Grumiller, Merbis '15]

- Construction and matching of Entanglement Entropy
  [AB, Basu, Grumiller, Riegler '14; Jiang, Song, Wen '17; Hijano, Rabideau '17.].

  SEE DANIEL'S TALK AFTER THIS FOR MORE ON FLAT SPACE ENTANGLEMENT ENTROPY!

- Construction of Characters and matching with 1-loop partition function.
  [Oblak '15; Barnich, Oblak, Maloney '15; AB, Saha, Zodinmawia '19.].

- Construction of bulk-boundary dictionary, matching of correlation functions of primary operators
  [Hijano, Rabideau '17; Hijano '18.]

- Generalisations
  (*) Flat Space Chiral Gravity: CS Gravity dual to chiral half of CFT. (+SUSY generalisation)
  [AB, Detournay, Grumiller '12, (AB, Basu, Detournay, Parekh '18).]
  (*) Higher spin theories in flat space. [Afshar, AB, Fareghbal, Grumiller, Rosseel '13,
  Gonzalez, Matulich, Pino, Troncoso '13.]

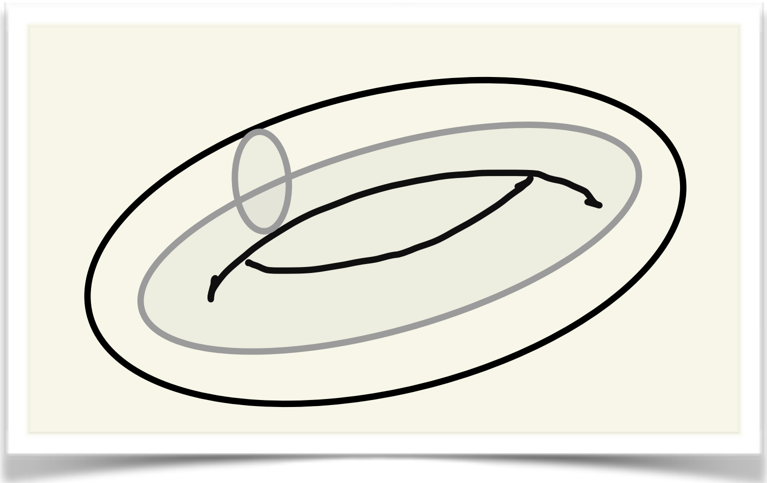
- Higher dimensional explorations [AB, Basu, Kakkar, Mehra, Nandi '16, '19, '20].

- Other important relevant work by Barnich et al.

FLAT COSMOLOGY & ENTROPY

AB, Detournay, Fareghbal, Simon 1208.4372.

MODULAR INVARIANCE AND 2D CFT



- Conformal field theories defined on tori enjoy covariance under the modular transformations of the torus:

$$\tau \mapsto \frac{a\tau + b}{c\tau + d} \qquad with \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})/\mathbb{Z}_2$$

- Of particular importance is S transformation: $\tau \mapsto -\frac{1}{\tau}$

- This maps the low temperature spectrum of the theory to the high temperature spectrum.

[Remember that this is just $\beta \mapsto \frac{4\pi^2}{\beta}$ where $\beta = \frac{1}{T}$.]

- Using the modular invariance of the partition function of a 2d CFT, one famously arrives at the Cardy formula that counts the entropy of the states of the theory.

$$S_{\text{Cardy}} = 2\pi \left(\sqrt{\frac{ch}{6}} + \sqrt{\frac{\bar{c}\bar{h}}{6}} \right).$$

BMS FIELD THEORIES AND MODULAR TRANSFORMATIONS

- Symmetry of the putative dual 2d boundary theory:

$$[L_n, L_m] = (n-m)L_{m+n} + \frac{c_L}{12}\delta_{n+m,0}(n^3 - n)$$
$$[L_n, M_m] = (n-m)M_{m+n} + \frac{c_M}{12}\delta_{n+m,0}(n^3 - n)$$
$$[M_n, M_m] = 0.$$

- Label states of the theory with $L_0|\Delta,\xi\rangle = \Delta|\Delta,\xi\rangle,\ M_0|\Delta,\xi\rangle = \xi|\Delta,\xi\rangle$

