>Panorama A

>Non-perturbative aspects

Plan

Plan

- D-brane instantons

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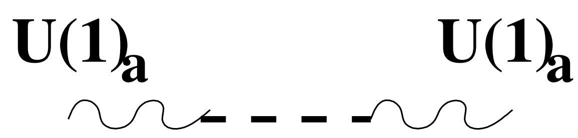
- D-brane instantons
- Discrete gauge symmetries

Flash back

Tadpoles, anomalies and all that

- Due to BF couplings, all ‘anomalous’ and some ‘non-anomalous’ U(1)’s become massive, with mass of order the string scale

$$\sum_{k,a} \int_{4d} B_k \wedge \mathrm{tr}\, F_a = - \sum_{k,a} \int_{4d} \partial_\mu a_k \, A_\mu^a$$



$$= m^2 A_\mu^2$$

- Consequences

  - Impose that hypercharge generator remains massless

  - Additional U(1)’s removed
    remain as global symmetries exact in perturbation theory

  - Operators violating the latter can appear non-perturbatively
    D-brane instantons, see later

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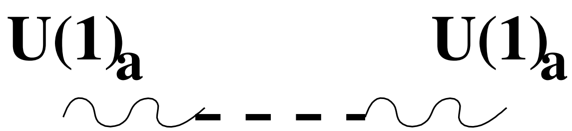

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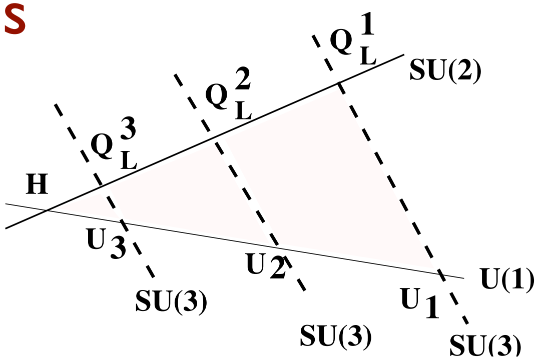
Couplings

- Arise from worldsheet instantons

  e.g. SM Yukawa couplings



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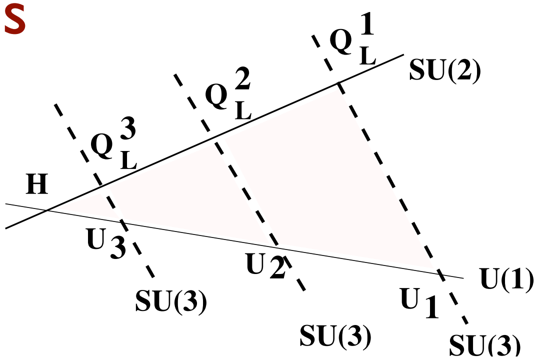

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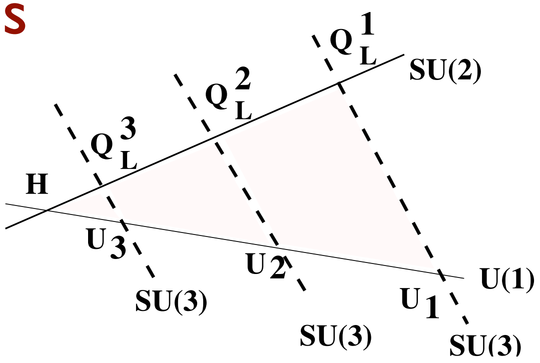

- Preserve the global symmetries associated to massive U(1)'s
- Useful to avoid e.g. proton decay....
- But prevent some phenomenologically interesting couplings

  - Right-handed neutrino masses

  - top quark Yukawa in SU(5) GUTs

  - ...

Neutrino masses

- Tiny neutrino masses are nicely explained by the see-saw mechanism, which requires right-handed neutrinos with Majorana and Dirac masses

$$\lambda L H \nu_R + M \nu_R \nu_R \longrightarrow M_\nu \simeq \frac{\lambda^2 \langle H \rangle^2}{M}$$

with $M_D \simeq$ lepton mass, need $M_M \simeq 10^{11} - 10^{13} \mathrm{GeV}$

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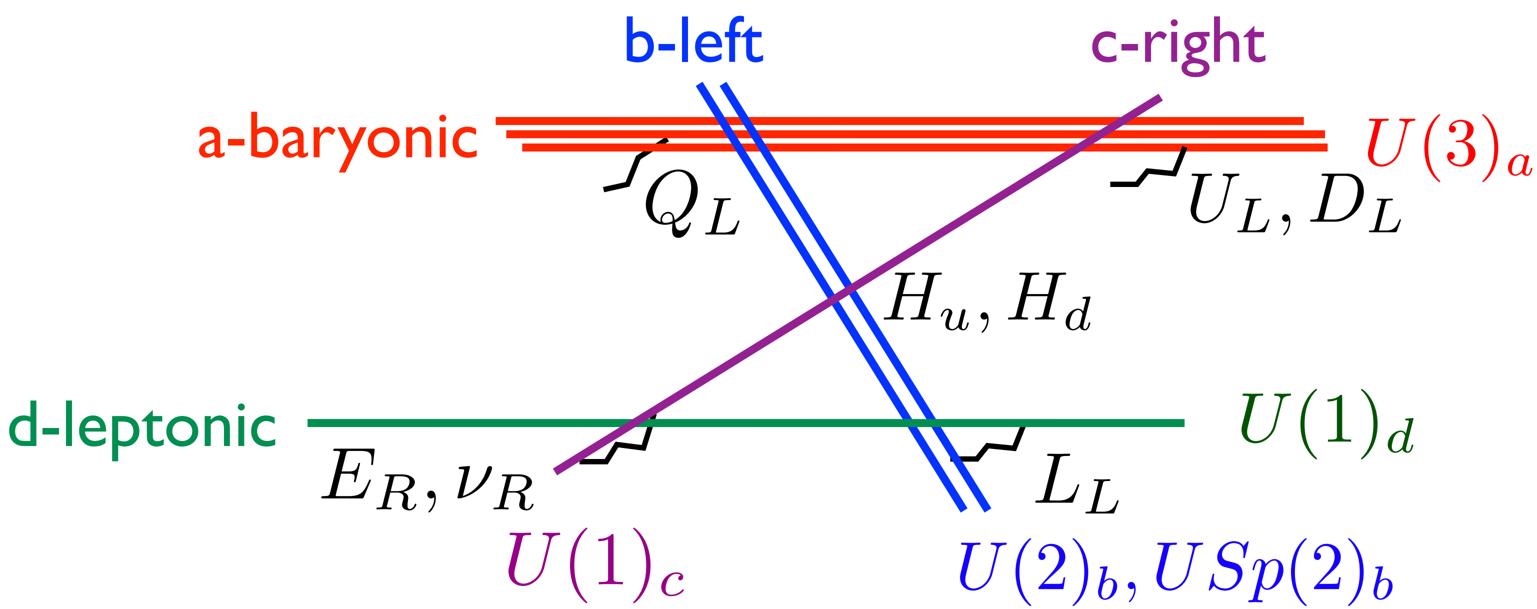

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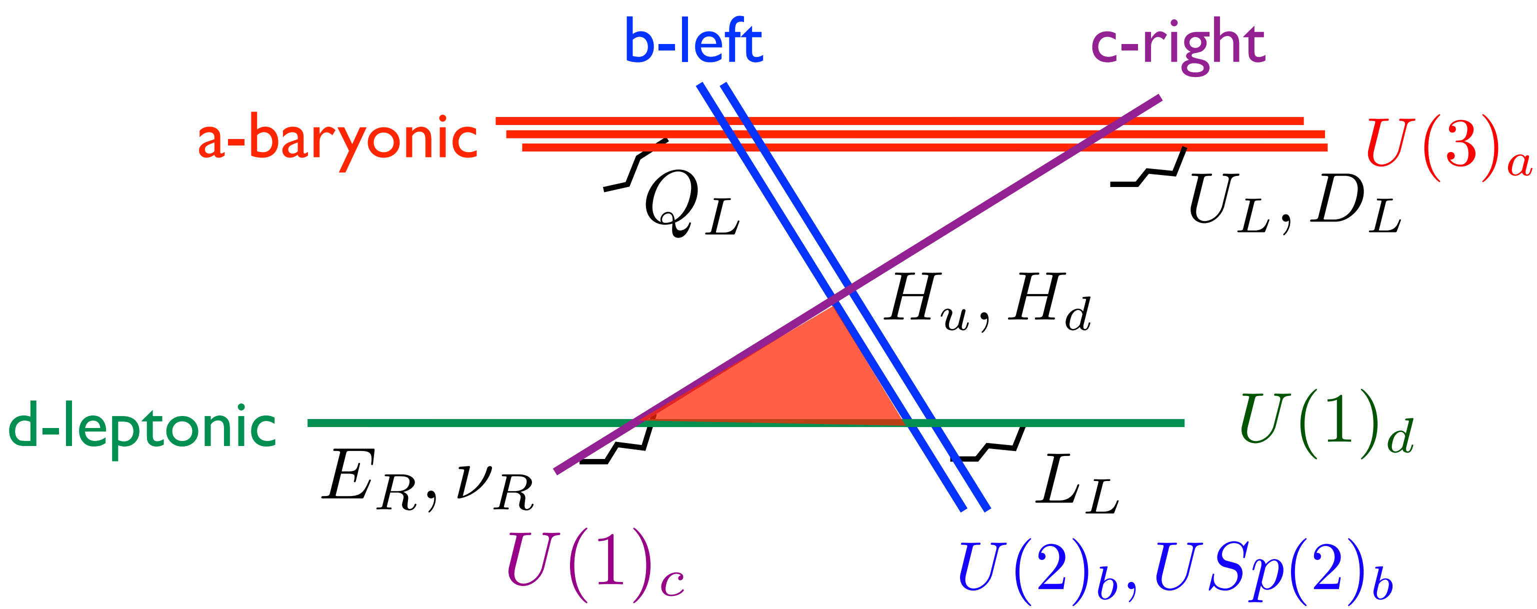

