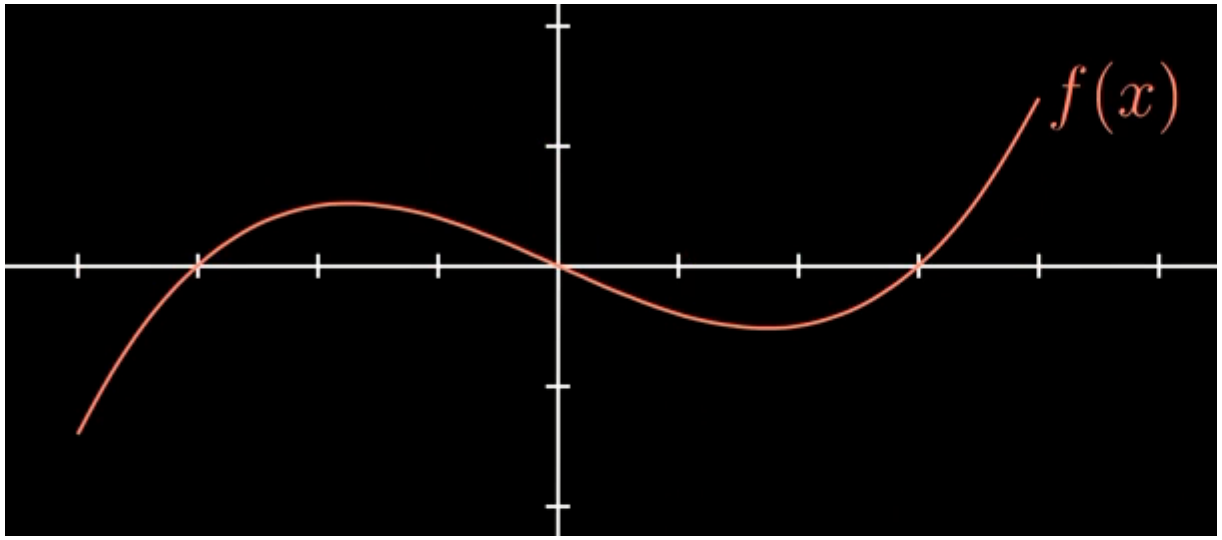
SKILL PILLS

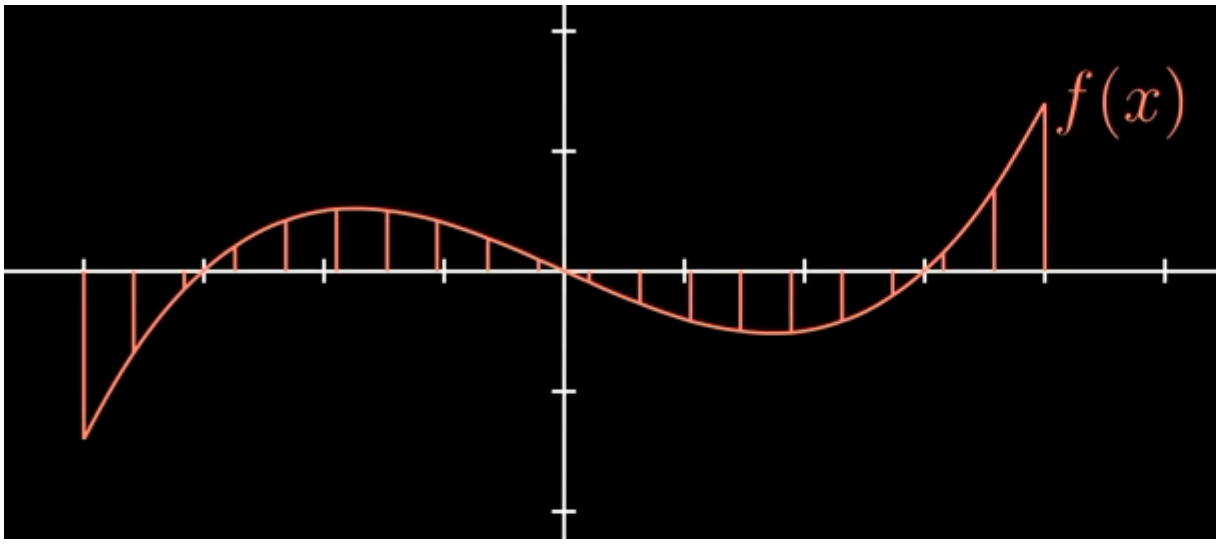
SKILL PILL: Fourier Transforms

Discretising and Application

OIST

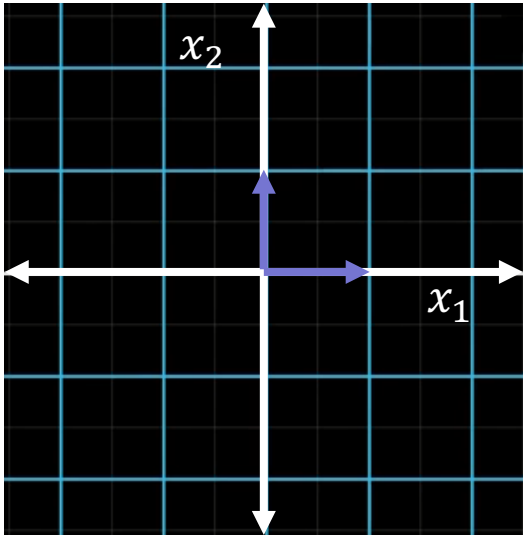
Discretising

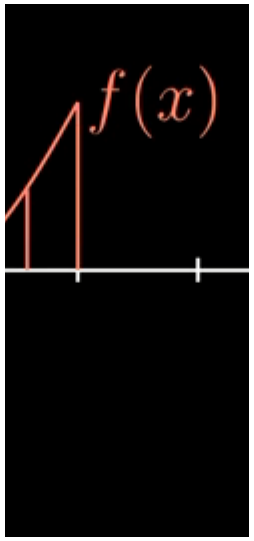




$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_{n-2} \\ x_{n-1} \\ x_n \end{bmatrix}$$

