6d superconformal field theories I

(lecture 3/4)

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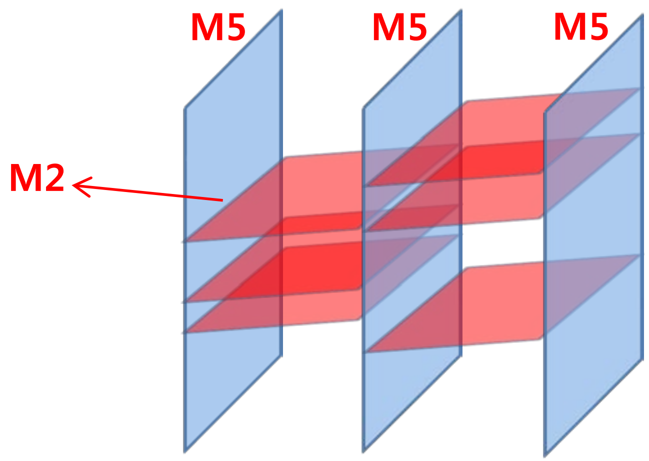
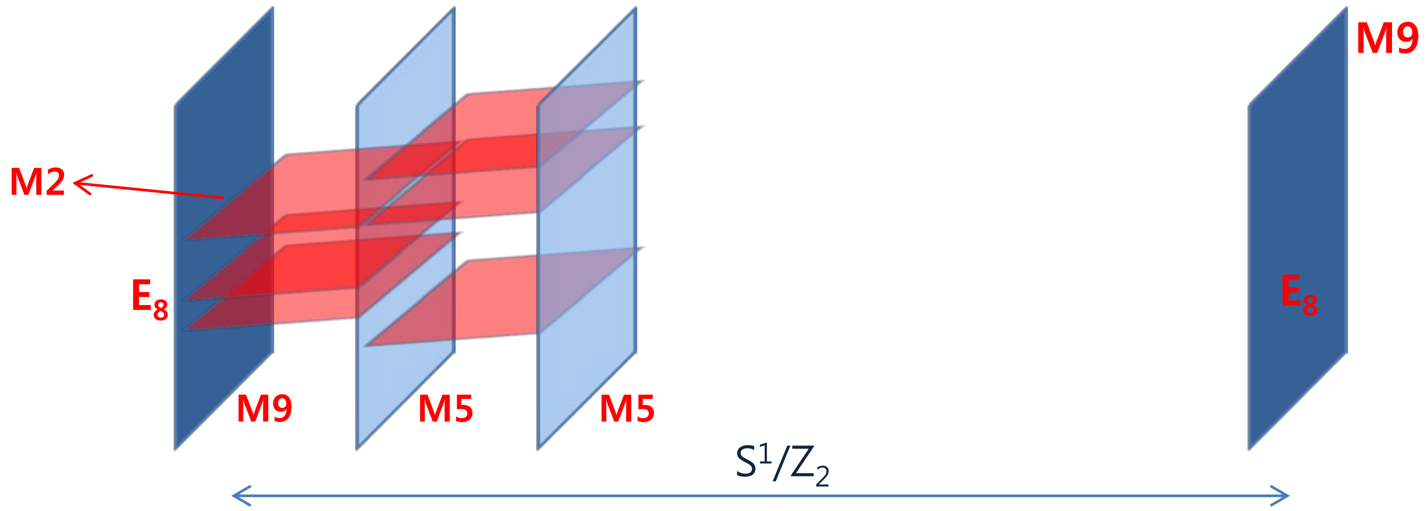
Two 6d SCFTs of our interest today

- (2,0) SCFT



- (1,0) SCFT on M5-M9: "E-string theory" (non-perturbative sector of HE)



