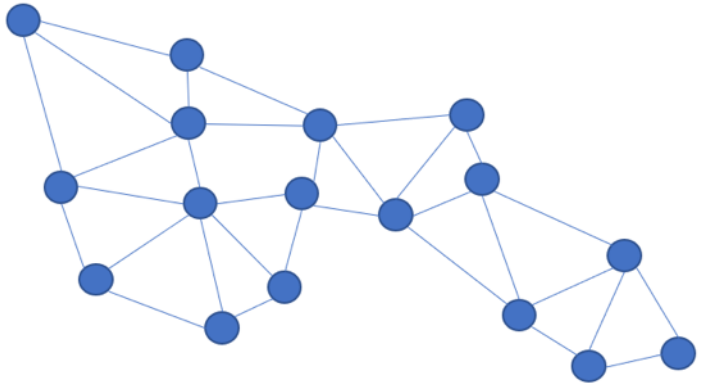
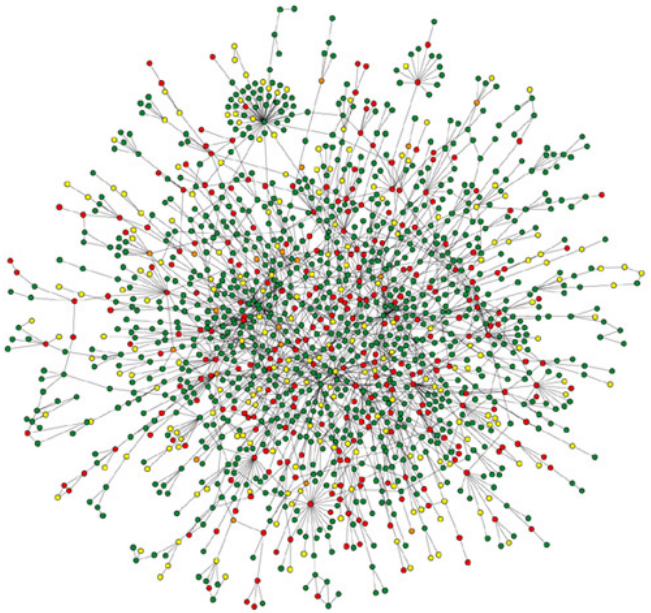
Mini course: Graph theory Introduction

2nd March 2021

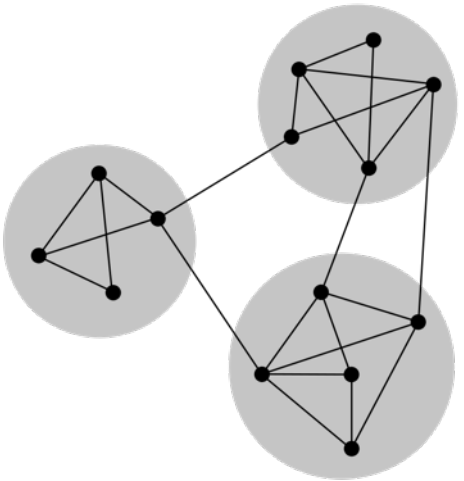
How do you characterize graphs? What to measure?



https://radiocrafts.com/why-is-multicasting-becoming-essential-for-mesh-networks/



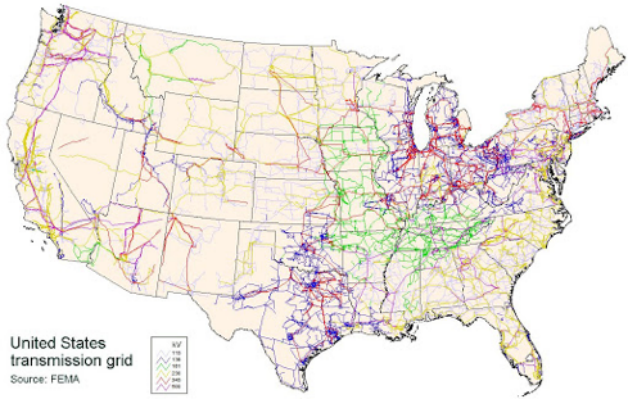
https://physics-complex-systems.fr/complex-networks.html