- CFT partition function: $Z^0_{\rm CFT}(\tau,\bar\tau) = \mathrm{Tr}_{\mathcal{H}}\left(q^{\mathcal{L}_0 - c/24}\,\bar q^{\bar{\mathcal{L}}_0 - \bar c/24}\right), \quad \text{where } q = e^{2\pi i\tau}.$

- Similarly, define $Z^0_{\rm BMS}(\sigma,\rho) = \mathrm{Tr}_{\mathcal{H}}\left(e^{2\pi i\sigma(L_0 - c_L/2)}\,e^{2\pi i\rho(M_0 - c_M/2)}\right).$

- Algebras go into each other as $\ell\to\infty$. Natural to assume same for partition functions.

- This means $\tau = \sigma + \frac{\rho}{\ell},\ \bar\tau = \sigma - \frac{\rho}{\ell}$

- BMS modular transformations: $\sigma \mapsto \frac{a\sigma+b}{c\sigma+d}, \quad \rho \mapsto \frac{\rho}{(c\sigma+d)^2}$

INVARIANCE OF PARTITION FUNCTION

- Demand that the BMS partition function is invariant under BMS modular transformation and attempt to find its consequences.

$$Z^0_{\rm BMS}(\sigma,\rho) = \text{Tr}\;\; e^{2\pi i\sigma(L_0-\frac{c_L}{2})}e^{2\pi i\rho(M_0-\frac{c_M}{2})} = e^{\pi i(\sigma c_L+\rho c_M)}Z_{\rm BMS}(\sigma,\rho)$$

- S-transformation in BMS: $(\sigma,\rho)\to\left(-\frac{1}{\sigma},\frac{\rho}{\sigma^2}\right)$

- Invariance of the above quantity: $Z^0_{\rm BMS}(\sigma,\rho) = Z^0_{\rm BMS}\left(-\frac{1}{\sigma},\frac{\rho}{\sigma^2}\right)$
- This translates to a statement about the partition function.

$$Z_{\rm BMS}(\sigma,\rho) = e^{2\pi i\sigma\frac{c_L}{2}}\,e^{2\pi i\rho\frac{c_M}{2}}\,e^{-2\pi i(-\frac{1}{\sigma})\frac{c_L}{2}}\,e^{-2\pi i(\frac{\rho}{\sigma^2})\frac{c_M}{2}}\,Z_{\rm BMS}\left(-\frac{1}{\sigma},\frac{\rho}{\sigma^2}\right)$$

- The density of states can be found with an inverse Laplace transformation

$$d(\Delta,\xi) = \int d\sigma d\rho\;\; e^{2\pi i\tilde{f}(\sigma,\rho)}Z\left(-\frac{1}{\sigma},\frac{\rho}{\sigma^2}\right). \quad \text{where} \quad \tilde{f}(\sigma,\rho) = \frac{c_L\sigma}{2}+\frac{c_M\rho}{2}+\frac{c_L}{2\sigma}-\frac{c_M\rho}{2\sigma^2}-\Delta\sigma-\xi\rho.$$

- In limit of large charges, this integration can be done with a saddle point approximation.

BMS CARDY FORMULA

- In the large charge limit, $\tilde{f}(\sigma,\rho) \to f(\sigma,\rho) = \frac{c_L}{2\sigma} - \frac{c_M \rho}{2\sigma^2} - \Delta\sigma - \xi\rho.$
- Value at the extremum is $f^{max}(\sigma,\rho) = -i\left(c_L\sqrt{\frac{\xi}{2c_M}} + \Delta\sqrt{\frac{c_M}{2\xi}}\right).$
- BMS-Cardy formula is given by

$$\boxed{S^{(0)} = \ln d(\Delta,\xi) = 2\pi\left(c_L\sqrt{\frac{\xi}{2c_M}} + \Delta\sqrt{\frac{c_M}{2\xi}}\right).}$$

Bagchi, Detournay, Fareghbal, Simon 2012.

- One can calculate leading logarithmic corrections to this.

$$S = 2\pi\left(c_L\sqrt{\frac{\xi}{2c_M}} + \Delta\sqrt{\frac{c_M}{2\xi}}\right) - \frac{3}{2}\log\left(\frac{\xi}{c_M^{1/3}}\right) + \text{constant} = S^{(0)} + S^{(1)}.$$

Bagchi, Basu 2013.