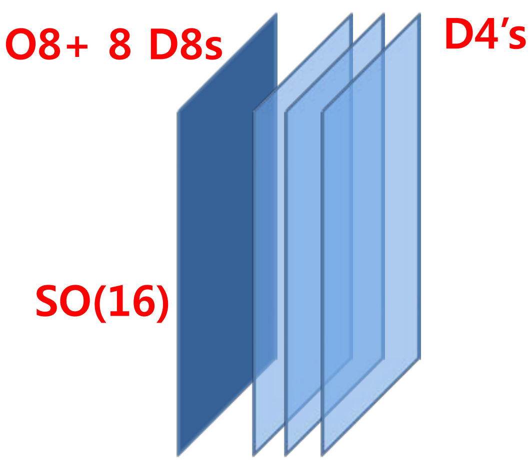
- important building block in the F-theory models

Properties

- N^3 d.o.f.: in various observables (thermal free energy, anomalies, vacuum Casimir energy)
- Strings in the tensor branch, tensionless in CFT

- S^1 reduction: 5d SYM descriptions vs. strong-coupling 5d CFT

- (2,0) theory: 5d MSYM w/ ADE gauge groups
- E-string theory: S^1 reduction yields strongly coupled 5d SCFT w/ E_8 (yesterday)
- Reduce E-string theory on S^1 w/ E_8 background Wilson line: HM to type I'

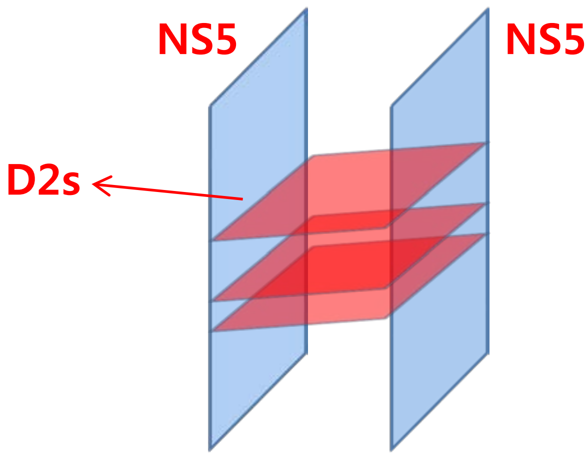


$$248 \to 120 + 128 \qquad RA_9 = (0,0,0,0,0,0,0,2\pi)$$

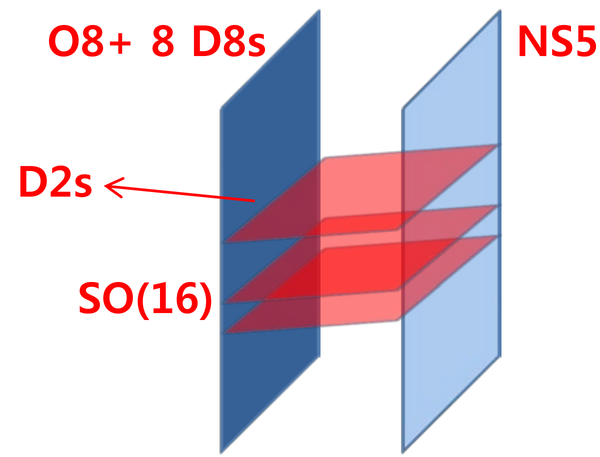
- 5d Sp(N) super-Yang-Mills w/ Nf = 8 fundamental + 1 antisymm. hypers

6d CFTs in tensor branch

- M-strings & E-strings: admits 2d QFT approaches



and more useful variant (later)



	0	1	2	3	4	5	6	7	8	9
NS5	•	•		•	•	•	•			
D8-O8	•	•		•	•	•	•	•	•	•
D2	•	•	•							

- Partition functions after S^1 compactifications: e.g. elliptic genera
- If 5d SYM indeed sees 6d physics,

$$Z^{\mathbb{R}^4\times T^2}(q,v) = Z_{\rm NS5}(q)\sum_{n=0}^{\infty} Z_n^{T^2}(q)e^{-n\lambda} \stackrel{?}{=} Z^{\mathbb{R}^4\times S^1}(q,v) = Z_{\rm pert}(v)\sum_{k=0}^{\infty} q^k Z_k(v)$$

Instanton partition functions: (2,0)