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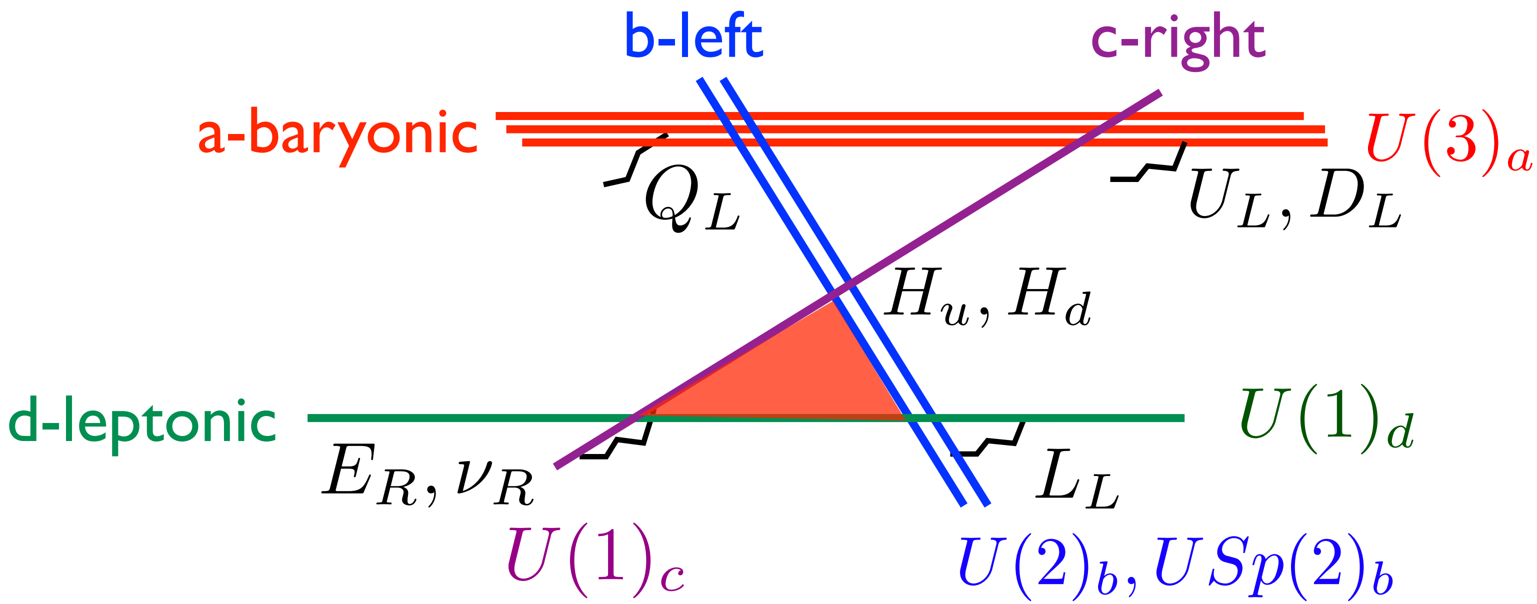

Towards the SM

- Explicit realization in toroidal models

N_α	(n^1_α, m^1_α)	(n^2_α, m^2_α)	(n^3_α, m^3_α)
$N_a = 3$	(1,0)	(1,3)	(1,-3)
$N_b = 1$	(0,1)	(1,0)	(0,-1)
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Flash back

$I_{ab} = I_{ab'} = 3 \qquad I_{ac} = I_{ac'} = -3$

$I_{db} = I_{db'} = -3 \qquad I_{cd} = 3; I_{cd'} = -3$

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Intersection	Matter fields		Q_a	Q_c	Q_d	Y	Y'	$3Q_a - Q_d$
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ac	U_R	$3(\bar{3},1)$	-1	1	0	-2/3	2/3	-3
ac'	D_R	$3(\bar{3},1)$	-1	-1	0	1/3	-4/3	-3
$bd, b'd$	L	$3(1,2)$	0	0	-1	-1/2	-1	1
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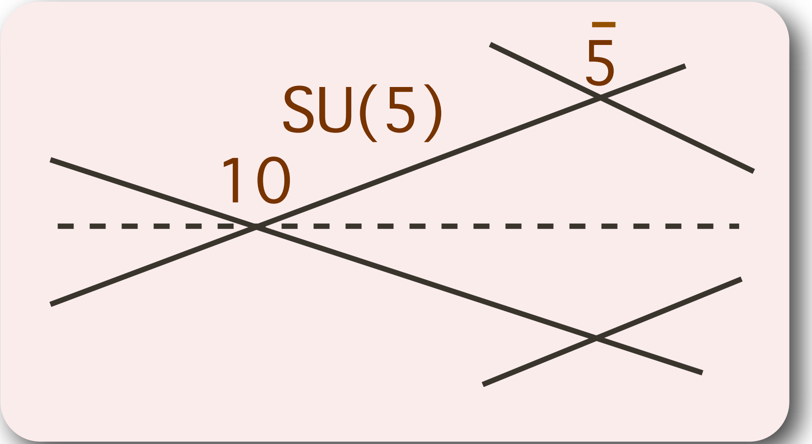
Majorana mass terms NOT allowed

Top Yukawa problem

- Possible to build SU(5) unifications models

  - 5 overlapping Dbranes
  - Intersection with image gives 10
  - Intersection with extra brane gives $5, \bar{5}$



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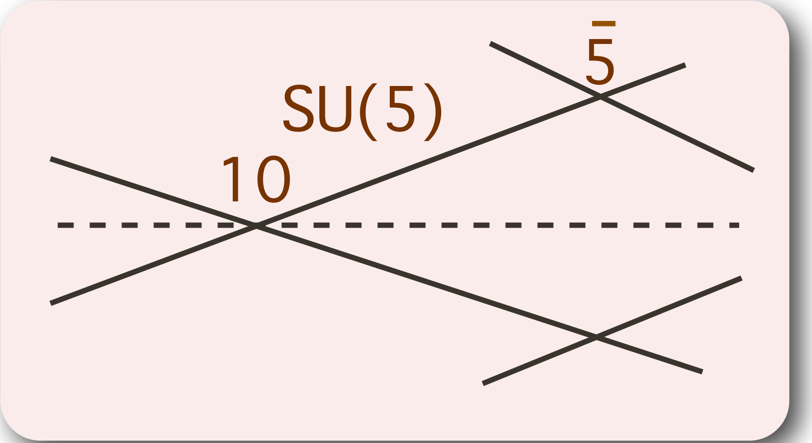