THE BULK SIDE

- Most generic metric compatible with bdy cond that gives BMS at the null boundary:

$$ds^2 = \Theta(\psi)du^2 - 2dudr + \left[\Xi(\psi) + \partial_\psi\Theta(\psi)\right]dud\psi + r^2 d\psi^2.$$

Here $\Xi(\psi), \Theta(\psi)$ are the angular momentum aspect and mass aspect respectively.

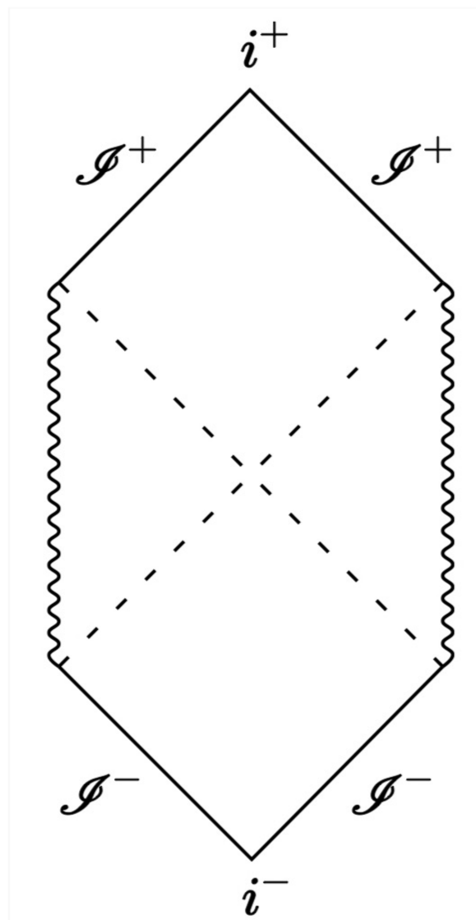
Barnich, Gomberoff, Gonzalez 2010.

- Zero modes of this solution: $ds^2 = M du^2 - 2dudr + J dud\psi + r^2 d\psi^2.$

- Another form of the metric:

$$ds^2_{\text{FSC}} = \hat{r}_+^2 dt^2 - \frac{r^2\, dr^2}{\hat{r}_+^2 (r^2 - r_0^2)} + r^2 d\phi^2 - 2\hat{r}_+ r_0 dt d\phi$$

Here $\hat{r}_+ = \sqrt{8GM},\ r_0 = \sqrt{\frac{2G}{M}}J.$



- Cosmological solution with horizon. Flat Space Cosmology (FSC).

- Also called shifted boost orbifolds. *Cornalba, Costa 2002.*

- Entropy: $$S_{\text{FSC}} = \frac{\text{Area of horizon}}{4G} = \frac{\pi r_0}{2G} = \frac{\pi J}{\sqrt{2GM}}$$

FSC ENTROPY FROM DUAL THEORY

- For the BTZ black hole $h = \frac{1}{2}(\ell M + J) + \frac{c}{24}, \quad \bar{h} = \frac{1}{2}(\ell M + J) + \frac{\bar{c}}{24}, \quad c = \bar{c} = \frac{3\ell}{2G}$
- Following the limit, the weights for the FSC:

$$\xi = M + \frac{c_M}{24} = M + \frac{1}{8G} \sim M, \quad \Delta = J.$$

- Putting this back into the BMS-Cardy formula, we get

$$S_{\text{FSC}} = \frac{\pi J}{\sqrt{2GM}}$$

Bagchi, Detournay, Fareghbal, Simon 2012; Barnich 2012

which is precisely what we obtained from the gravitational analysis.

- The log-correction is of the form $S^{(1)}_{\text{FSC}} = -\frac{3}{2}\log(2GM)$
- Total entropy can be put in the following form:

$$\boxed{S_{\text{FSC}} = \frac{2\pi r_0}{4G} - \frac{3}{2}\log(\frac{2\pi r_0}{4G}) - \frac{3}{2}\log\kappa + \text{ constant}}$$

Bagchi, Basu 2013.