- Witten index: $Z_{\rm Nek} = \mathrm{Tr}\left[(-1)^F q^k e^{-\beta\{Q,Q^\dagger\}} e^{-\epsilon_+\left(2J_R^3+J_1+J_2\right)} e^{-\epsilon_-(J_1-J_2)} e^{-\mathrm{tr}(v\Pi)} \boxed{(\text{flavor fugacities})}\right]$

$$e^{2mJ_F}$$

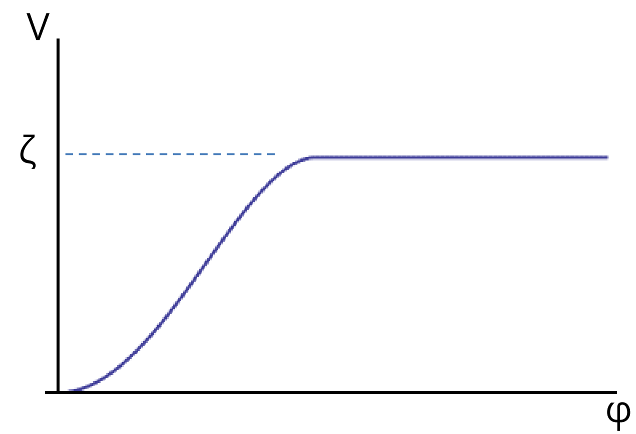
- SU(2)$_F$ in SO(5) chemical potential plays a role similar to 5d N=1* mass
- index of 1d N=(4,4) mechanics: U(k) or Sp(k) gauge groups

$$\begin{aligned} L_{QM} \;=\; & \frac{1}{g_{QM}^2}\mathrm{tr}_{k,N}\left[\frac{1}{2}(D_t\varphi^I)^2 + \frac{1}{2}(D_t a_m)^2 + \frac{1}{4}[\varphi^I,\varphi^J]^2 + \frac{1}{2}[a_m,\varphi^I]^2\right. \\ & \left. + D_t q_{\dot\alpha} D_t \bar{q}^{\dot\alpha} - (\varphi^I\bar{q}^{\dot\alpha} - \bar{q}^{\dot\alpha}v^I)(q_{\dot\alpha}\varphi^I - v^I q_{\dot\alpha}) - D^{\dot\alpha}{}_{\dot\beta}D^{\dot\beta}{}_{\dot\alpha} + \cdots\right] \end{aligned}$$

$$D^{\dot\alpha}{}_{\dot\beta} = \bar{q}^{\dot\alpha}q_{\dot\beta} - \frac{1}{2}\zeta^A(\tau^A)^{\dot\alpha}{}_{\dot\beta} + \frac{1}{4}(\bar\sigma^{mn})^{\dot\alpha}{}_{\dot\beta}[a_m,a_n] - \frac{1}{2}\delta^{\dot\alpha}_{\dot\beta}(\mathrm{trace})$$

- U(N): [Nekrasov] 2002 [H.-C.Kim, Koh, SK, K. Lee, S. Lee] 2011
- decoupled sector (D0 escaping D4 into R^5)
- no $Z_{\rm extra}$: introduce FI deformation

$$V \leftarrow |\phi^i q_{\dot\alpha}|^2 + \left((\tau^I)^{\dot\beta}{}_{\dot\alpha} q_{\dot\alpha}\bar{q}^{\dot\beta} - \zeta^I\right)^2$$



- SO(2N): $Z_{\rm extra}$ from k D0 moving on R^5/Z_2 [Y. Hwang, J. Kim, SK] work in progress
- E_n: little to say systematically, but some statements later

The answers

- The results are given in terms of residue sums:

- U(N): nonzero JK-Res classified using N-colored Young diagrams w/ k boxes

[Flume, Poghossian] [Bruzzo, Fucito, Morales, Tanzini] 2002, [Hwang, J. Kim, SK, Park] 2014

$$Z_k = \sum_{\sum_i |Y_i| = k} \prod_{i,j=1}^{N} \prod_{s \in Y_i} \frac{\sinh \frac{E_{ij}+m-\epsilon_+}{2} \sinh \frac{E_{ij}-m-\epsilon_+}{2}}{\sinh \frac{E_{ij}}{2} \sinh \frac{E_{ij}-2\epsilon_+}{2}}$$

$$E_{ij} = \alpha_i - \alpha_j - \epsilon_1 h_i(s) + \epsilon_2 (v_j(s) + 1)$$

$h_i(s)$: distance from box "s" to the right end of i'th Young diagram

$v_j(s)$: distance from box "s" to the lower end of the j'th Young diagram

- SO(2N): very complicated, if one lists all nonzero JK-Res.