- But problems with top Yukawa coupling ⇒ U(1) global symmetries

  5 has U(1) charge +1 , $\bar{5}$ has charge -1 , 10 has charge +2

  $10 \,.\, \bar{5} \,.\, \bar{5}$ is allowed by U(1) symmetry

  $10 \,.\, 10 \,.\, 5$ is NOT allowed by U(1) symmetry

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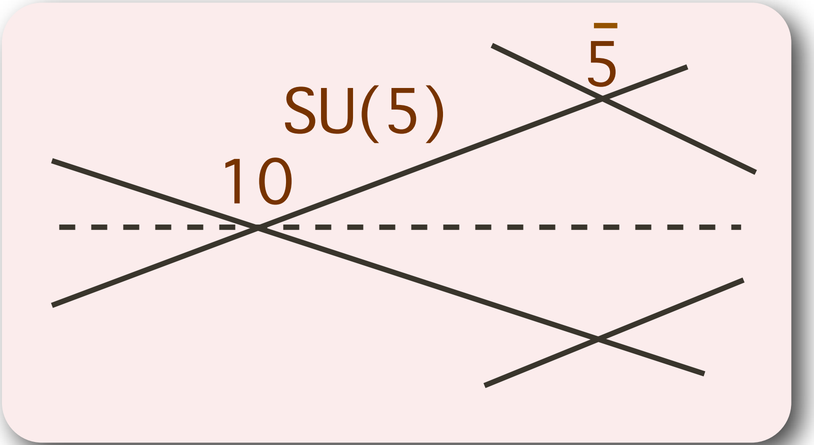

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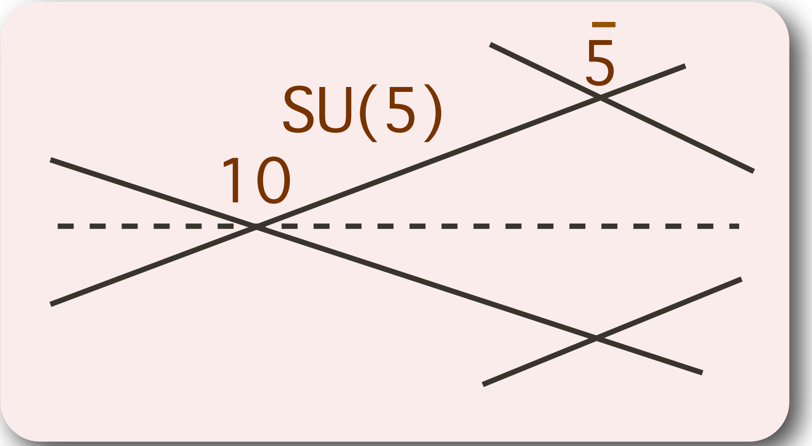

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- Geometrically, no triangles bounding worldsheet instantons
- Need to go non-perturbative...

D-brane instantons

Non-perturbative effects in string theory are relevant in a number of phenomenological model building issues

- Moduli stabilization (see lecture 4)
- Generation of perturbatively forbidden couplings

Neutrino masses, Yukawas, mu term, ...

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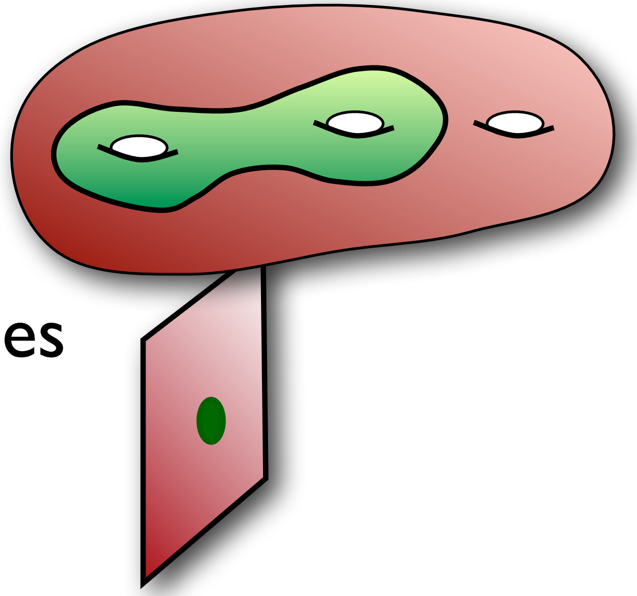
Focus on type II (or orientifolds):
D-brane instantons

Euclidean D-branes wrapped on CY cycles and localized on 4d Minkowski



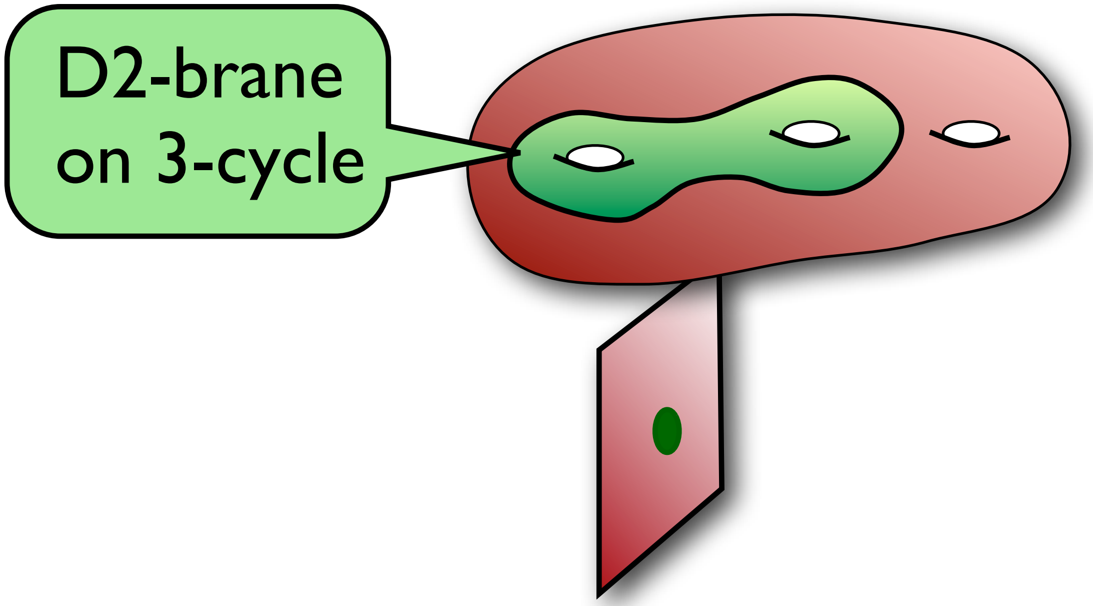
D-brane instantons

Violate certain perturbatively exact U(1) global symmetries

- Ex: Take one complex structure modulus in IIA CY orientifold

$$T = t + i\,a = \int_C \mathrm{Re}\,\Omega + i \int_C C_3$$

PQ symmetry $a \to a + \lambda$ violated by D2-brane instanton $\simeq e^{-T}$



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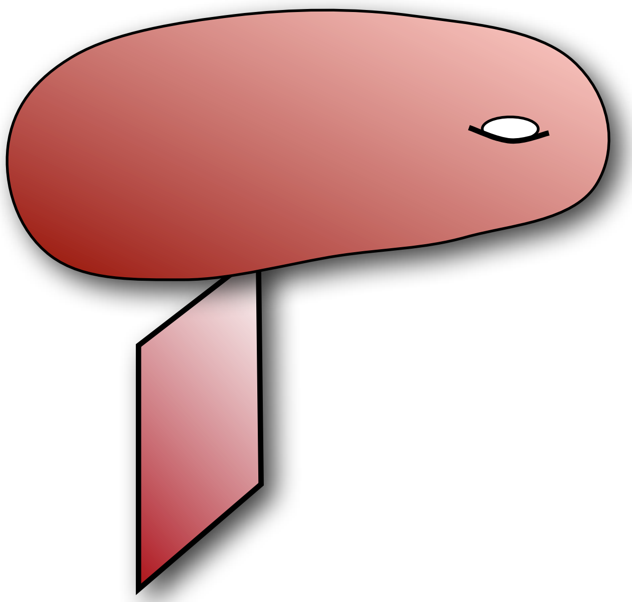