- Here $\kappa = \frac{\hat{r}^2}{r_0} = \frac{8GM}{r_0}$ is the surface gravity of FSC.

BMS STRUCTURE CONSTANTS

With A.Saha, P. Nandi and Zodinmawia (2007.11713)

MORE FROM MODULARITY

- Can we use the modular properties of BMS field theories to push this correspondence further?
- Yes!

- Remember: Partition function = zero point function on the torus.
- We will now turn our attention to the one-point function and its modular properties following Kraus and Maloney (2016).
- Result: Asymptotic formula for BMS structure constants.

TORUS ONE POINT FUNCTIONS IN 2D CFT

- Torus 1 point function: $\langle \phi(\omega,\bar{\omega}) \rangle_{(\tau,\bar{\tau})} = \mathrm{Tr}_{\mathcal{H}} \left(\phi(\omega,\bar{\omega})\, q^{\mathcal{L}_0 - \frac{c}{24}}\, \bar{q}^{\bar{\mathcal{L}}_0 - \frac{\bar{c}}{24}} \right)$

- Modular transformation: $\tau \to \gamma.\tau \equiv \dfrac{a\tau + b}{c\tau + d}, \quad \bar{\tau} \to \gamma.\bar{\tau} \equiv \dfrac{a\bar{\tau} + b}{c\bar{\tau} + d}$

- Elliptic coordinates of torus transform as: $w \to \gamma.w \equiv \dfrac{w}{c\tau + d}, \quad \bar{w} \to \gamma.\bar{w} \equiv \dfrac{\bar{w}}{c\bar{\tau} + d}.$

- Primary fields transform as: $\phi_{h,\bar{h}}(\gamma.w, \gamma.\bar{w})|_{\gamma.\tau} = \left(\dfrac{\partial(\gamma.w)}{\partial w} \right)^{-h} \left(\dfrac{\partial(\gamma.\bar{w})}{\partial \bar{w}} \right)^{-\bar{h}} \phi_{h,\bar{h}}(w,\bar{w})$

- Using this & modular invariance of partition function, 1-point functions transform as:

$$\langle \phi_{h,\bar{h}}(\gamma.w, \gamma.\bar{w}) \rangle_{(\gamma.\tau,\gamma.\bar{\tau})} = (c\tau + d)^{-h} (c\bar{\tau} + d)^{-\bar{h}} \langle \phi_{h,\bar{h}}(w,\bar{w}) \rangle_{(\tau,\bar{\tau})}$$

BMS TORUS ONE POINT FUNCTIONS

- Torus 1 point function:

$$\langle \psi_{\text{cyl}}(u,\varphi)\rangle_{(\sigma,\rho)} = \text{Tr}_{\mathcal{H}}\left(\psi_{\text{cyl}}(u,\varphi)\, e^{2\pi\sigma(L_0-\frac{c_L}{2})}\, e^{2\pi\rho(M_0-\frac{c_L}{2})}\right)$$

- Using translational invariance and the plane-cylinder map

$$\langle \psi_{\text{cyl}}(u,\varphi)\rangle_{(\sigma,\rho)} = \text{Tr}_{\mathcal{H}}\left[\psi(0,1)\, e^{2\pi\sigma(L_0-\frac{c_L}{2})}\, e^{2\pi\rho(M_0-\frac{c_L}{2})}\right]$$

- Modular transformation: $\sigma \to \gamma.\sigma = \dfrac{a\sigma+b}{c\sigma+d}, \qquad \rho \to \gamma.\rho = \dfrac{\rho}{(c\sigma+d)^2}.$

- Elliptic coordinates of BMS torus transform as: $\gamma.u = \dfrac{u}{c\sigma+d} - \dfrac{\phi\rho}{(c\sigma+d)^2},\ \gamma.\phi = \dfrac{\phi}{c\sigma+d}.$

  Note that this is a finite BMS transformation of the form: $u \to u\partial_\phi f(\phi) + g(\phi),\ \phi \to f(\phi).$