- Question: Are they really 6d partition functions at all?
- Consider U(N) in detail. Separate the analysis to U(1) and SU(N). [H.-C.Kim, SK, Koh, K. Lee, S. Lee] 2011, [Haghighat, Iqbal, Kozcaz, Lockhart, Vafa] 2013

(Similar results for SO(2N) are partly available [Y. Hwang, J. Kim, SK] work in progress)

The KK momenta: U(1)

- This is the 5d/6d version of a fundamental problem in M-theory:
- "bulk" problem: 11d SUGRA is free in IR, making one multiplet of KK fields at each k.
- Do D0's make unique threshold bounds? [Yi] [Sethi, Stern] (1997)

$$L_{\rm bulk} = \frac{1}{g_{QM}^2}{\rm tr}_k\left[\frac{1}{2}(D_tX^M)^2 + \frac{1}{4}[X^I, X^J]^2 + \cdots\right] \qquad X^M = (a_m, \varphi^I)$$

- Similar question holds for 5d U(1) instantons, since 6d Abelian (2,0) is free.

- U(1) partition function [Iqbal, Kozcaz, Shabbir] [Awata, Kanno] [H.-C. Kim, SK, Koh, K. Lee, S. Lee]

$$Z[q] = PE\left[I_-(\epsilon_{1,2}, m)\frac{q}{1-q}\right] = 1+qI_-+q^2\left(\frac{I_-^2 + I_-(\cdot^2)}{2} + I_-\right)+q^3\left(\frac{I_-^3 + 3I_-I_-(\cdot^2) + I_-(\cdot^3)}{6} + I_-^2 + I_-\right)+\cdots$$

$$\boxed{I_-(e^{-\epsilon_{1,2}}, e^m)} = \frac{\sinh\frac{m+\epsilon_-}{2}\sinh\frac{m-\epsilon_-}{2}}{\sinh\frac{\epsilon_1}{2}\sinh\frac{\epsilon_2}{2}} \qquad PE\left[f(x)\right] = \exp\left[\sum_{n=1}^{\infty}\frac{1}{n}f(x^n)\right]$$

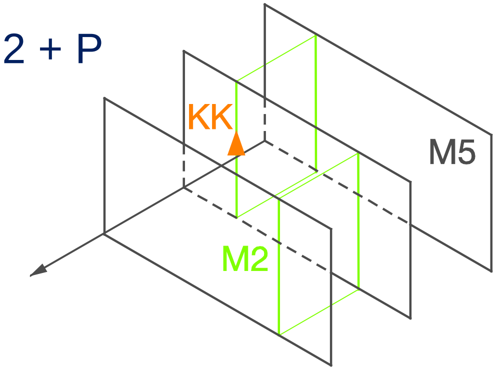
- Single particle index:

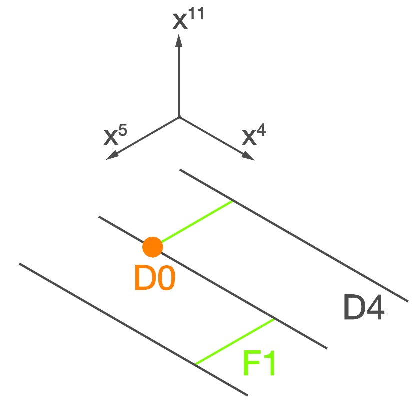
one 5d massive tensor multiplet for every k.