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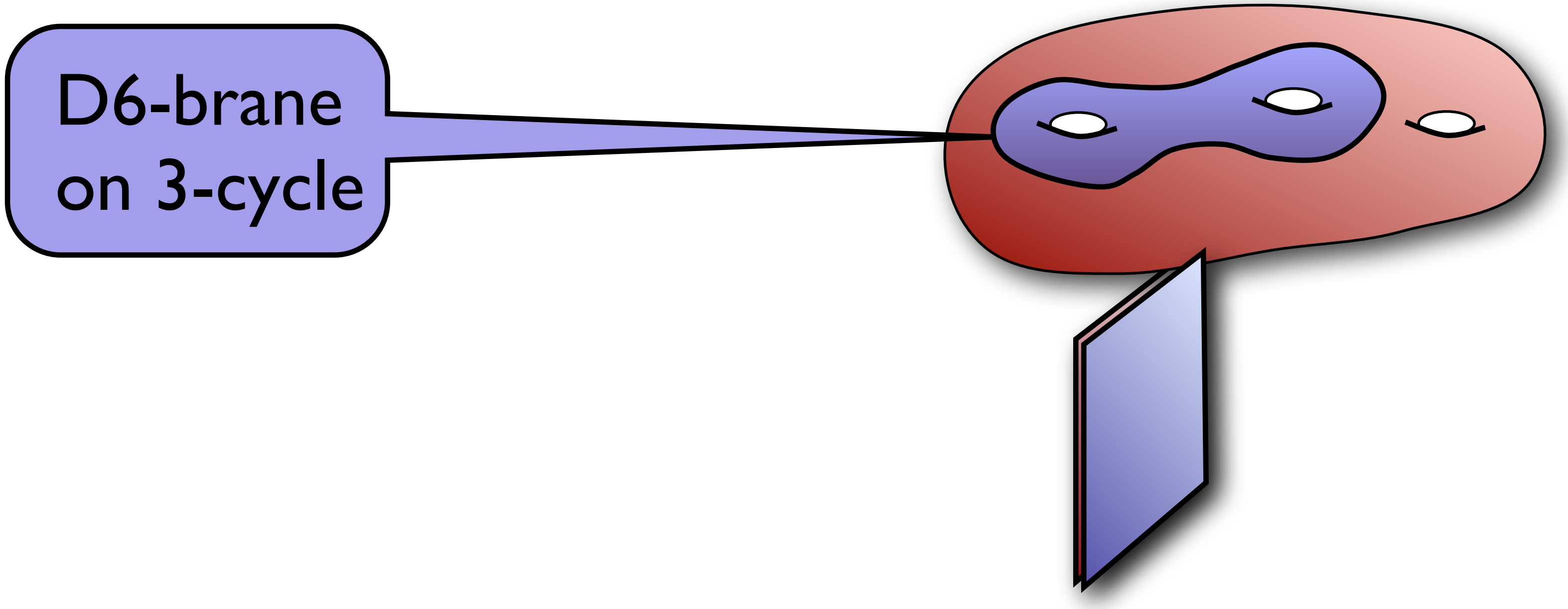

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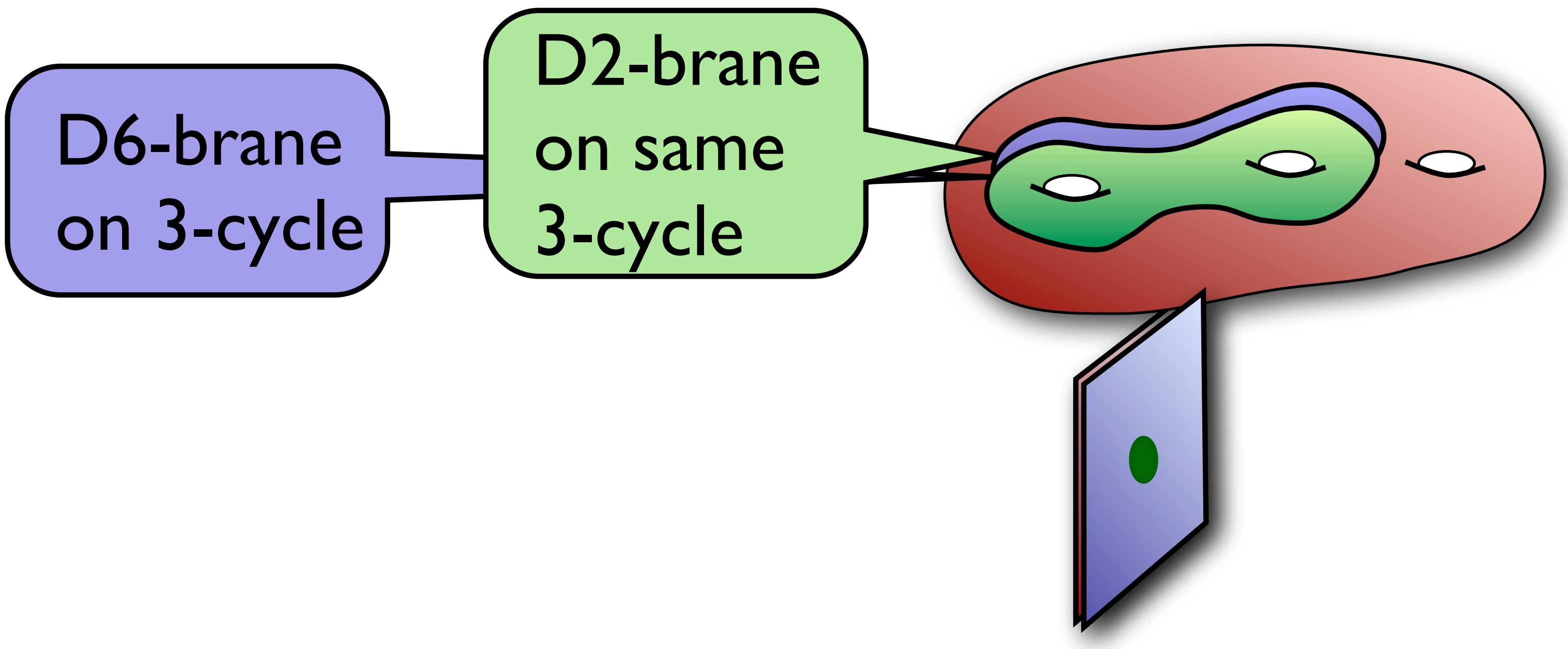

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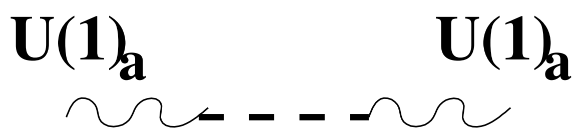

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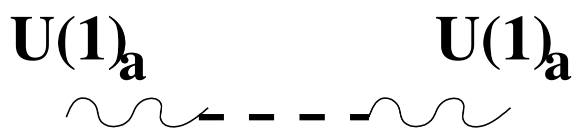

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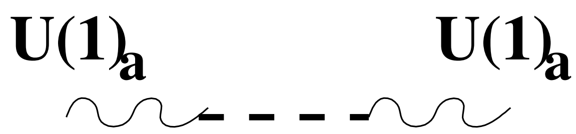

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⇒ Exponential must be dressed by charged field $\quad e^{-T}\,\Phi$

⇒ Role in generating perturbatively forbidden couplings

$$e^{-T}\,\Phi_1 \ldots \Phi_n$$

D-brane instantons

- Microscopic explanation

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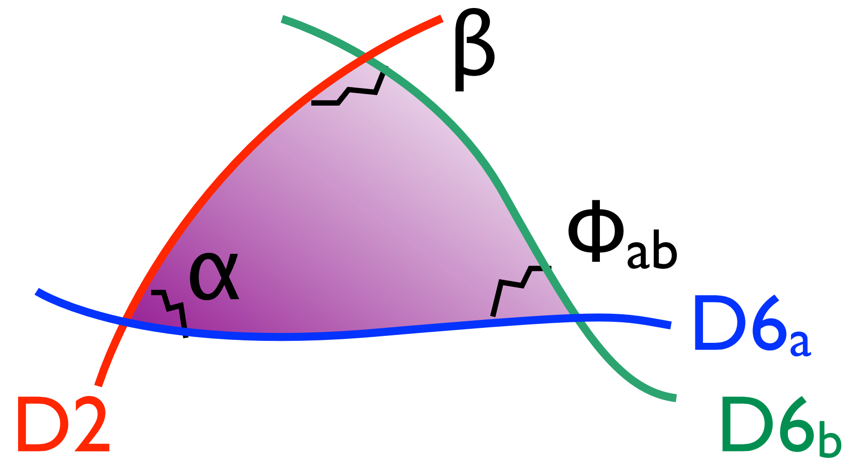
  ⇒ D-brane instanton has charged fermion zero modes