- Using the transformation rule for BMS primary fields and modular invariance of partition function, we get the transformation of BMS torus 1-point functions:

$$\langle \phi_{\Delta,\xi}(\gamma.u,\gamma.\phi)\rangle_{(\gamma.\sigma,\gamma.\rho)} = (c\sigma+d)^{\Delta} e^{-\frac{\xi c\rho}{(c\sigma+d)}} \langle \phi_{\Delta,\xi}(u,\phi)\rangle_{(\sigma,\rho)}.$$

ASYMPTOTIC STRUCTURE CONSTANTS

- States: $L_0|\Delta_i,\xi_i\rangle = \Delta_i|\Delta_i,\xi_i\rangle,\ M_0|\Delta_i,\xi_i\rangle = \xi_i|\Delta_i,\xi_i\rangle$
- One point function

$$\langle\phi_p\rangle_{(\sigma,\rho)} = \sum_i \langle\Delta_i,\xi_i|\phi_p(0,1)|\Delta_i,\xi_i\rangle D(\Delta_i,\xi_i) e^{2\pi i\sigma(\Delta_i-\frac{c_L}{2})} e^{2\pi i\rho(\xi_i-\frac{c_M}{2})}$$

$$= e^{-2\pi i\left(\sigma\frac{c_L}{2}+\rho\frac{c_M}{2}\right)} \sum_i D(\Delta_i,\xi_i) C_{ipi} e^{2\pi i(\sigma\Delta_i+\rho\xi_i)}.$$

- We will determine $C_{ipi} \equiv \langle\Delta_i,\xi_i|\phi_p(0,1)|\Delta_i,\xi_i\rangle$ for large Δ_i and ξ_i.
- Define $\widetilde{\langle\phi_p\rangle}_{(\sigma,\rho)} = e^{2\pi i\left(\sigma\frac{c_L}{2}+\rho\frac{c_M}{2}\right)}\langle\phi_p\rangle_{(\sigma,\rho)} = \sum_i D(\Delta_i,\xi_i) C_{ipi} e^{2\pi i(\sigma\Delta_i+\rho\xi_i)}$
- Use modular properties derived earlier and saddle point integration to get:

$$\widetilde{\langle\phi_p\rangle}_{(-\frac{1}{\sigma_c},\frac{\rho_c}{\sigma_c^2})} = \sum_i D(\Delta_i,\xi_i) C_{ipi} e^{2\pi i(-\frac{1}{\sigma_c}\Delta_i+\frac{\rho_c}{\sigma_c^2}\xi_i)} = \sum_i D(\Delta_i,\xi_i) C_{ipi}\, e^{-2\pi\sqrt{\frac{2\xi}{c_M}}\Delta_i-2\pi\sqrt{\frac{\xi}{2c_M}}\left(\frac{\Delta}{\xi}-\frac{c_L}{c_M}\right)\xi_i}.$$

