Graph Gang DynamicsGraph gang session four : Dynamics

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OVERVIEW

- A quick digression to what dynamical systems are
- Components of a graph dynamical system
- Understanding the dynamics on a graph
- Symbolic dynamics, Partitioning and modeling state trajectories as directed graphs

Dynamical systems

- A dynamical system refers to any physical system whose behaviour can be described as a trajectory of time-dependent states in state-space.
- The state-space of a dynamical system refers to the space of all possible states that a system can achieve.
- The family of all possible state-trajectories of a dynamical system is known as the phase-portrait of the system.

Discrete and continuous dynamical systems

Discrete dynamical system are described by mappings of the form:

$$x_{n+1} = f(x_n)$$

Each subsequent state is uniquely determined by the previous state.

The Logistics Map:

$$x_{n+1} = r x_n (1 - x_n)$$

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Psst…for a cooler example look up the cat map :3

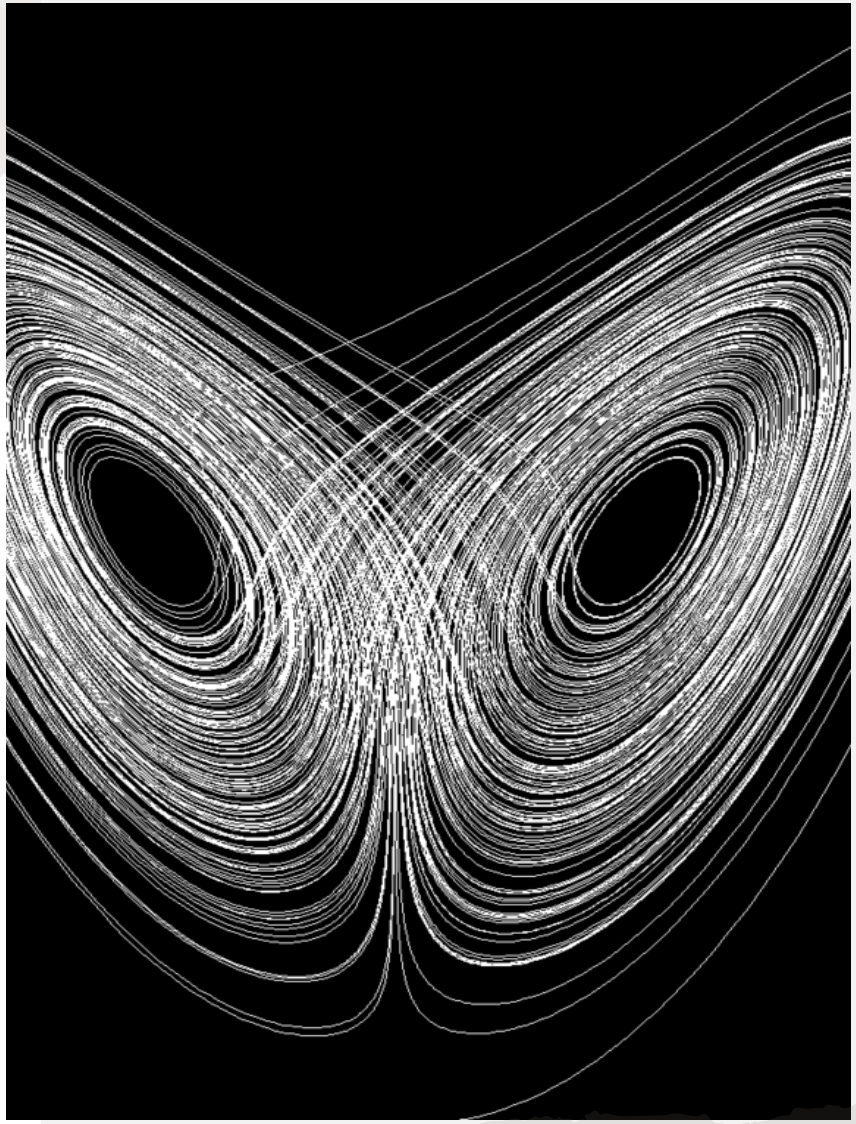
Discrete and continuous dynamical systems

A continuous dynamical system is generally described by:

$$\frac{dx}{dt} = F(x)$$

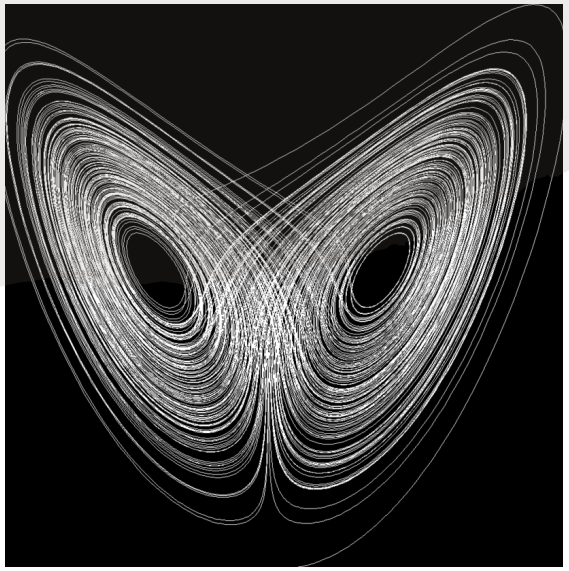
The Lorenz system:

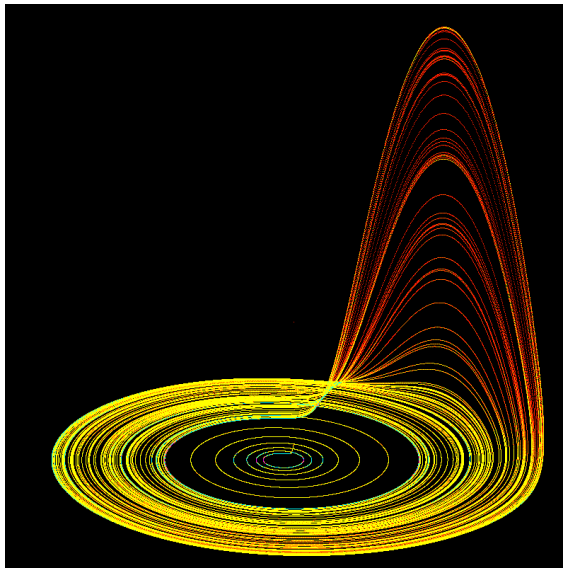
$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = x(\rho - z) - y$$
$$\frac{dz}{dt} = xy - \beta z$$

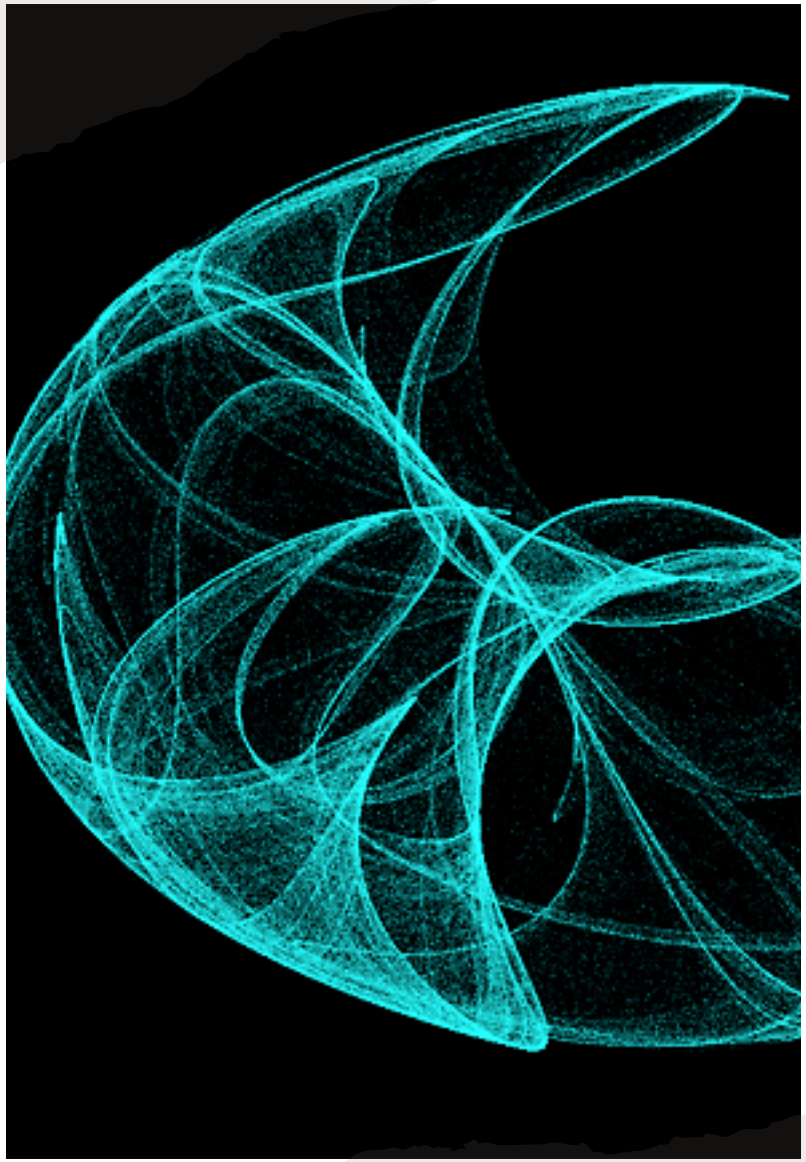


Attractor states

- An attractor state is a state or set of states towards which a system tends to evolve.







Graph Dynamical Systems

A dynamical system with a graph representation.

It consists of:

- A graph with vertices and edges
- States that each node can take
- A vertex function
- An update scheme

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- Sequential dynamical systems

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Symbolic dynamics and modeling state transitions as graphs

(CHALK BREAK)

So why do we care?

- Phase transitions!
- Stability of flow
- Criticality