Stenotic Channel Flow using the Lattice Boltzmann Method

Alexandru Mihai
alexandru.mihai@oist.jp
Okinawa Institute of Science and Technology
Research Advisors: More Names, Hopefully Spelled Correctly

Abstract

The Lattice Boltzmann Method provides an accessible avenue to analyze and model complex fluid dynamics. We analyze the flow through a constricted channel in two dimensions using the LBGK collision model. There are currently no analytic solutions known for the flow through such a geometry. We investigated the transition period of Reynolds numbers where the flow is neither laminar nor turbulent as well as the boundary conditions necessary to produce a stable simulation. Initial results show that this transition period occurs for Reynolds numbers of approximately 1000 to 1500 and that velocity boundaries produce less numerical aberrations. Simulations were also conducted with the open source library OpenLB. Two dimensional as well as three dimensional results using the turbulent Smagorinski Turbulent Model were analyzed.

The Boltzmann Transport Equation

$$\frac{\partial f\left(x, \vec{u}, t\right)}{\partial t} + \vec{u} \cdot \nabla f\left(x, \vec{u}, t\right) = \Omega \tag{1}$$

where $f\left(x, \vec{u}, t\right)$ is the particle distribution function, $\vec{u}$ is the particle velocity, and Ω is the collision operator. We also know that collisions of particles tend to relax the particle distribution function. Thus we have that

$$\Omega = \frac{-1}{\tau}\left[f\left(x, \vec{u}, t\right) - f^{eq}\left(x, \vec{u}, t\right)\right]$$

where τ is the relaxation time and f^{eq} is the equilibrium distribution function. The equilibrium distribution is derived from the solution of the Maxwell Equations and the conservation of energy and momentum laws. From this we see that our continuous time equation is

$$\frac{\partial f\left(x, \vec{u}, t\right)}{\partial t} + \vec{u} \cdot \nabla f\left(x, \vec{u}, t\right) = -\frac{1}{\tau}\left[f\left(x, \vec{u}, t\right) - f^{eq}\left(x, \vec{u}, t\right)\right]$$

Following a simple space and time discretization the explicit Lattice Boltzmann Equation is found:

$$f_i\left(x + e_i \Delta t, t + \Delta t\right) + f_i\left(x, t\right) = \frac{-\left[f_i\left(x, t\right) - f_i^{eq}\left(x, t\right)\right]}{\tau} \tag{2}$$