  ⇒ Arise from D2/D6 intersections



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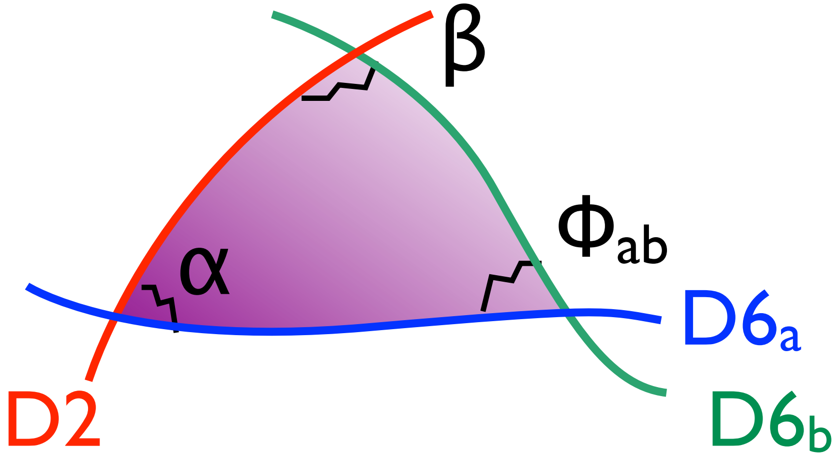

- Integration over instanton fermion zero modes leads to insertions of charged 4d fields

$$\int d\alpha \, d\beta \, \exp\left(- S_{\rm inst} + \alpha \Phi \beta \right) \sim e^{-S_{\rm inst}} \det \phi$$

Degree of "det" is number of intersections

D-brane instantons

- Microscopic explanation

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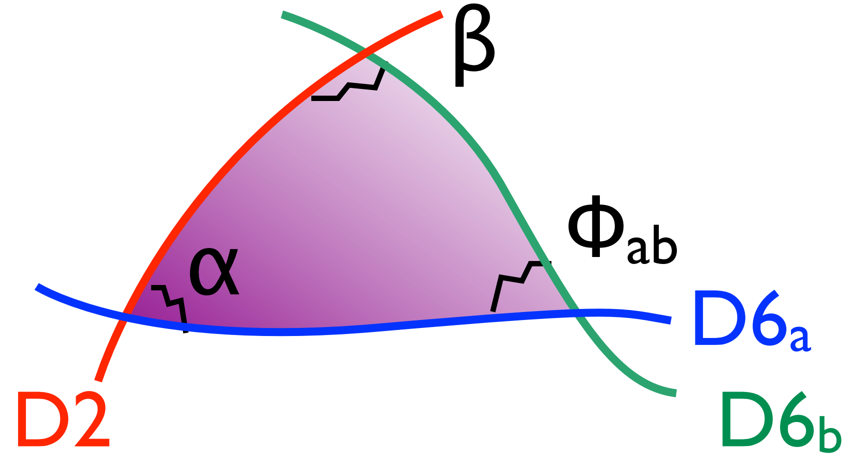
  ⇒ Arise from D2/D6 intersections

(different from the familiar
2 neutral fermion zero modes
required for the instanton to
induce a superpotential term)



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Flash back

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4d couplings $N_a q_{ak} \int_{4d} (B_2)_k \text{tr}\, F_a$ with $q_{ak} = [\Pi_a] \cdot [\Lambda_k]$

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⇒ For a D2-brane instanton on $[\Pi] = \sum_k p_k [\Lambda_{\tilde{k}}]$

$$W \sim \exp\left(- S_{\text{inst}} - i \sum_k p_k \phi_k \right) \to W \exp\left(- i \sum_k p_k I_{a,\Pi} \lambda_a \right)$$

- Phase rotation of exponential cancels against operator charge

Neutrino masses

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- Right handed neutrinos are in cd’ sector

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N_α	(n_α^1, m_α^1)	(n_α^2, m_α^2)	(n_α^3, m_α^3)
$N_a = 3$	(1,0)	(1,3)	(1,-3)
$N_b = 1$	(0,1)	(1,0)	(0,-1)
$N_c = 1$	(0,1)	(0,-1)	(1,0)
$N_d = 1$	(1,0)	(1,3)	(1,-3)

$$I_{ab} = I_{ab'} = 3 \qquad I_{ac} = I_{ac'} = -3$$

$$I_{db} = I_{db'} = -3 \qquad I_{cd} = 3; I_{cd'} = -3$$

Intersection	Matter fields		Q_a	Q_c	Q_d	Y	Y'	$3Q_a - Q_d$
ab, ab'	Q_L	$3(3,2)$	1	0	0	1/6	1/3	3
ac	U_R	$3(\bar{3},1)$	-1	1	0	-2/3	2/3	-3
ac'	D_R	$3(\bar{3},1)$	-1	-1	0	1/3	-4/3	-3
$bd, b'd$	L	$3(1,2)$	0	0	-1	-1/2	-1	1
cd	E_R	$3(1,1)$	0	-1	1	1	0	-1
cd'	ν_R	$3(1,1)$	0	1	1	0	2	-1

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  - Two uncharged fermion zero modes
  - Charged fermion zero modes with the pattern

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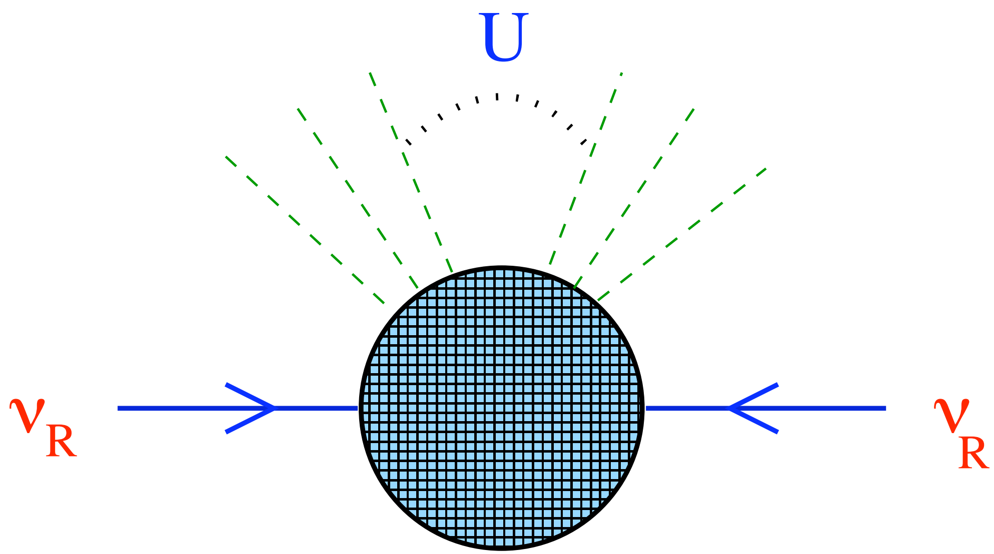

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$$\int d^2\theta \, d^2\alpha \, d^2\beta \exp(-S_{\rm inst} + \alpha_1 \nu_R \beta_1 + \alpha_2 \nu_R \beta_2) \sim e^{-S_{\rm inst}} \int d^2\theta \nu_R \nu_R$$

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  - Exponential suppresion wrt string scale may be ok i.e. may have 10^{11} Gev << Ms

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  - The top Yukawa is better understood in F-theory, see lecture 3
  - Can we try a non-perturbative geometrization in IIA?

M-theory on G2

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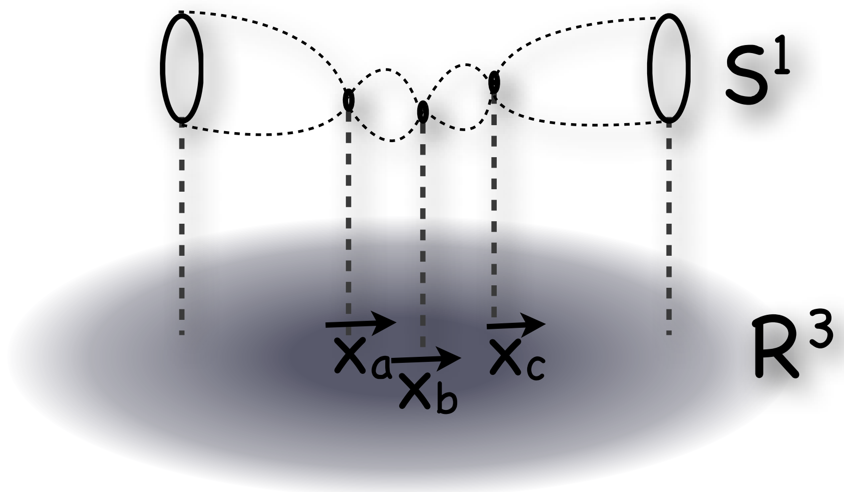
- Lift of IIA D6-brane to M-theory is purely geometrical: Taub-NUT space

    Circle fibration over 3d space with fiber shrinkint over "centers"