	$SU(2)_{1L}$	$SU(2)_{1R}$	$SU(2)_{2L}$	$SU(2)_{2R}$
B_2	**3**	**1**	**1**	**1**
ϕ_I	**1**	**1**	**2**	**2**
	1	**1**	**1**	**1**
λ	**2**	**1**	**2**	**1**
	2	**1**	**1**	**2**

SU(N): “M-strings”

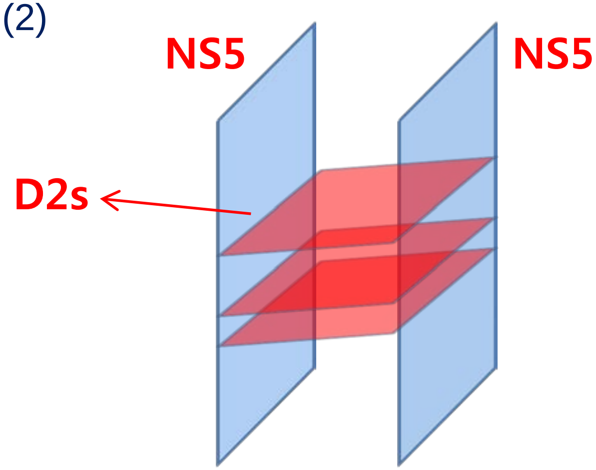
- bound states of D0+F1: ~ open M2 + P



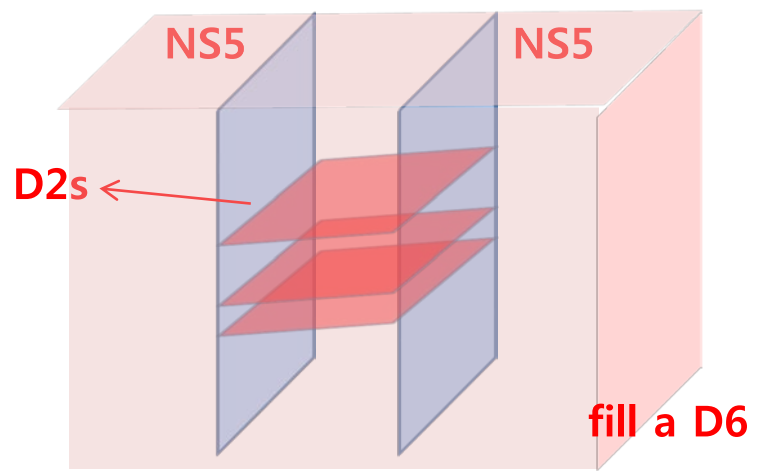


- Seeing 6d physics ~ seeing 2d QFT on open M2’s.

- Branes: e.g. SU(2)



2d N=(4,4) gauge theories



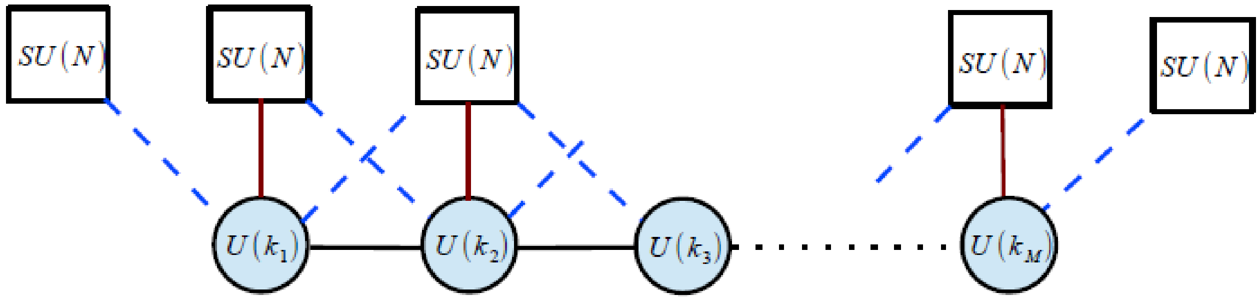
2d N=(0,4) gauge theories

	0	1	2	3	4	5	6	7	8	9
NS5	•	•	•	•	•	•	–	–	–	–
D6	•	•	•	•	•	•	•	–	–	–
D2	•	•	–	–	–	–	•	–	–	–

[2d IR limit ~ strong coupling limit ~ decompactifying]