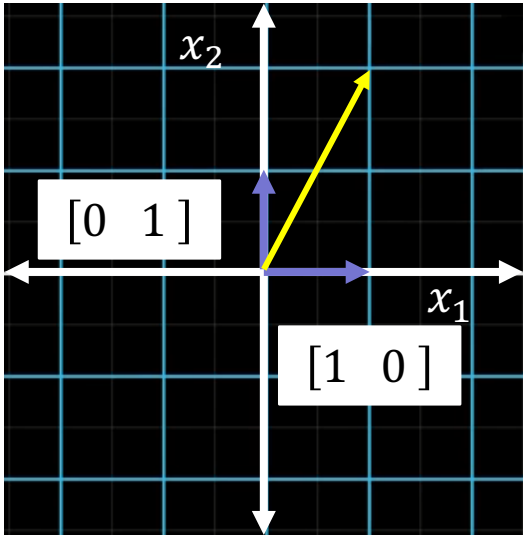
Discretising

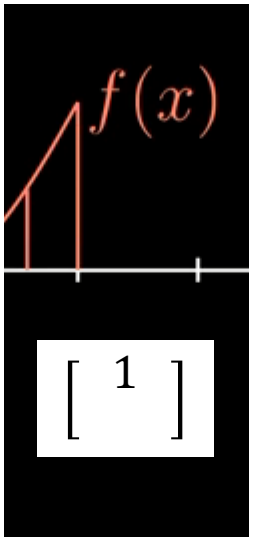




$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_{n-2} \\ x_{n-1} \\ x_n \end{bmatrix}$$

Discretising



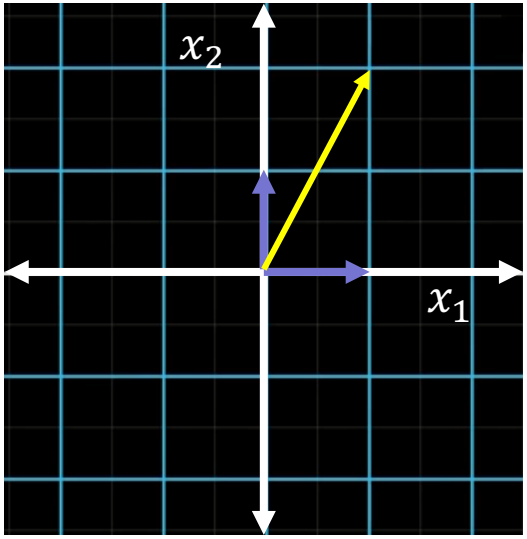


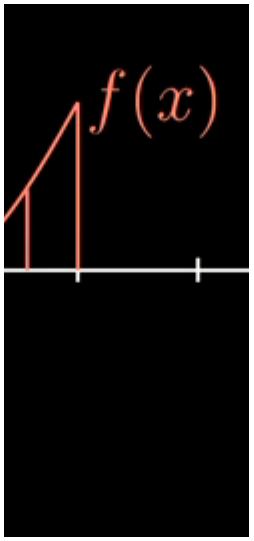
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } = \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

Discretising



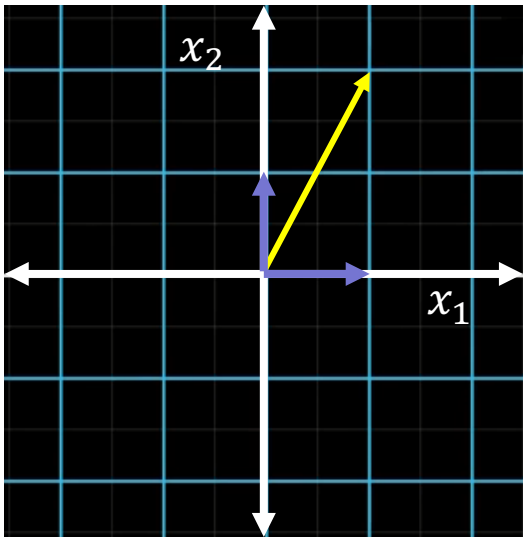


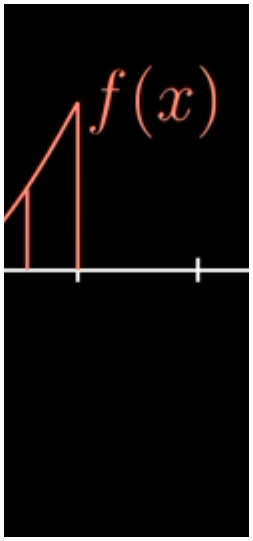
Linear Transformations:

Must fulfill additivity and scalability criteria

- ~~Reflection~~
- Rotation
- ~~Scaling~~
- ~~Shearing~~

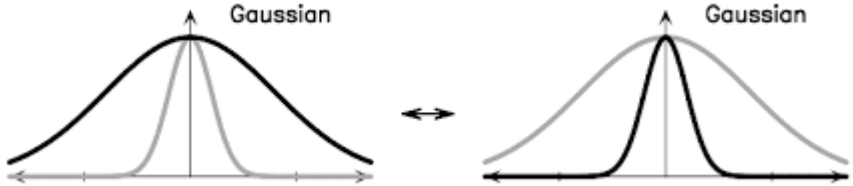
Discretising





Try in matlab:

```
>> dftmtx(2)
```



$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Discretising

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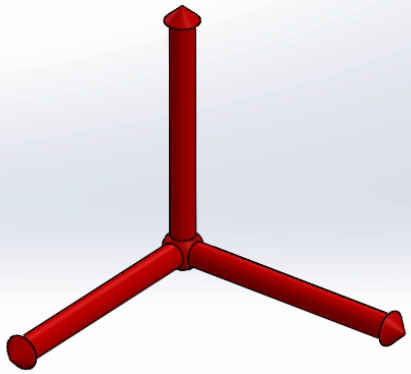