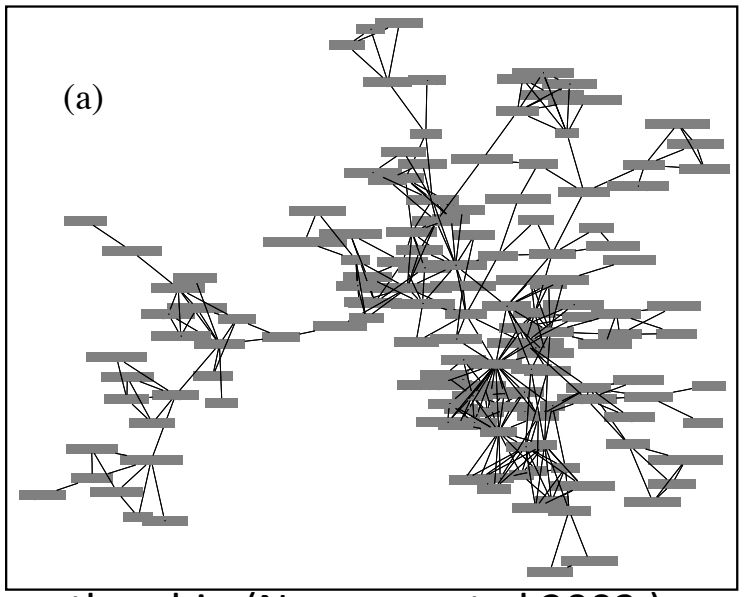
https://en.wikipedia.org/wiki/Connectivity_(graph_theory)

How do you characterize graphs? What to measure?

(Q. does network topology represent/affect dynamics?)



US electricity grid (Global Energy Network Institute)



Co-authorship (Newman et al 2003)

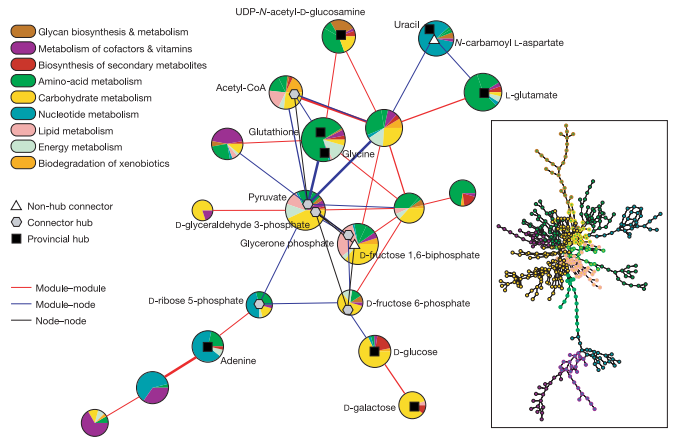
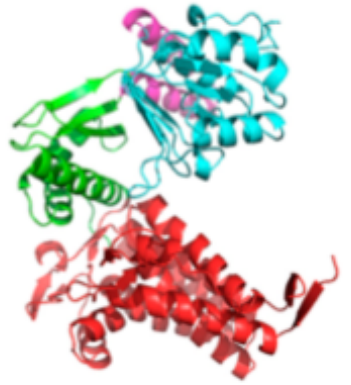


Figure 3 Cartographic representation of the metabolic network of *E. coli*. Each circle represents a module and is coloured according to the KEGG pathway classification of the metabolites it contains. Certain important nodes are depicted as triangles (non-hub connectors), hexagons (connector hubs) and squares (provincial hubs). Interactions between modules and nodes are depicted using lines, with thickness proportional to the number of actual links. Inset: metabolic network of *E. coli*, which contains 473 metabolites and 574 links. This representation was obtained using the program Pajek. Each node is coloured according to the 'main' colour of its module, as obtained from the cartographic representation.

Metabolic network of E. coli (Guimera and Amaral 2005)



Protein structure (Suzuki and Yura 2016)

Concepts and terminology

- Vertex (node), edge (link), loop
- Subgraph
- Degree
- Path
- Directed vs undirected graph
- Weighted graph
- Classical graph examples

What is a graph

- G = (V, E) where V: a set of vertices, E: a set of edges

- Vertex (node), edge (link), loop
- Subgraph

Degree (of a vertex)

- The number of edges linked to a vertex
- Degree distribution
- (Degree *centrality*)

Paths

A (finite or infinite) sequence of edges which joins a sequence of vertices which are (by most definitions) all distinct.

- Average shortest path length
- Diameter

Directed vs undirected graph

Weighted graph

Classical graph examples

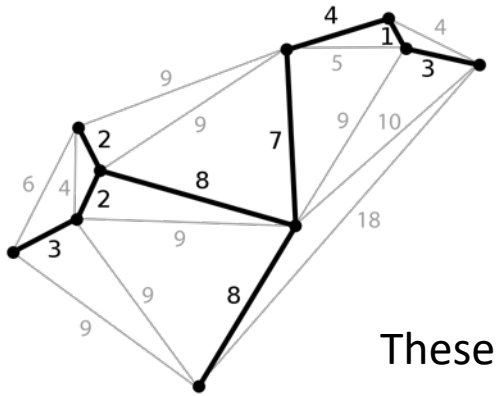
Examples

- Complete networks
- Random networks (Erdős–Rényi model); G(n,M) and G(n,p) versions
- Small-world networks (Watts-Strogatz model)
  e.g. social networks, neural system in *C. elegance*, electricity network in the US, collaboration network of Hollywood actors