SU(N) instantons vs. M-strings

- 2d N=(0,4) quiver: [Haghighat, Iqbal, Kozcaz, Lockhart, Vafa] [HKLV] 2013



vector : $A_\mu,\ \lambda_{A\dot\alpha}$ hyper : $\varphi_{\dot\alpha},\ \psi_A$ twisted hyper : $\varphi_A,\ \psi_{\dot\alpha}$ Fermi : Ψ

- M-strings' elliptic genera: [Benini, Eager, Hori, Tachikawa]

$$Z = \frac{1}{(2\pi i)^r}\oint \frac{1}{|W(G)|} Z_{\text{1-loop}} \qquad Z_{\text{vec}} = \prod_{I=1}^{r}\frac{2\pi\eta^2 du_I}{i}\cdot\prod_{\alpha\in\text{root}}\frac{\theta_1(\alpha\cdot u)}{i\eta} \quad Z_{\text{chiral}} = \frac{i\eta}{\theta_1(Q\cdot u+\rho\cdot z)}\ ,\quad Z_{\text{Fermi}} = \frac{\theta_1(Q\cdot u+\rho\cdot z)}{i\eta}$$

- Up to high orders in q & Coulomb VEVs,

$$Z^{\mathbb{R}^4\times T^2}(q,v) = Z_{\text{NS5}}(q)\sum_{n=0}^{\infty} Z_n^{T^2}(q)e^{-n\lambda} \overset{?}{=} Z^{\mathbb{R}^4\times S^1}(q,v) = Z_{\text{pert}}(v)\sum_{k=0}^{\infty} q^k Z_k(v)$$

- So, the 5d MSYM partition function is indeed 6d partition function.

[Often, the 1d (rather than 2d) expressions are more useful to study the CFT in symmetric phase.]

Application: the superconformal index

- Put the theory on $S^5 \times R$: energy E ; SO(6) j_1, j_2, j_3 ; $SO(5)_R$ R_1, R_2

- Choose a pair of Q, S (= Q^+)

$$Q^{(R_1,R_2)}_{(j_1,j_2,j_3)} \rightarrow \quad Q^{(\frac{1}{2},\frac{1}{2})}_{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}} \quad : \quad \text{BPS bound } E = 2R_1 + 2R_2 + j_1 + j_2 + j_3$$

[Bhattacharya,Bhattacharyya,Minwalla,Raju] 2008

- Index on $S^5 \times S^1$: counts local BPS operators on R^6

$$I(\beta, m, \epsilon_1, \epsilon_2) = \mathrm{Tr}\left[(-1)^F e^{-\beta'\{Q,S\}} e^{-\beta(E-\frac{R_1+R_2}{2})} e^{\beta m(R_1-R_2)} e^{-\gamma_1(j_1-j_3)} e^{-\gamma_2(j_2-j_3)}\right]$$

$$e^{-\gamma_1(j_1-j_3)} e^{-\gamma_2(j_2-j_3)} = e^{-\beta(aj_1+bj_2+cj_3)} \qquad a+b+c=0$$

The superconformal index: result

- Result: (even technically, some issues on general saddle points are open)

$\lambda = r\phi$ (W: Weyl group, r: rank)

Each factor takes the form of Nekrasov partition function on $R^4 \times S^1$

$$\boxed{Z(\beta, m, a_i) = \frac{1}{|W|}\int_{-\infty}^{\infty}\left[\prod_{i=1}^{r} d\lambda_i\right]\exp\left[-\frac{2\pi^2 \mathrm{tr}\lambda^2}{\beta(1+a)(1+b)(1+c)}\right]\boxed{Z^{(1)}_{\rm pert}Z^{(1)}_{\rm inst}}\cdot Z^{(2)}_{\rm pert}Z^{(2)}_{\rm inst}\cdot Z^{(3)}_{\rm pert}Z^{(3)}_{\rm inst}}$$

$$1 \ : \ (\epsilon_1,\epsilon_2) = \left(\frac{2\pi i(b-a)}{1+a}, \frac{2\pi i(c-a)}{1+a}\right) \ , \ m_0 = \frac{2\pi i(m+\frac{3}{2}(1+a))}{1+a} \ , \ \mu = \frac{2\pi\phi}{1+a} \ , \ q = e^{-\frac{4\pi^2}{\beta(1+a)}}$$