Discretising

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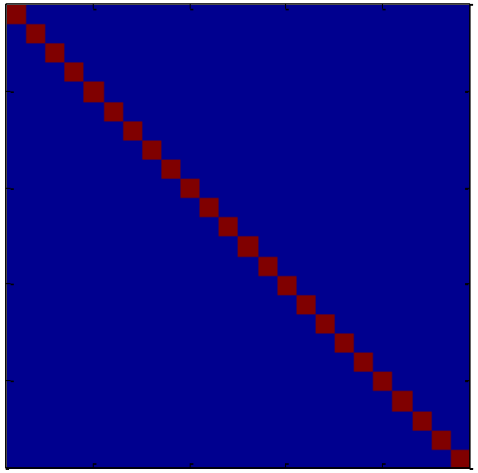
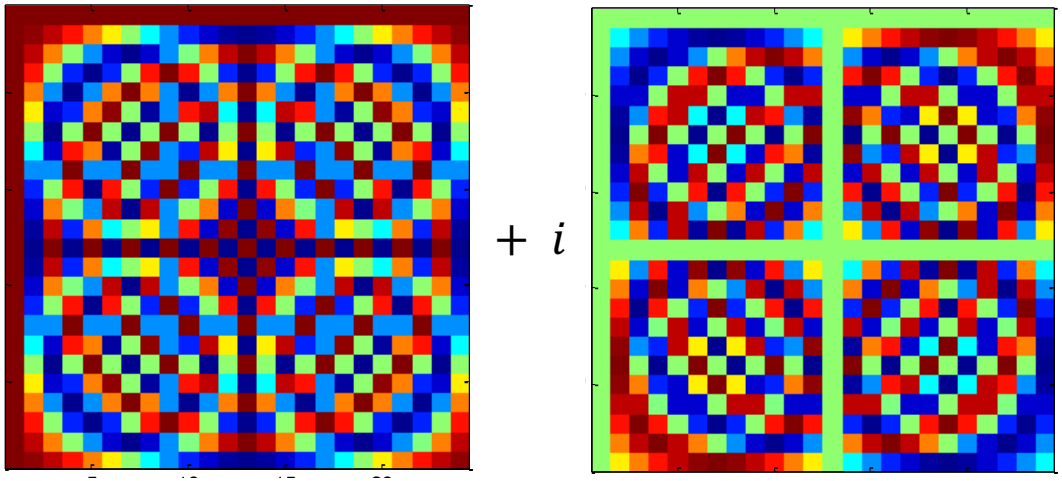
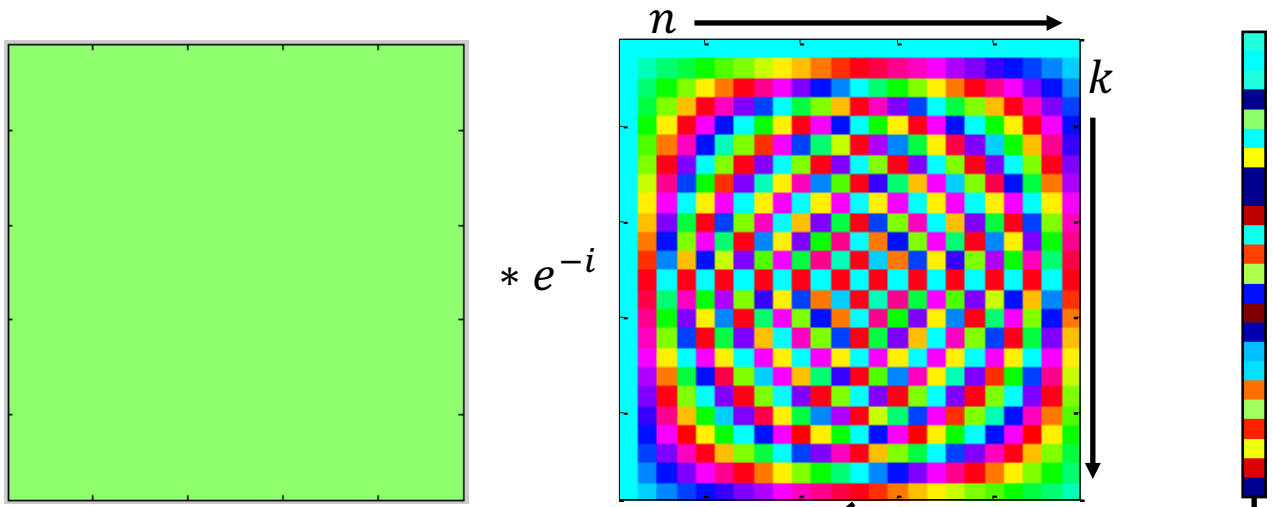
Time Domain Sampling Matrix



Frequency Domain Sampling Matrix



Discretising



$$X_k \equiv \sum_{n=0}^{N-1} x_n \cdot 1e^{-2\pi i k n/N}, \qquad k \in \mathbb{Z}$$

$$F(\xi) \equiv \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i x \xi}\, dx, \qquad \xi \in \mathbb{R}$$