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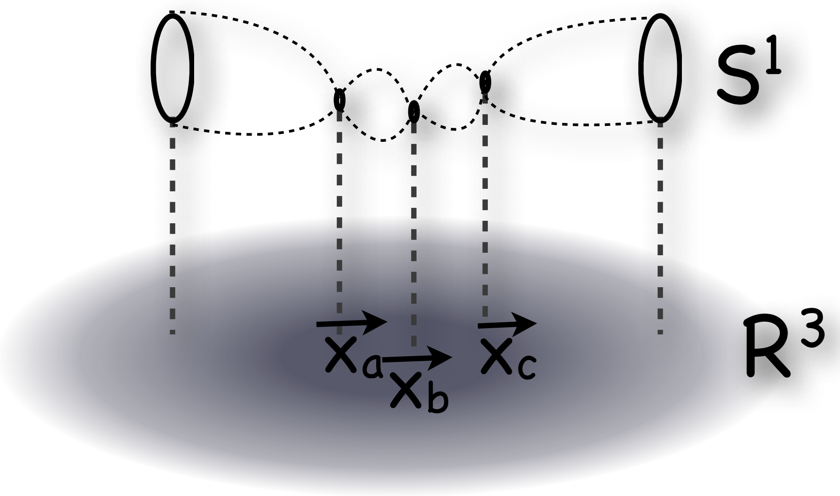

- Dictionary

  D6-brane positions $\Longleftrightarrow$ Taub-NUT centers

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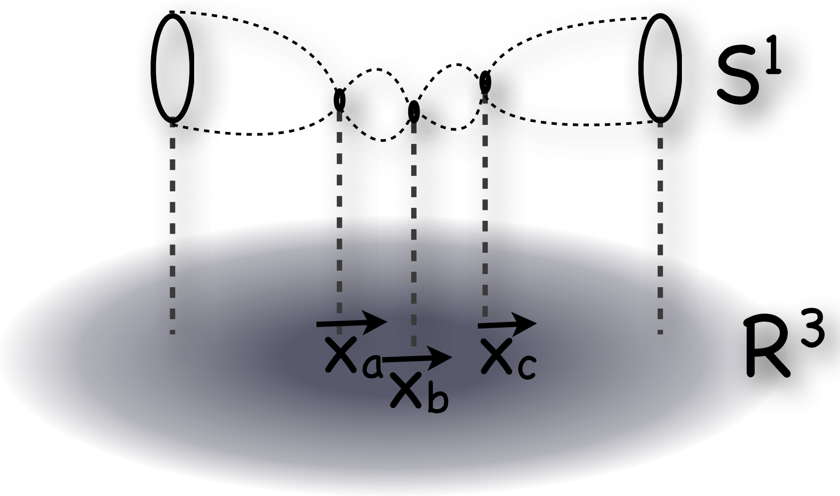

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(O6 lifts to Atiyah-Hitchin space)

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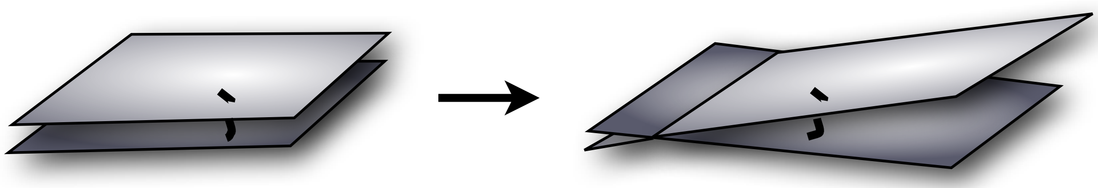
Gauge symmetry from codimension-4 singularity (singular TN)
Chiral fermions from codimension-7 singularity (intersection of TN)

Representation determined by 'unfolding' of degenerate fiber locus



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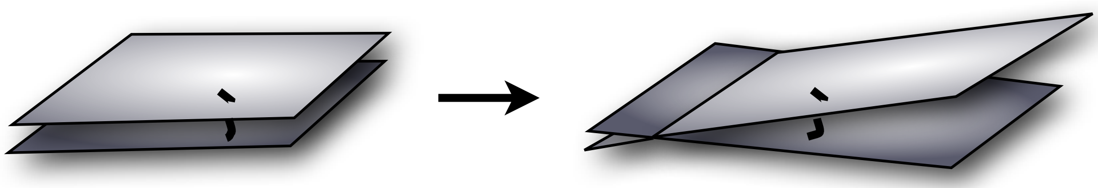

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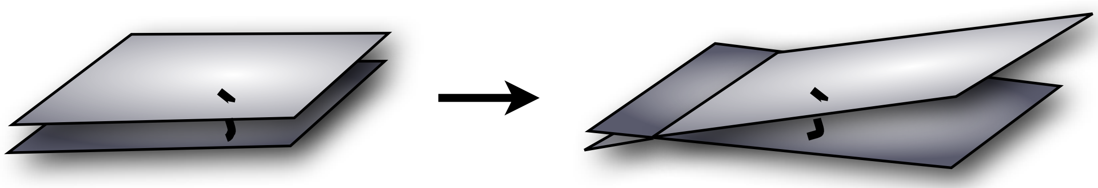

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But similar ideas hold in a tractable way
in F-theory, see lecture 3

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Discrete gauge symmetries

- Let us stay a bit more on D-brane models & U(1)s

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- We said "U(1)s disappear as gauge symmetries but remain as global symmetries, exact in perturbation theory"

- Actually, a discrete subgroup of the U(1)s can remain as discrete gauge symmetry, thus exact even non-perturbatively.

Why discrete gauge symmetries?

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• For instance in the MSSM, the most general supo, up to dim 4

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\begin{aligned}
W_{\rm MSSM} \;\; &= \;\; Y_U^{ij}\, Q_i U_j H_u \,+\, Y_D^{ij}\, Q_i D_j H_d \,+\, Y_L^{ij}\, L_i E_j H_d \,+\, \mu\, H_u H_d \,+ \\
&+ \;\; \lambda^{ijk}\, U_i D_j D_k \,+\, \lambda^{ijk\prime}\, Q_i D_j L_k \,+\, \lambda^{ijk\prime\prime}\, L_i L_j E_k \,+\, \mu_R^i\, L_i H_u
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Ops in second line can combine and induce fast proton decay

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Ops in second line can combine and induce fast proton decay

Introduce discrete symmetries that forbid

- all such ops, e.g. R-parity (Q, D, L, E odd, Hu, Hd even)
- dangerous combinations, e.g. baryon triality B_3 (Q,U, D, L, E, Hu, Hd have charge 0,-1/3,+1/3, -1/3, +1/3,+1/3,-1/3)

Experimental signatures (at the LHC) depend on the symmetry!

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- Quantum gravity does not like global symmetries
  - Microscopic arguments in string theory
  - General black hole arguments

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  - Also true for discrete gauge symmetries:
    lasso black hole with charged strings and measure holonomy

Zn gauge symmetry

- Realize Z_n as "U(1) Higgssed by field of charge n"

  Lagrangian for gauge field and phase of scalar field

$$|d\phi - nA|^2 + \frac{1}{2} F \wedge *F$$

  i.e. gauge transformation is

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- Can be dualized to BF theory

schematically

$$\frac{1}{2} H \wedge *H \ + nB \wedge F \ + \frac{1}{2} F \wedge *F$$

Zn symmetry read from coefficient of BF coupling

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- Dualizing also the gauge field

schematically

$$|d\tilde{A}_1 - nB_2|^2 + \frac{1}{2} H \wedge * H$$

Dual gauge invariance

$$B_2 \to B_2 + d\Lambda_1 \quad ; \quad \tilde{A}_1 \to \tilde{A}_1 + n\Lambda_1$$

Zn gauge symmetry

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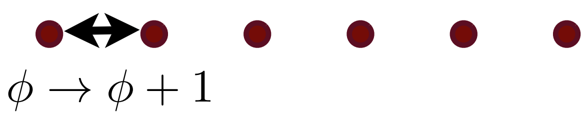
Periodic axion $\phi \simeq \phi + 1$ defines lattice Γ



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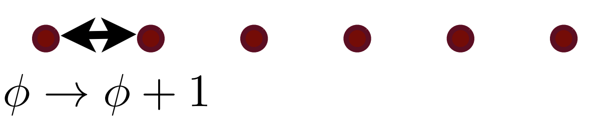

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Embedding of U(1) S^1 into axion S^1 with winding n

Defines lattice Γ’=nZ

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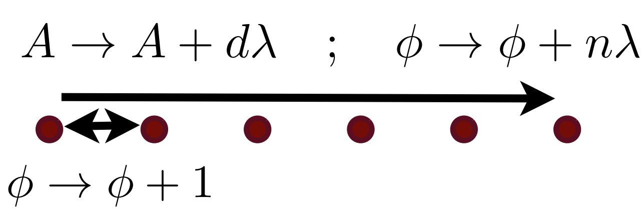