The Particle Distribution Function

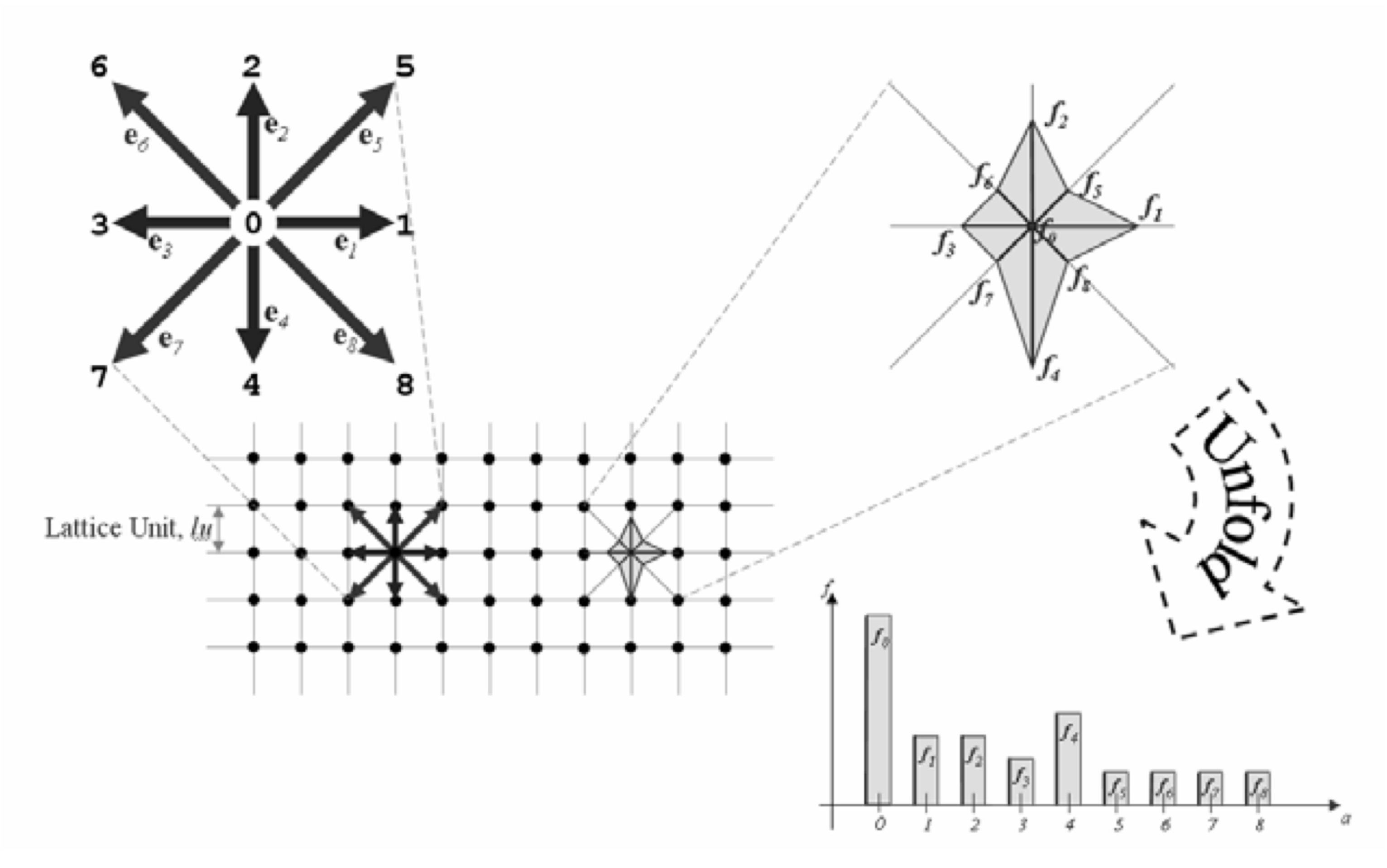

Fig. 1: Particle distributions of the Lattice Boltzmann Method. Source: Michael C. Sukop

Generalized Geometry

Fig. 2: General Geometries of Simulation (Left: America Heart Association, Second from Left: Aortic Segmentation from Fraunhofer MEVIS)

Implementation

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// Define Simulation Parameters

Conversion of Physical Units to Lattice Units

Initialize all active nodes to Equilibrium

Set fixed boundary conditions

for ( ti = 0; ti < tMax; ti++ )
{
        Set time dependent boundary conditions:
                Inlet— Velocity Boundary
                Outlet— Pressure Boundary

        BounceBack Conditions on given nodes

        Collision Step (RHS of Equation 2)
        Steaming Step (LHS of Equation 2)
}
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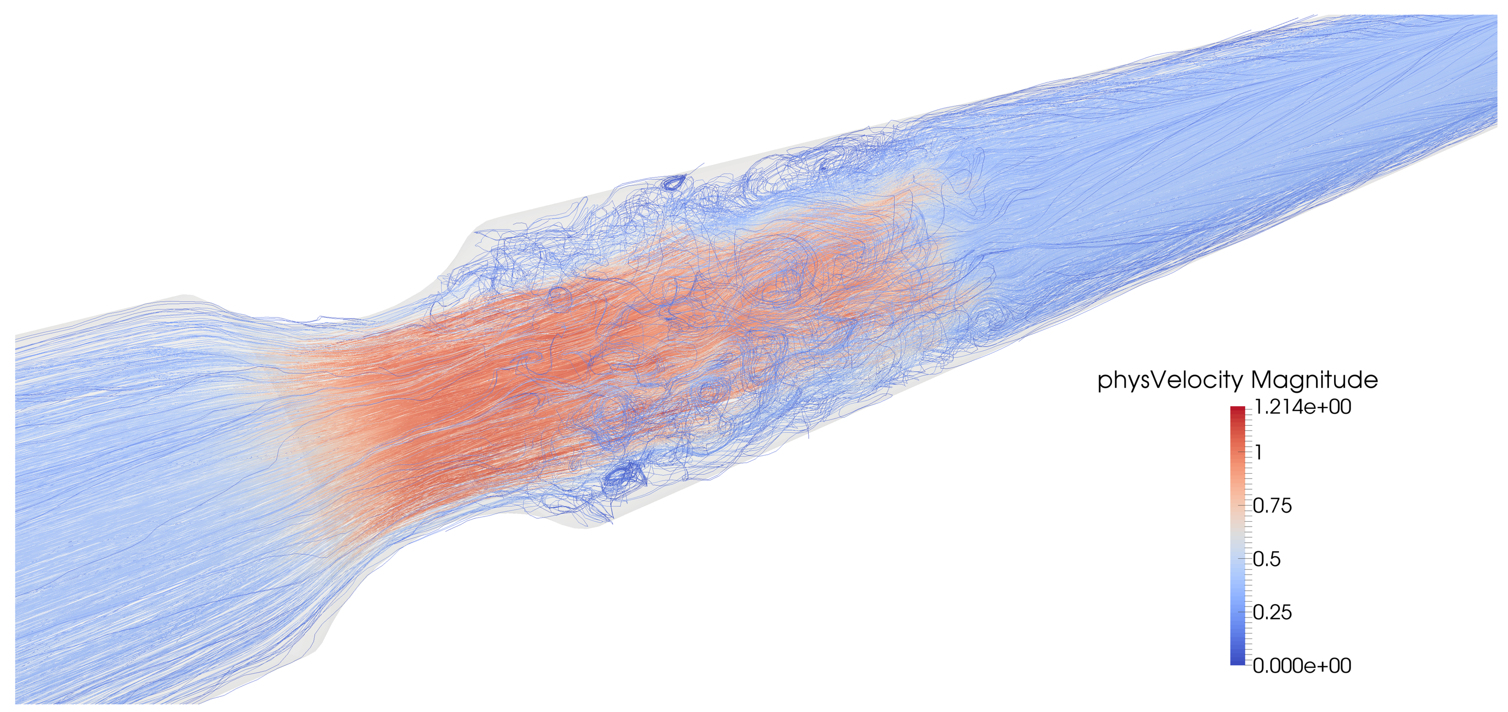
Macroscopic Quantities