Sampling

Sampling

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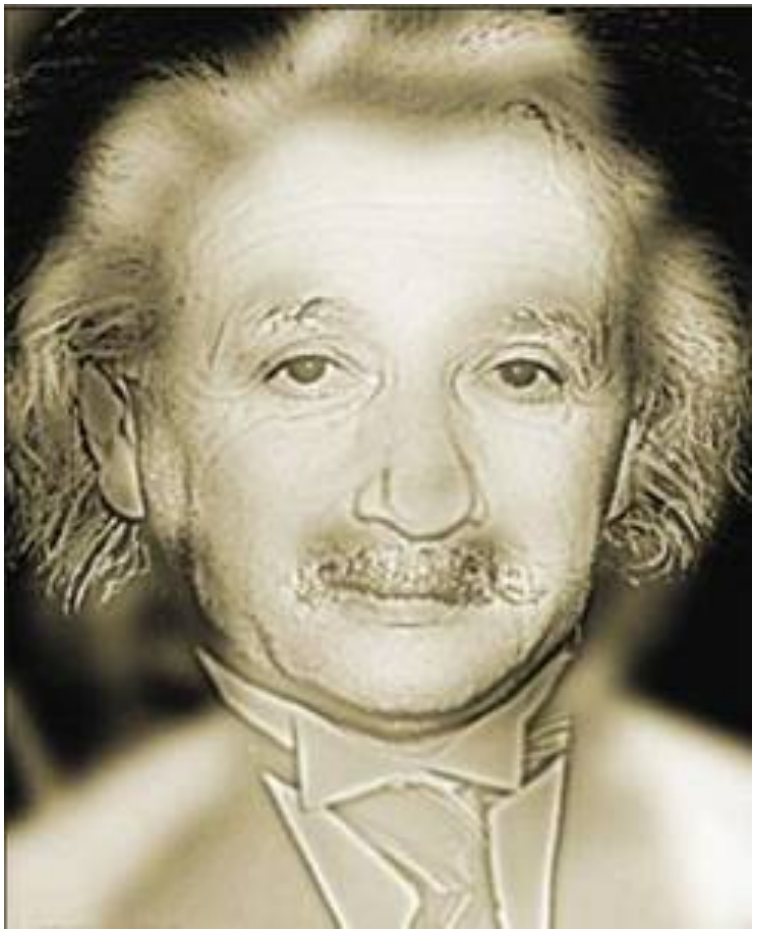
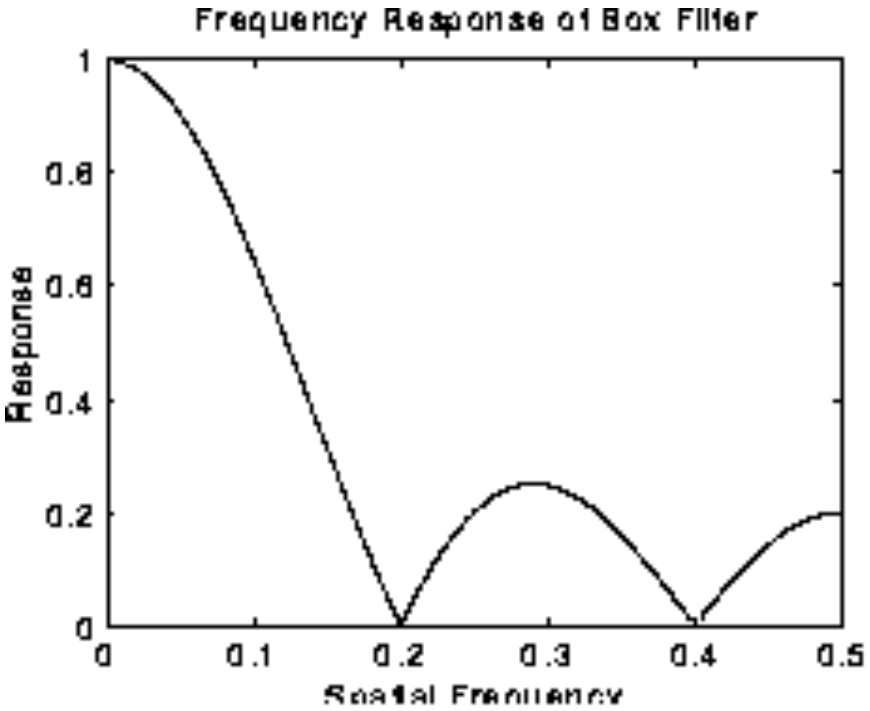
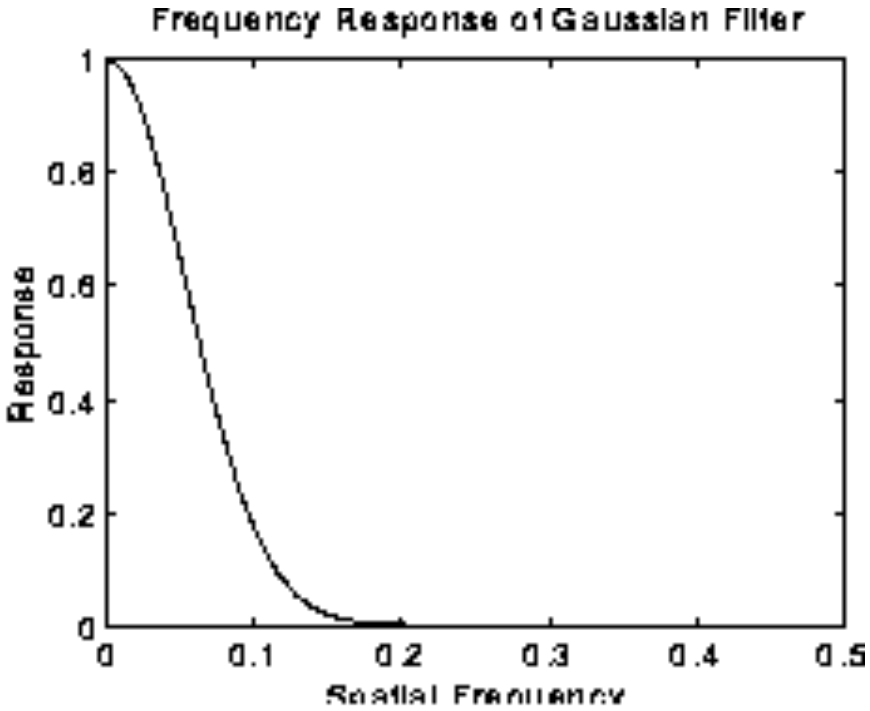
To Matlab !

Filtering



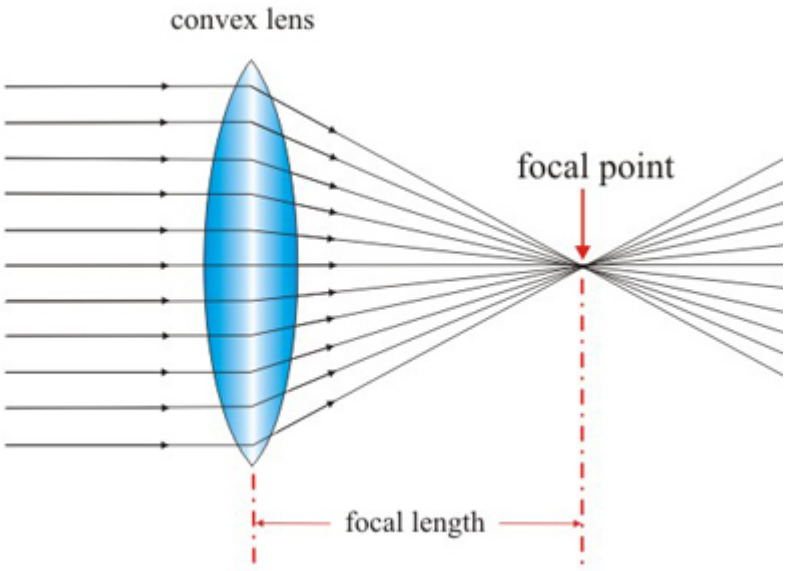
Filtering


Frequency Response of Box Filter
Response
Spatial Frequency
0
0.2
0.4
0.6
1
0.1
0.3
0.5