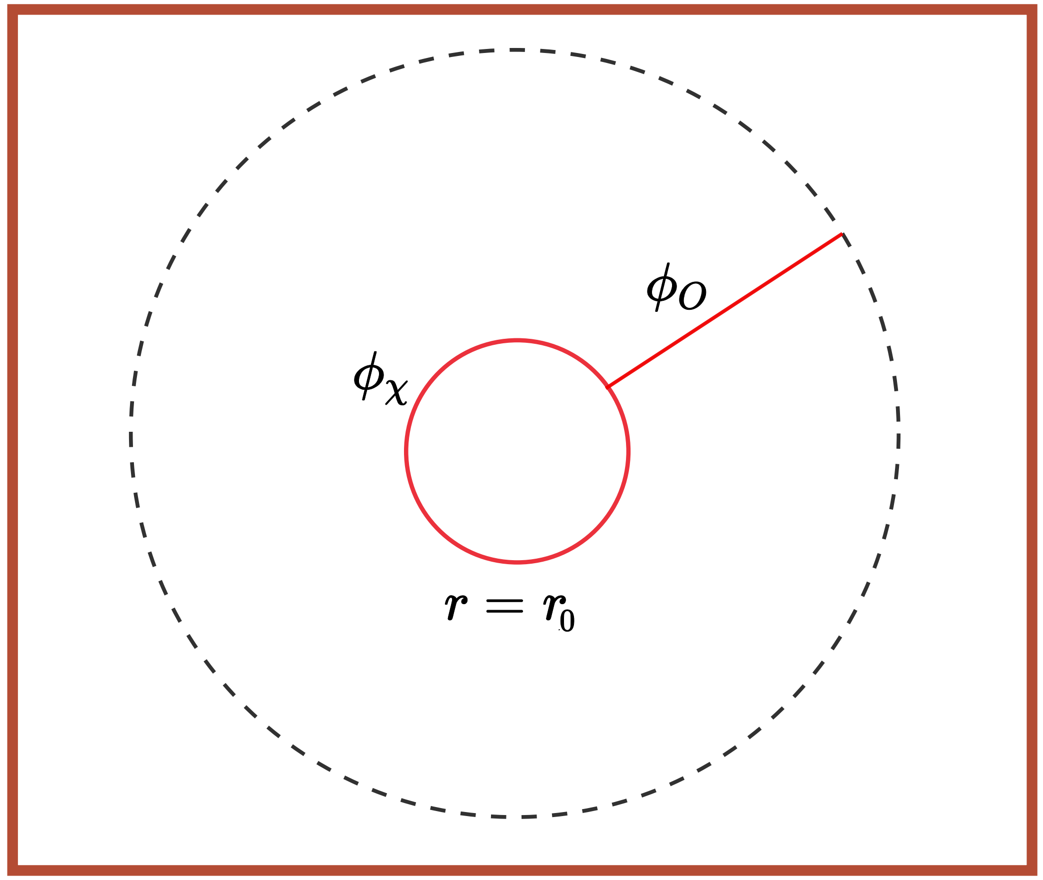
ASYMPTOTIC STRUCTURE CONSTANTS

- Now $C_{ipi} = 0 \quad \text{for} \ |\Delta, \xi\rangle = |0, 0\rangle = |\text{vacuum}\rangle$
- So leading contribution comes from lightest state above the vacuum $|\chi\rangle = |\Delta_\chi, \xi_\chi\rangle$
- Putting everything together, for the special case of $c_L = 0$

$$C_{ipi} \approx D(\Delta_\chi, \xi_\chi) C_{\chi p \chi} \exp\left(-\frac{\Delta_i \xi_p}{2\xi_i}\right) \exp\left(-2\pi \frac{\Delta_i \xi_\chi}{\sqrt{2c_M \xi_i}}\right).$$

- Here $D(\Delta_\chi, \xi_\chi)$ is the degeneracy of $|\chi\rangle$ and we have chosen the vacuum to have no degeneracy.
- This is the main result of 2007.11713. In the paper, we find ways to improve on the result and extend it.
- We also derive this for purely primaries and the highest weight representation. We find expressions for BMS torus blocks on the way.

A QUICK BULK ANALYSIS: SET UP



One-loop contribution to $\langle E|O|E\rangle$

- We wish to calculate $\langle E|O|E\rangle$ for high energy E from the bulk side.
- Dual to finite temperature BMS field theory state is the FSC solution.
- Calculation: one point function of a probe $\boldsymbol{O}$ in the FSC background.
- Field ϕ_O comes in from the boundary and splits into a pair of ϕ_χ s which wrap the cosmological horizon.

A QUICK BULK ANALYSIS

- We work in the probe limit: $\xi_O, \xi_\chi \ll c_M, \;\; \xi_{\text{FSC}} \gg c_M$
- **Geodesic approximation:** also holds for flat space. [Hijano-Rabideu 2018]

  2-point function for bulk field with mass $m \sim e^{-mL}$ (L = length of geodesic linking two points.)