From the distribution function f, we can easily extract the typical macroscopic quantities which are used to analyze flow dynamics. Namely Density/Pressure and Velocity.

$$\rho\left(x, t\right) = \sum_{i=0}^{8} f_i\left(x, t\right)$$

$$\vec{u}\left(x, t\right) = \frac{1}{\rho}\sum_{i=0}^{8} c_i \cdot f_i \cdot \vec{e}_i$$

Flow Through Idealized Stenosis using OpenLB



Results in Two Dimensions

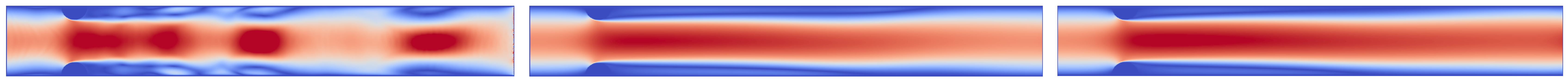


Fig. 3: Flow patterns with Pressure(Left), Velocity with equivalent inflow and outflow(Middle), and Velocity with outflow = 1.2·inflow (Right) boundary conditions

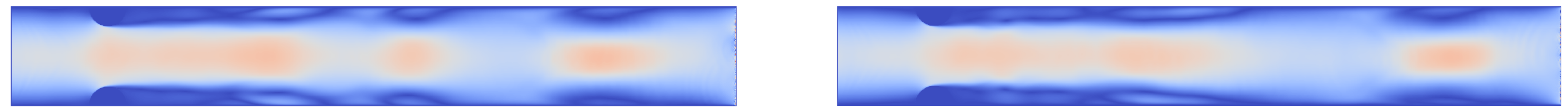


Fig. 4: Simulation of Reynolds Number 1000 geometry with Pressure Boundary and Smagorinsky constants 0.2(Left) and 0.1(Right)

Fig. 5: Flow with a 0 velocity outflow profile and Smagorinski constant of 0.2, demonstrating stabilization of flow modeled with turbulent aspects in mind

Future Research

Currently, the stability of the Zou-He[4] Pressure boundary conditions for a stenotic geometry, as well as the transition period of Reynolds Numbers where periodic flow is seen, are incomplete. As such more work needs to be done in the direction of classification of flow patterns with regards to Reynolds numbers as well as a thorough analysis to the boundary conditions needed in a geometry with increased outward velocity. Also as seen in the generalization of geometries above there exists a geometry, the arched stenosis, which has not yet been studied and may provide interesting results.

References

[1] M.-P.-I. fur Eisenforschung GmbH. Lattice Boltzmann Fluid Dynamics, 2008.

[2] S. Ianos Bernad, R. Susan-Resiga, and D. Bernad, Elenaand Gaita. 3d simulation of the blood flow in arterial stenosis. *Tohoku Mathematical*, 55:No. 2, 2005.

[3] Z. Stankovic, B. D. Allen, J. Garcia, K. B. Jarvis, and M. Markl. 4d flow imaging with mri. *Cardiovascular Diagnosis and Therapy*, 4(2), 2014.

[4] Q. Zou and X. He. On pressure and velocity boundary conditions for the lattice Boltzmann BGK model. *Phys. Fluids*, 9:1591–1598, 1997.