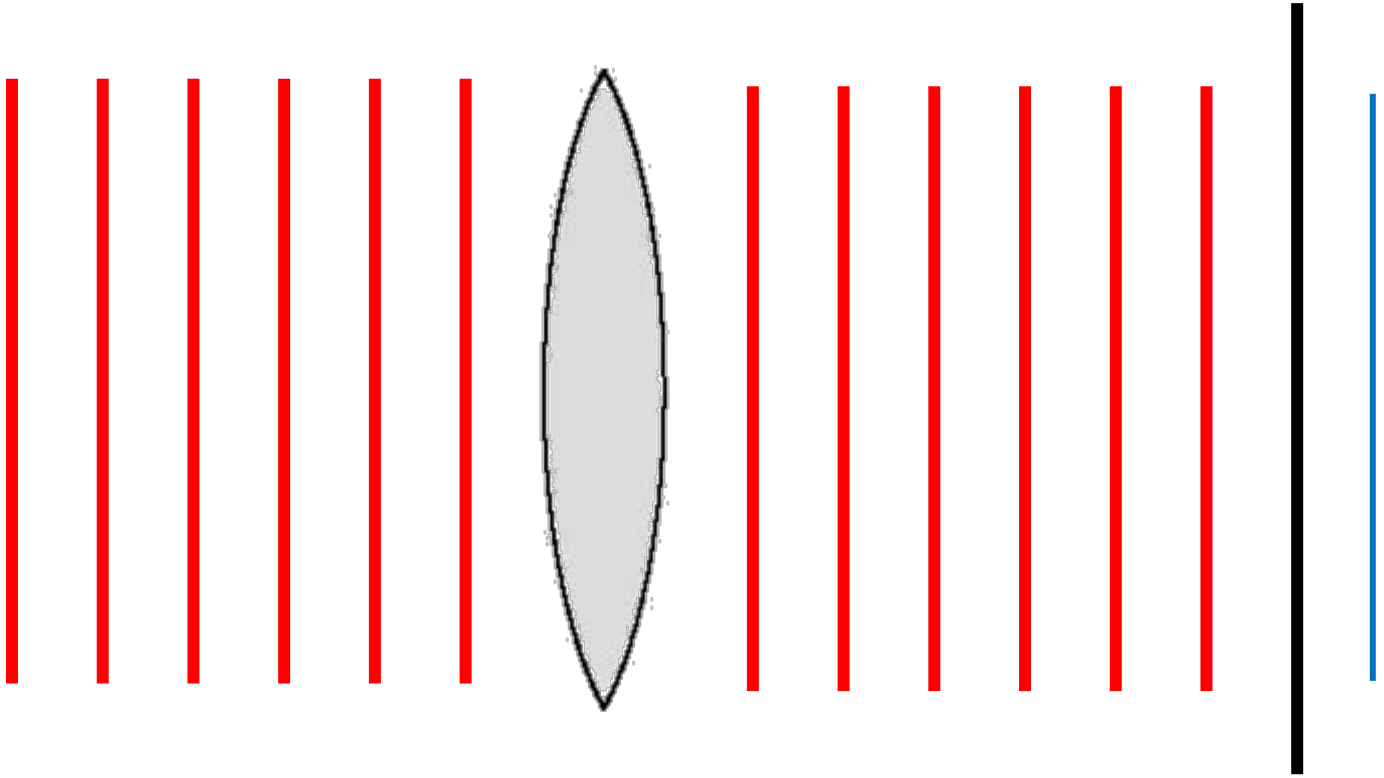

Frequency Response of Gaussian Filter
Response
Spatial Frequency
0
0.2
0.4
0.6
1
0.1
0.3
0.5

Plane wave electric field:

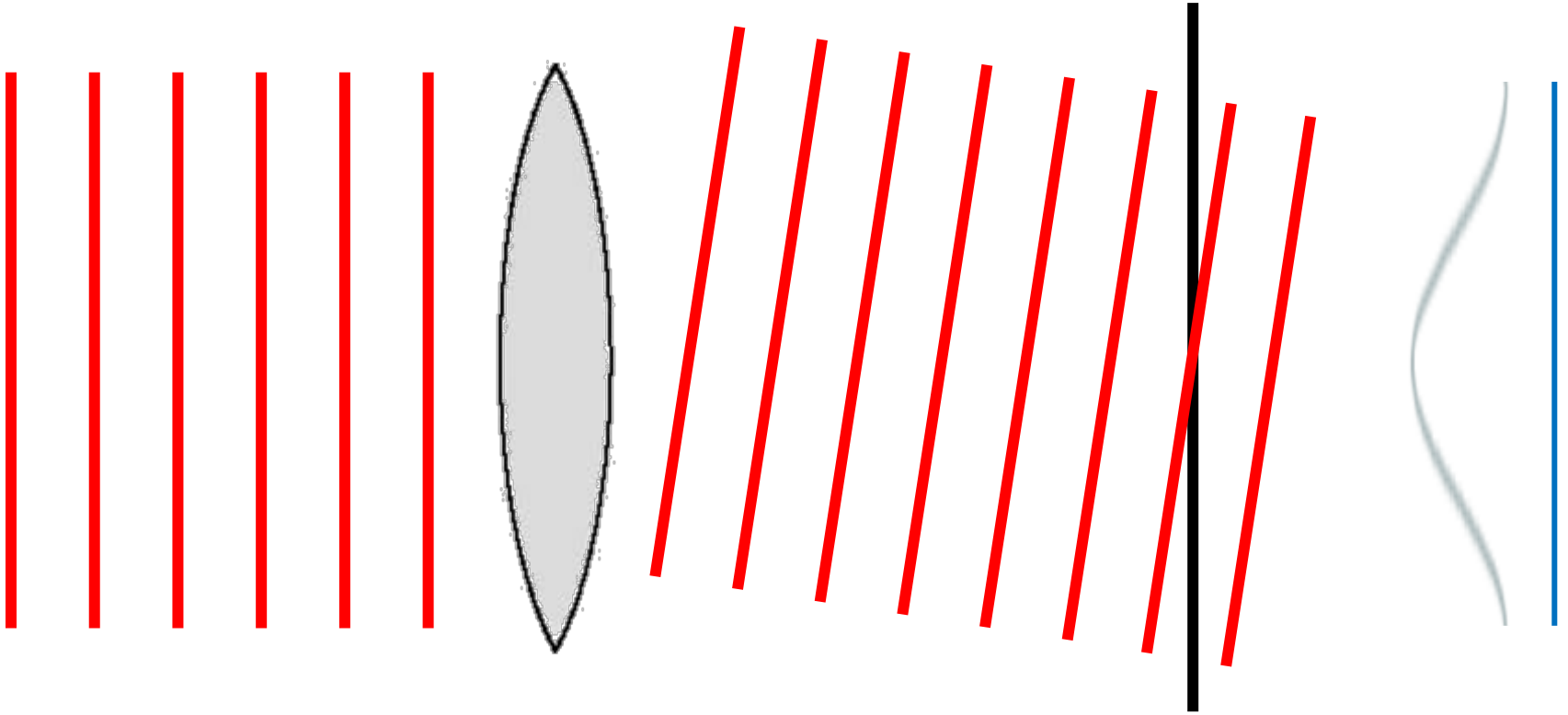
$$E = E_0 e^{-i(\omega t - kz)}$$



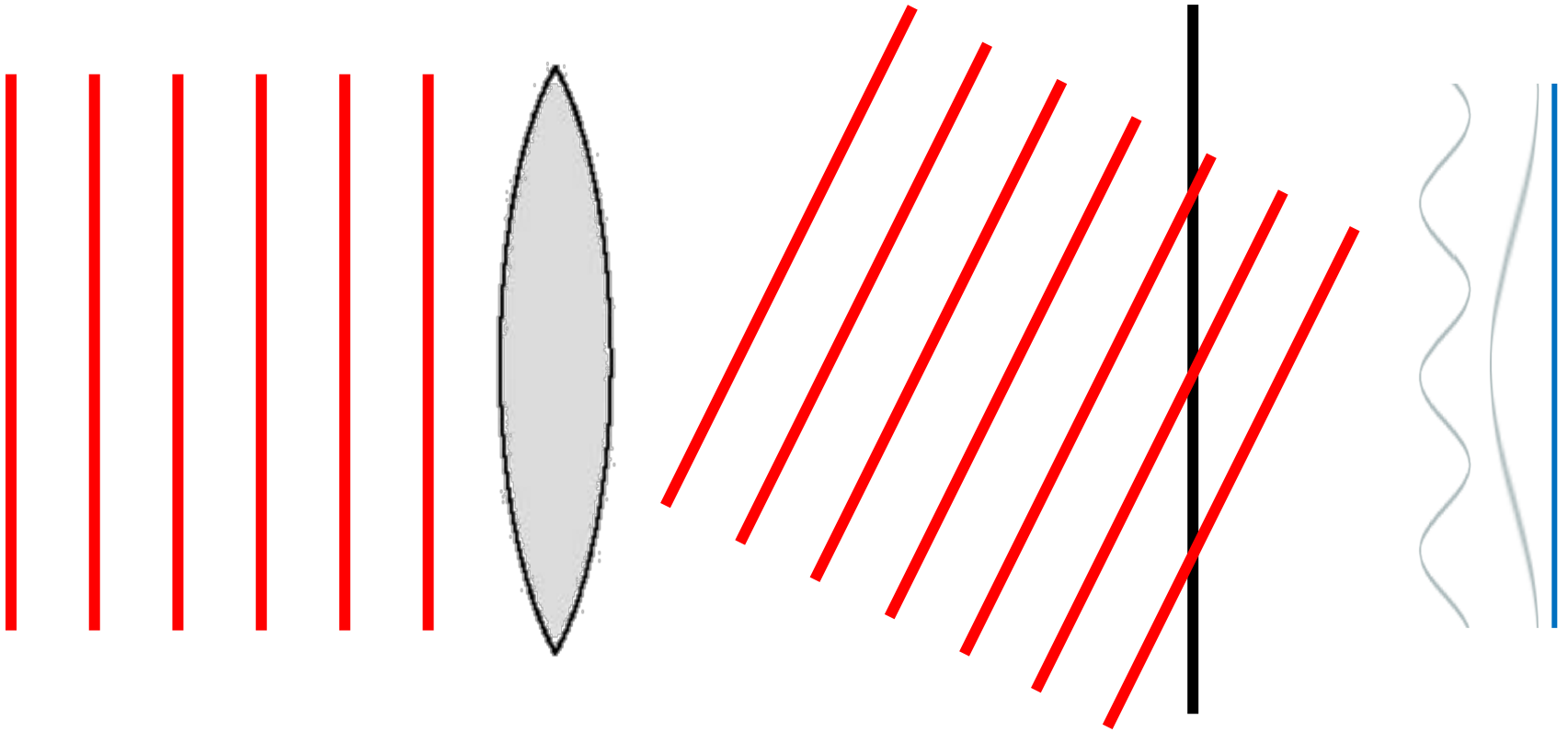
Optics

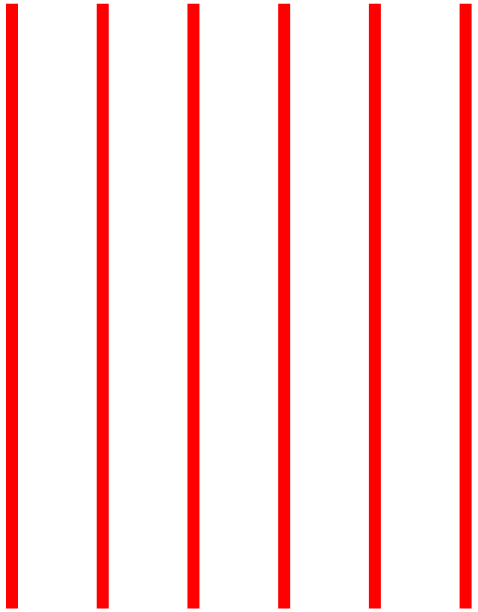


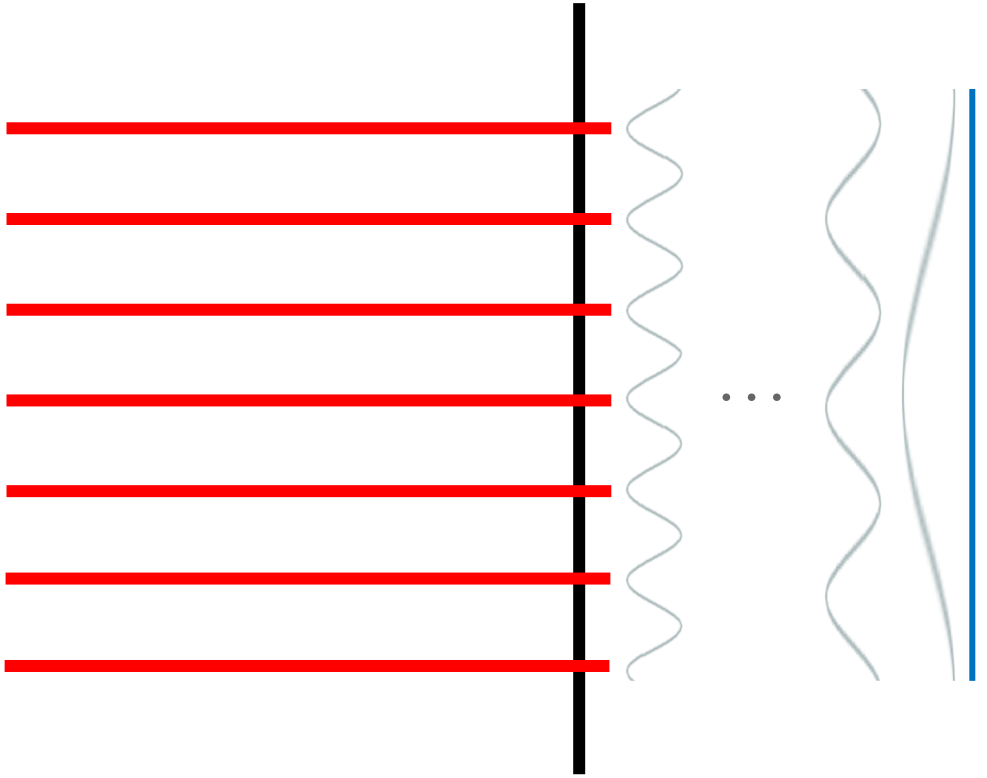
Optics



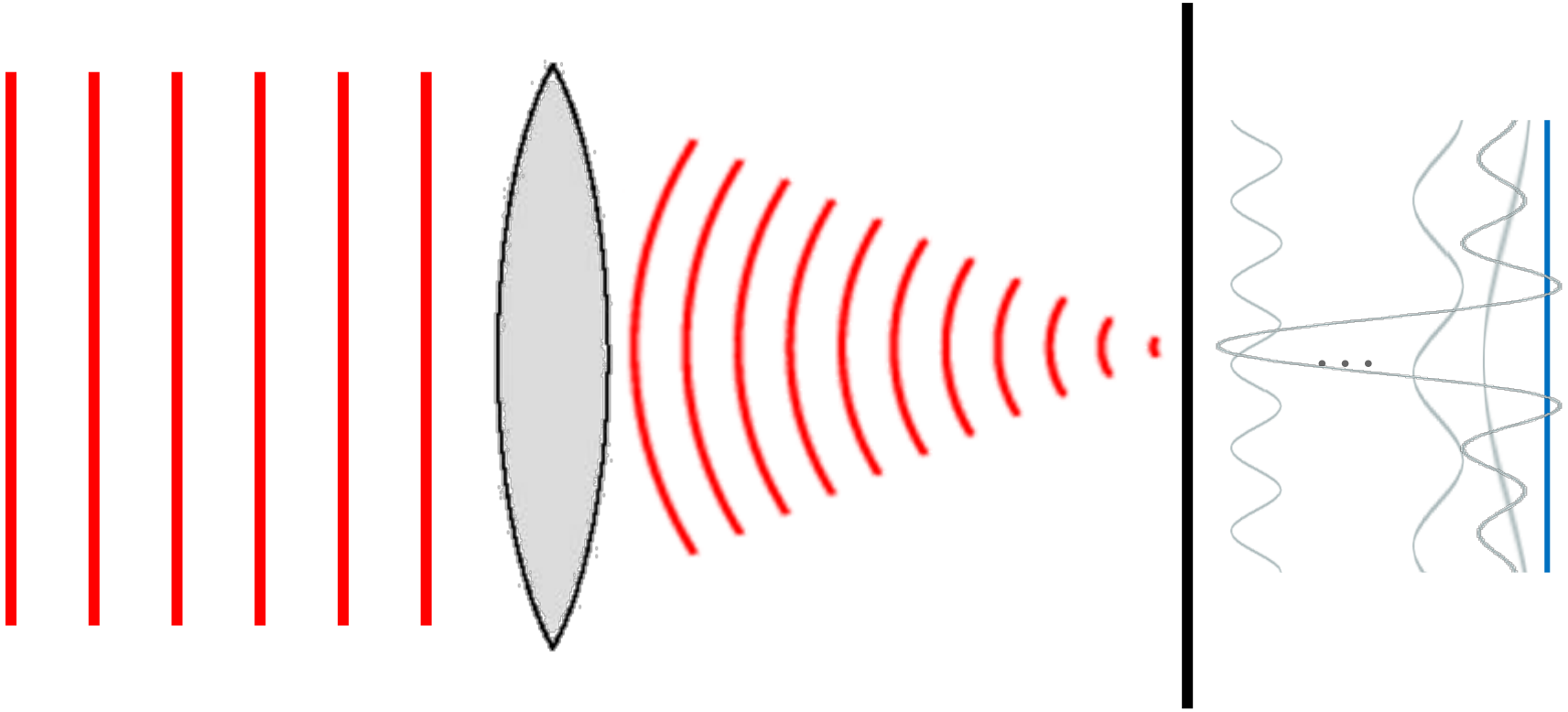
Optics







Optics



Convolution (Time Domain)