- Contribution from $\phi_\chi = e^{-\xi_\chi 2\pi r_0} = \exp\left(-2\pi\xi_\chi \frac{\Delta_{\text{FSC}}}{\sqrt{2c_M \xi_{\text{FSC}}}}\right)$.
- Contribution from $\phi_O = \exp(-\xi_O L)$, L: length of the geodesic from r_0 to boundary.

$$L = \int_{r_0}^{\Lambda} \frac{r dr}{\hat{r}_+ \sqrt{r^2 - r_0^2}} = \frac{\sqrt{\Lambda^2 - r_0^2}}{\hat{r}_+^2} \Rightarrow \log L = \frac{1}{2}\log\left(\frac{\Lambda^2}{r_0^2} - 1\right) - \log\left(\frac{\hat{r}_+}{r_0}\right).$$

- We drop the divergent piece and get $\log L = -\log\left(\frac{\hat{r}_+}{r_0}\right) \implies L = \frac{r_0}{\hat{r}_+} \approx \frac{\Delta_{\text{FSC}}}{2\xi_{\text{FSC}}}$.
- Putting everything together

$$\langle E|O|E\rangle \approx \langle\chi|O|\chi\rangle \exp\left(-\frac{\xi_O \Delta_{\text{FSC}}}{2\xi_{\text{FSC}}} - \frac{2\pi\xi_\chi \Delta_{\text{FSC}}}{\sqrt{2\xi_{\text{FSC}} c_M}}\right).$$

Matches the field theory analysis!

A MORE REFINED ANALYSIS

- Trace in 1-pt fn did not assume much (except states = eigenstates of L_0 and M_0). Now specialise to highest weight representation.
- For this, we need the form of the BMS torus blocks.

$$\mathcal{F}^{\Delta_p,\xi_p}_{\Delta_A,\xi_A,c_L,c_M}(\sigma,\rho) = \frac{q^{-\Delta_A+\frac{c_L}{2}}y^{-\xi_A+\frac{c_M}{2}}}{C_{ApA}}\mathrm{Tr}_{\Delta_A,\xi_A}\left(\phi_p(0,1)q^{L_0-\frac{c_L}{2}}y^{M_0-\frac{c_M}{2}}\right).$$

- Here the trace is over all primaries with weights (Δ_A, ξ_A). Also $q = e^{2\pi i\sigma},\ y = e^{2\pi i\rho}$
- In a limit of large weights, we can calculate these blocks

$$\mathcal{F}^{\Delta_p,\xi_p}_{\Delta_A,\xi_A,c_L,c_M}(\sigma,\rho) \equiv \sum_N q^N \mathcal{F}_N(\Delta_p,\xi_p;\Delta_A,\xi_A|c_L,c_M|\rho), \quad \text{where}$$

$$\mathcal{F}_N = \left(1+\frac{\xi_p(\Delta_p-1)}{2\xi_A}N\right)\widetilde{\dim}_N + \pi i\rho\frac{\xi_p^2}{\xi_A}\sum_{k=0}^{N} p(N-k)p(k)(N-k)(N-2k) + \mathcal{O}(\xi_A^{-2}).$$

($p(N)$=partition of N. $\widetilde{\dim}_N$ = partition of N with two colours.)

- Leading piece = BMS-character.
- The asymptotic form of the structure constants remain almost identical with just primaries, with the replacement $c_L \to c_L - 1/6$.
- Matches with quantum shift from 1-loop partition function. **[Merbis-Riegler 2019]**
  One loop renormalisation of bulk effective central charge. Note: no shift in c_M.

CONCLUSIONS

- BMS field theories are putative duals to asymptotically flat spacetimes.
- We looked at the notion of modular invariance for 2d BMS field theories.
- This helped derive a Cardy-like formula for these field theories.
- Flat Space Cosmologies are solutions in 3d flat spacetime with a cosmological horizon.
- BMS-Cardy formula reproduced Bekenstein-Hawking entropy of FSCs.

- Then looked at modular properties of torus 1-point functions.
- This led to an asymptotic formula for BMS structure constants.
- A bulk calculation of a probe in the background of FSCs gave the same answer.
- More refined calculation with highest weight primaries can be performed. This also led us to expressions for torus blocks.
- Asymptotic formula for structure constants of primaries leads to shift in central charge observed in 1-loop partition functions.

FUTURE DIRECTIONS.

- Generalisations
  - BMS with U(1), Super-BMS, higher spins.
- Torus two-point functions *AB, Nandi, Pal, Zodinmawia (in progress)*
  - Eigenstate thermalisation?
  - Links to Quasi-normal modes of FSC?
- Modular Bootstrap in BMS field theories. *AB, Saha (in progress)*
- Applications to scattering of tensionless strings?
- Higher dimensions?

Thank you for listening!