Discrete gauge symmetry: field identifications not implemented by U(1) gauge transformations

$$\frac{\Gamma}{\Gamma'} = \mathbf{Z}_n$$

Generalizes to multiple U(1), non-abelian, etc

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- Charged particles and strings (conserved mod n)

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  - Particles couple to A_1, and end on instanton
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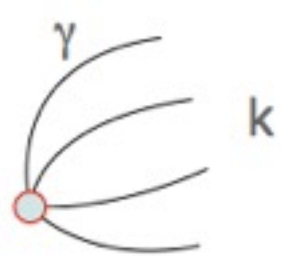
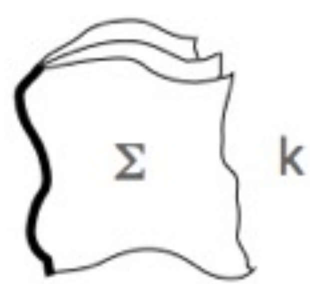
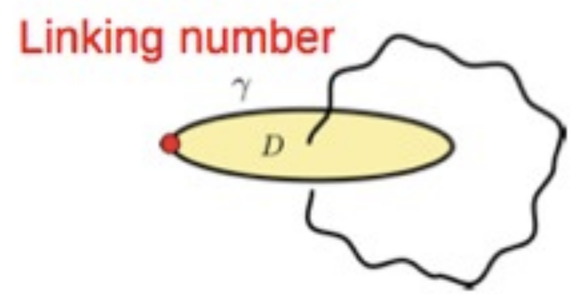
$$\mathcal{O}_{\text{particle}} \sim e^{2\pi i n \int_\gamma A} \qquad \mathcal{O}_{\text{string}} \sim e^{2\pi i m \int_\Sigma B_2}$$







$$e^{-2\pi i \phi} e^{2\pi i k \int_\gamma A} \qquad e^{-2\pi i \int_{\partial\Sigma} A} e^{2\pi i k \int_\Sigma B_2} \qquad \exp\left[2\pi i \frac{nm}{k} L(\Sigma,\gamma)\right]$$

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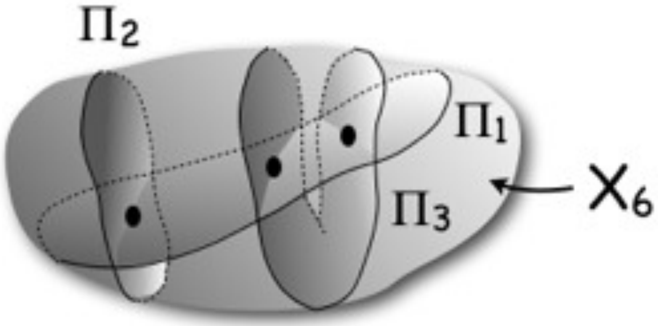

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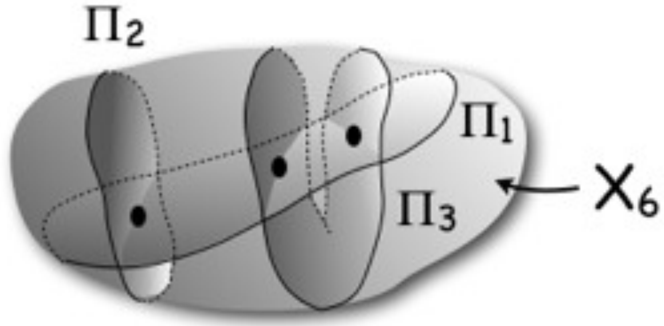

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Flash back

$$(B_2)_k = \int_{[\Lambda_k]} C_5 \quad , \quad \phi_{\tilde{l}} = \int_{[\Lambda_{\tilde{l}}]} C_3 \qquad \text{with } d\phi_{\tilde{l}} = -\delta_{k\tilde{l}} *_{4d} (B_2)_k$$

4d couplings $N_a q_{ak} \int_{4d} (B_2)_k \mathrm{tr}\, F_a$ with $q_{ak} = [\Pi_a] \cdot [\Lambda_k]$

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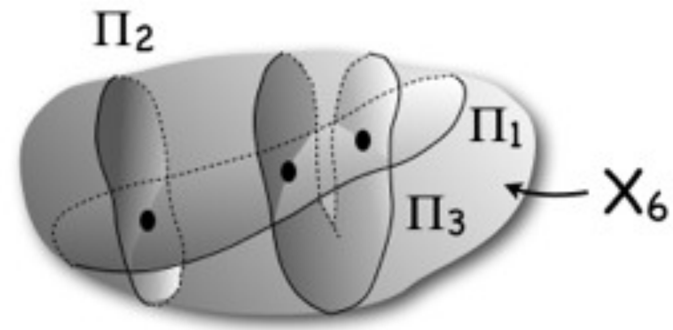

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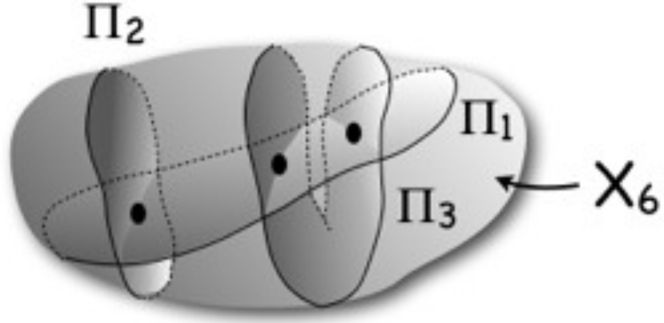

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$$\sum_A c_A N_A q_{ak} = 0 \text{ mod } n \,, \text{ for all } k$$

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- Instantons from euclidean D2-branes wrapped on 3-cycles [Π], violate U(1)'s

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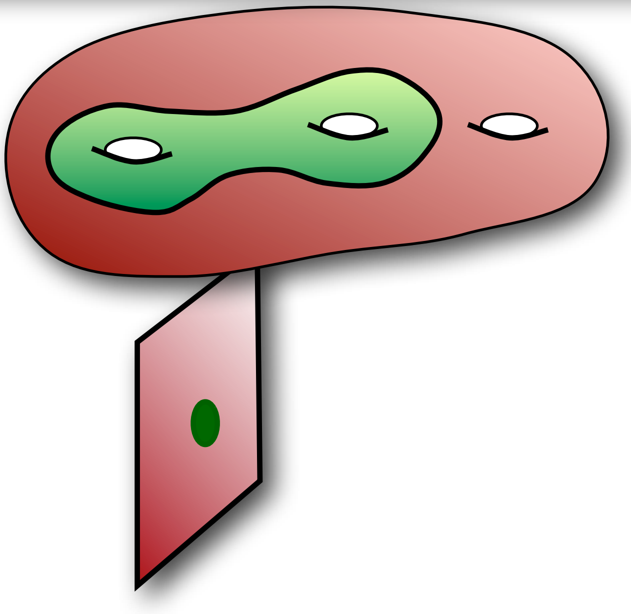

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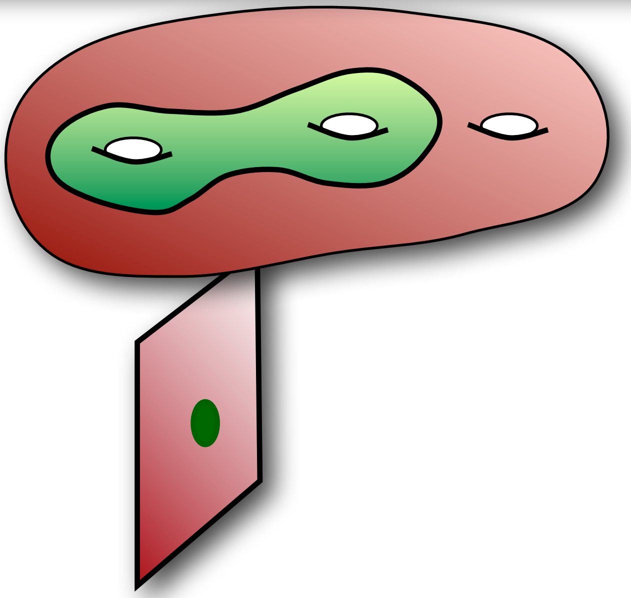