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0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	....

★

0	0	0
-1	2	-1
0	0	0

Kernel

0	0	0	0	0	0
0	0	0	0	0	0
0	-1	1	0	0	0
0	-1	1	0	0	0
0	-1	1	0	0	0
0	-1	1	0	0	0

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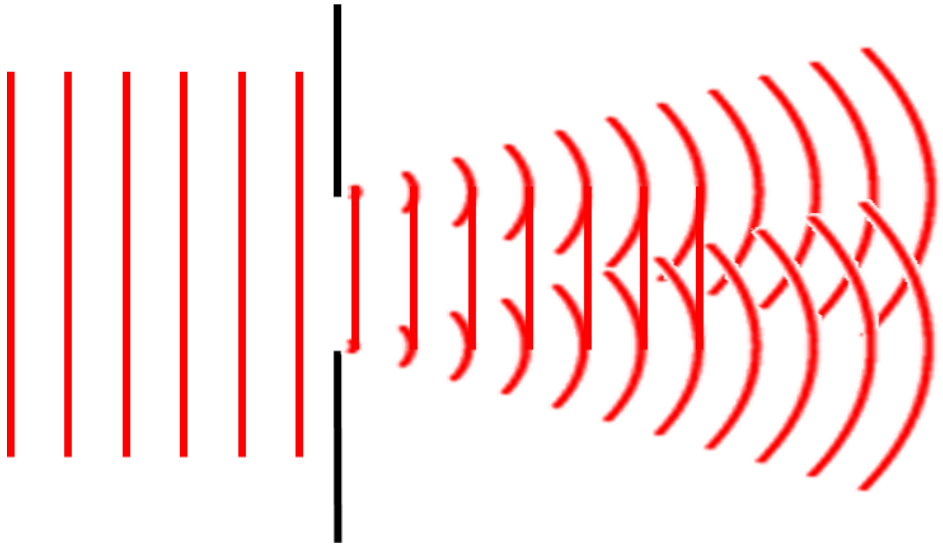
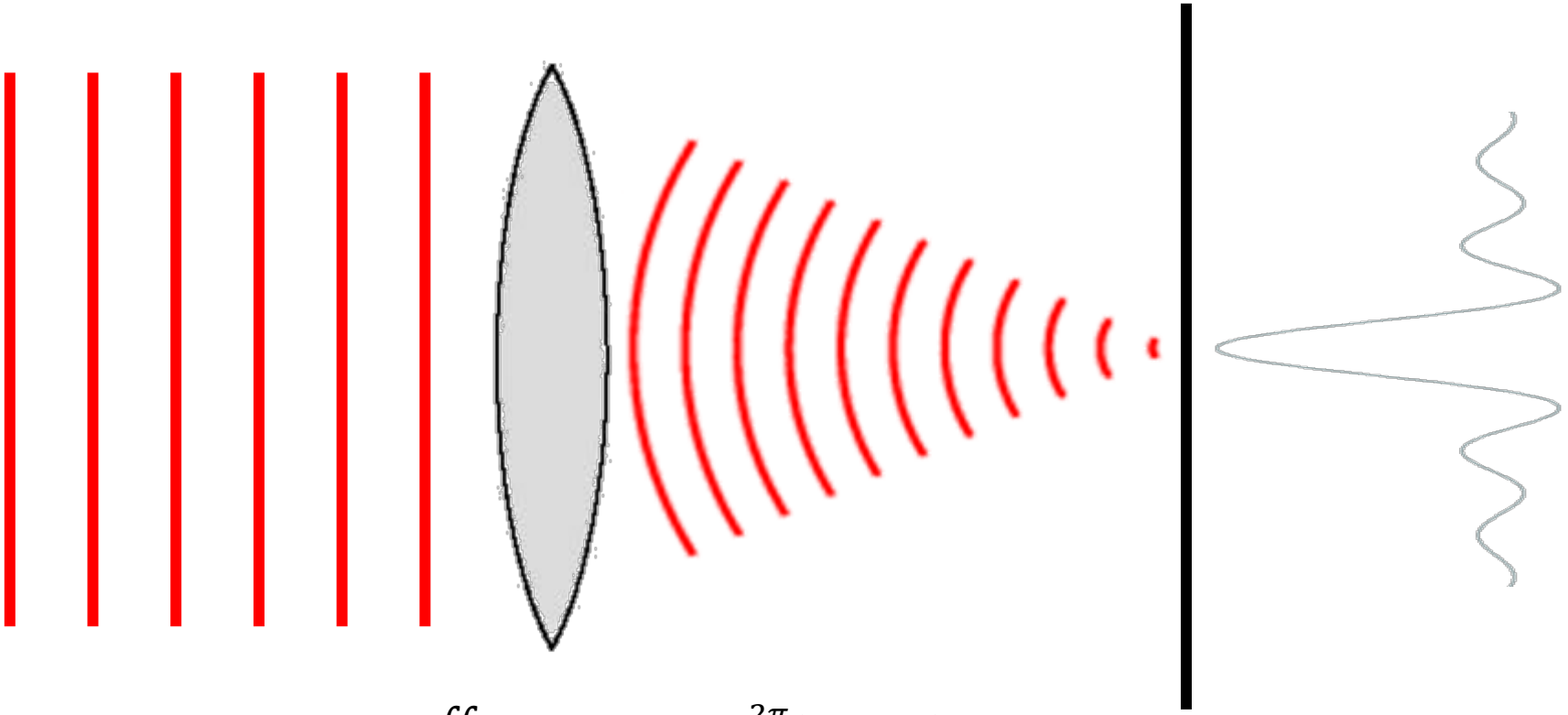
Fraunhofer diffraction pattern from an aperture:



$$U(\nu,\eta) \quad \propto \quad \iint\limits_{Aperture} E(x',y')e^{-i\frac{2\pi}{\lambda}(\nu x'+\eta y')}dx'dy' \qquad \nu,\eta \in \mathbb{R}$$

$$F(\xi) \equiv \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i x\xi}\, dx, \qquad \xi \in \mathbb{R}$$

Optics



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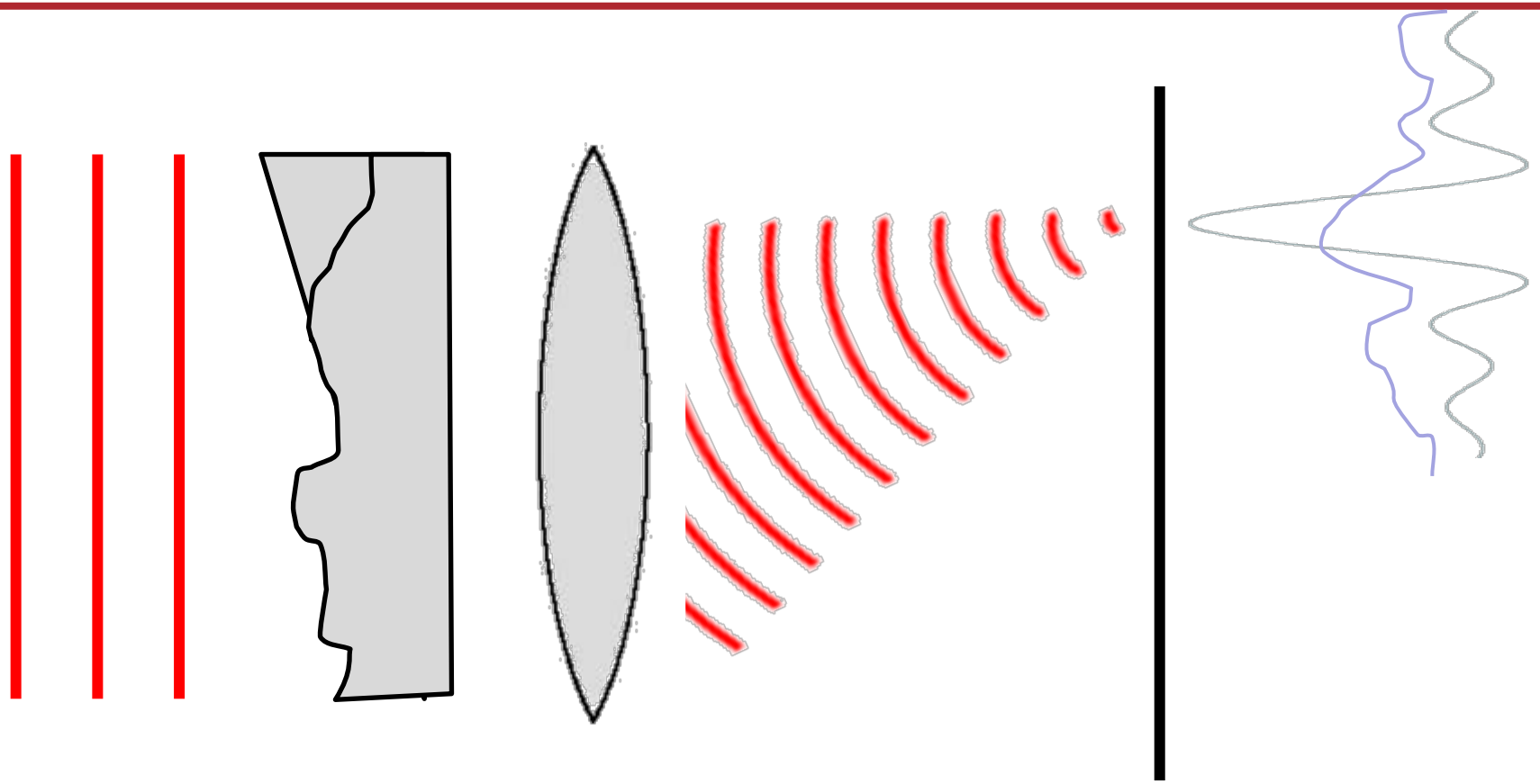
Optics



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