- Scale-free network (Barabási–Albert model)
  e.g. WWW

A little more advanced concepts

- Trees and forests
  Tree: an undirected graph in which any two vertices are connected by exactly one path.
  Forest: an undirected graph in which any two vertices are connected by at most one path, or equivalently an acyclic undirected path.
- Minimum spanning tree

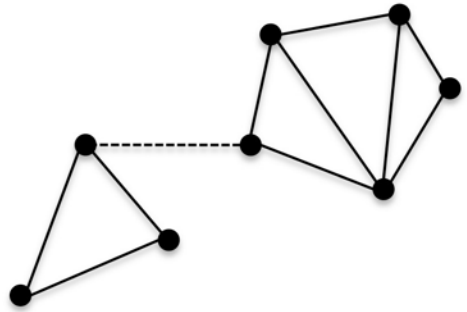


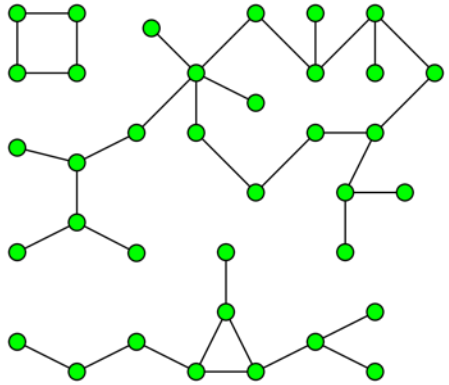
These edges, a subset of the edges of the connected graph, connect all the vertices.

https://en.wikipedia.org/wiki/Minimum_spanning_tree

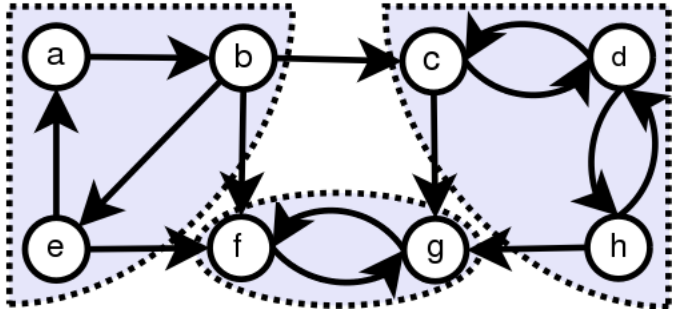
A little more

- Connectivity: an interest we may have in graph theory is whether nodes are **connected** or **disconnected**.





- Components
  (strongly, weakly connected components)



https://en.wikipedia.org/wiki/Connectivity_(graph_theory)

https://en.wikipedia.org/wiki/Strongly_connected_component
https://en.wikipedia.org/wiki/Component_(graph_theory)

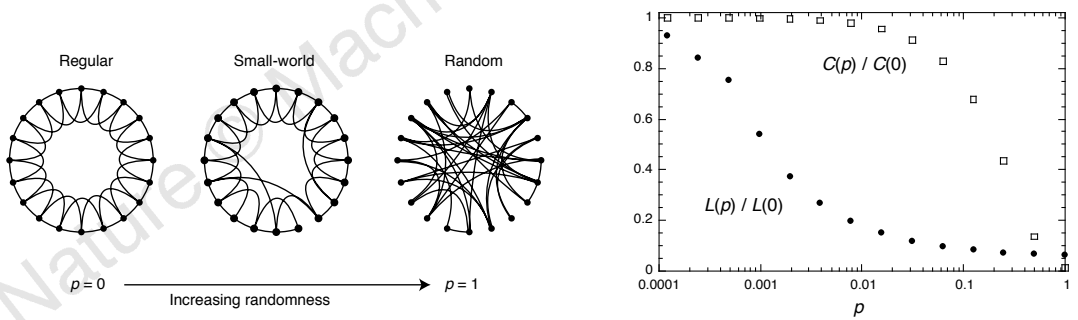
Link between dynamics and network topology

- In social networks (small-world-like networks), friends of friends are likely to be friends: High Transitivity → information diffusion is fast
- Metrics introduced:

$$\text{Transitivity} = 3\,\frac{\#\text{ of closed triplets}}{\#\text{ of all triplets (open and closed)}}$$

$$= \frac{\text{Your friends are friends}}{\#\text{ of a group of three people connected directly or indirectly}}$$

- Features in small-world networks
  - Average shortest path length
  - Transitivity (*clustering coefficient)



(Watts and Strogatz 1998 Nature)

Exercise (coding! Coding!!)

- https://colab.research.google.com/drive/1l0FZdYtdxJO4N6X-pJBH1cCRG9WGlwjL?usp=sharing

- Networkx (version 2.5) documentation https://networkx.org/documentation/stable/reference/index.html#
- Julia package LightGraphs https://juliagraphs.org/LightGraphs.jl/latest/
- R package igraph https://igraph.org/r/