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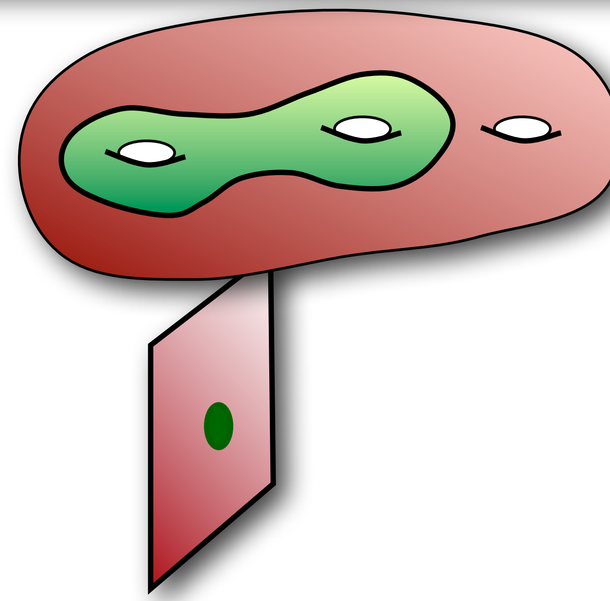

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Equivalently: **any** instanton intersects the U(1) brane with intersection number multiple of n

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- Massive U(1) associated to Zn-torsion classes in 7-manifold

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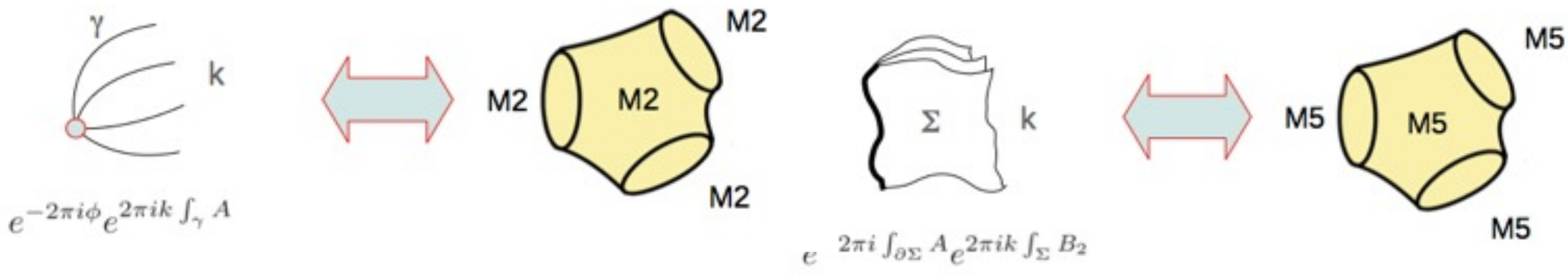

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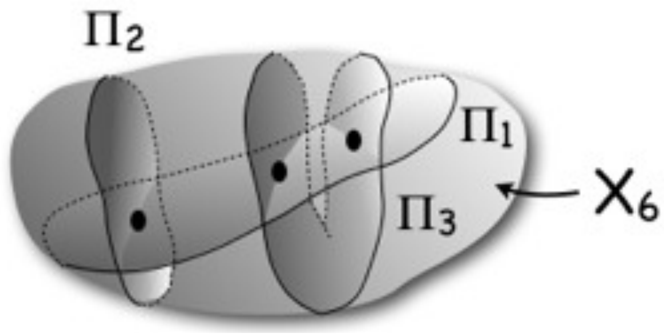
Zn symmetry in BSM

Flash back

- BF couplings permeate the physics of D-branes in compactifications to 4d, due to Chern-Simons action

- For D6-brane models...



Recall

$$(B_2)_k = \int_{[\Lambda_k]} C_5 \quad , \quad \phi_{\tilde{l}} = \int_{[\Lambda_{\tilde{l}}]} C_3 \qquad \text{with } d\phi_{\tilde{l}} = -\delta_{k\tilde{l}} *_{4d} (B_2)_k$$

4d couplings $N_a q_{ak} \int_{4d} (B_2)_k \mathrm{tr}\, F_a$ with $q_{ak} = [\Pi_a] \cdot [\Lambda_k]$

For U(1) comb: $Q = \sum_A c_A Q_A \Rightarrow (\sum_A c_A N_A q_{ak}) B_k \wedge F$

Z_n gauge symmetry iff

$$\sum_A c_A N_A q_{ak} = 0 \text{ mod } n \,, \text{ for all } k$$

Zn symmetry in BSM

- For instance, in toroidal models Ibáñez, Marchesano, Rabadán'01

N_i	(n_A^1, m_A^1)	(n_A^2, m_A^2)	(n_A^3, m_A^3)
$N_a = 3$	$(1/\beta^1, 0)$	$(n_a^2, \epsilon\beta^2)$	$(1/\rho, 1/2)$
$N_b = 2$	$(n_b^1, -\epsilon\beta^1)$	$(1/\beta^2, 0)$	$(1, 3\rho/2)$
$N_c = 1$	$(n_c^1, 3\rho\epsilon\beta^1)$	$(1/\beta^2, 0)$	$(0, 1)$
$N_d = 1$	$(1/\beta^1, 0)$	$(n_d^2, -\beta^2\epsilon/\rho)$	$(1, 3\rho/2)$

$$
\begin{aligned}
F^a &\wedge 3\left(\frac{1}{\rho}B_2^2 + n_a^2\frac{B_2^3}{2}\right)\\
F^b &\wedge 2\left(-B_2^1 + 3\rho n_b^1\frac{B_2^3}{2}\right)\\
F^c &\wedge 2n_c^1\frac{B_2^3}{2}\\
F^d &\wedge \left(-\frac{1}{\rho}B_2^2 + 3\rho n_d^2\frac{B_2^3}{2}\right)
\end{aligned}
$$

- R-parity (Qc) is automatic
- Baryon triality (Qa) if $\rho = 1/3 \text{ and } n_a^2 = 0 \bmod 3$
- Etc...

- Nicely dovetails the classification of discrete gauge symmetries in the MSSM (plus right-handed neutrinos)

Discrete Zn gauge symmetries

Discrete Zn gauge symmetries

- Let us stay a bit more with discrete gauge symmetries

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  Zn particles, Zn strings, ...

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  Zn symmetry of a 3-form that eats up a 2-form

$$\frac{1}{2}|F_4|^2 + |db_2 - n\, c_3|^2$$

Gauge invariance

$$c_3 \to c_3 + d\Lambda_2 \quad ; \quad b_2 \to b_2 + n\Lambda_2$$

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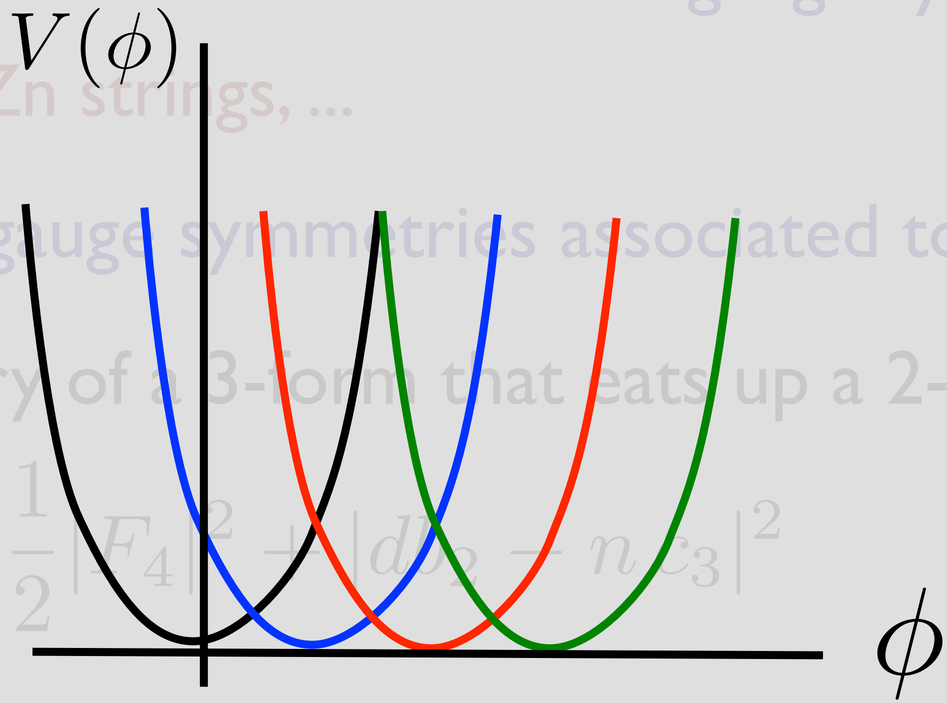

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Massive axion

Can arise in D-brane models. Will come back in lecture 4