$$2 \ : \ (\epsilon_1,\epsilon_2) = \left(\frac{2\pi i(c-b)}{1+b}, \frac{2\pi i(a-b)}{1+b}\right) \ , \ m_0 = \frac{2\pi i(m+\frac{3}{2}(1+b))}{1+b} \ , \ \mu = \frac{2\pi\phi}{1+b} \ , \ q = e^{-\frac{4\pi^2}{\beta(1+b)}}$$

$$3 \ : \ (\epsilon_1,\epsilon_2) = \left(\frac{2\pi i(a-c)}{1+c}, \frac{2\pi i(b-c)}{1+c}\right) \ , \ m_0 = \frac{2\pi i(m+\frac{3}{2}(1+c))}{1+c} \ , \ \mu = \frac{2\pi\phi}{1+c} \ , \ q = e^{-\frac{4\pi^2}{\beta(1+c)}}$$

- proposed from non-renormalizable QFT, but can phrase it in an intrinsic manner.
- All three Z_{Nekrasov} factors are replaced by intrinsic $Z_{R4 \times T2}$ of 6d QFT
- “tr” or “integral” over matrix ~ sum over the tensor branch moduli

- The integral is difficult to carry out in full generality
- But for (2,0) theory, can do so in a simple case.

The unrefined index

- 16 (maximal) SUSY at m = ½ or – ½ & a = b = c = 0;

$$\mathrm{tr}[(-1)^F e^{-\beta(E-R_1)}] \qquad Q^{(+\frac{1}{2},R_2)}_{(j_1,j_2,j_3)}$$

- Can work with simpler setting m = ½ - c: measure commutes w/ 4 SUSY

- Simplifications of three Z[R^4 x T^2]'s at this point: extra SUSY, cancelation.

$$Z(\beta) = \frac{1}{|W|}\int d\lambda\, e^{-\frac{2\pi^2 \mathrm{tr}(\lambda^2)}{\beta}} \prod_\alpha 2\sinh^2(\pi\alpha(\lambda)) \cdot \eta(2\pi i/\beta)^{-N}$$

The spectrum: W-algebra

- U(N):

$$Z^{U(N)} = e^{\beta\left(\frac{N(N^2-1)}{6}+\frac{N}{24}\right)} \prod_{n=0}^{\infty}\prod_{s=1}^{N} \frac{1}{1-e^{-\beta(n+s)}}$$

- SO(2N):

$$Z^{SO(2N)} = e^{\beta\left(\frac{c_2|G|}{6}+\frac{N}{24}\right)} \prod_{n=0}^{\infty}\left[\frac{1}{1-e^{-\beta(n+N)}}\prod_{s=1}^{N-1}\frac{1}{1-e^{-\beta(n+2s)}}\right]$$

- Large N limits tested against SUGRA indices on AdS7 x S4 (or x S4/Z2)

- General form: (for all ADE)

$$Z^{ADE} = e^{\beta\left(\frac{c_2|G|}{6}+\frac{r}{24}\right)} \prod_{n=0}^{\infty}\prod_{\text{Casimir op.}} \frac{1}{1-e^{-\beta(n+d)}}$$

d: orders of the Casimir invariants

- We only computed perturbative part of En, but is compatible with this full form.
- These turn out to be the so-called “vacuum characters” of the W_G algebra.
- subset of local operators isomorphic to W-algebra, playing important roles in the mini-bootstrap: [Beem, Rastelli, Van Rees] 2014
- generalized with 6d surface operators [Bullimore, Hee-Cheol Kim] 2014

The Casimir energy

- The prefactor takes the form of "vacuum energy." $e^{-\beta\epsilon_0} \equiv e^{\beta\left(\frac{c_2|G|}{6}+\frac{r}{24}\right)}$

- Consider a simple example of free QFT on S^n x R.

$$\epsilon_0 \equiv \mathrm{tr}\left[(-1)^F \frac{E}{2}\right] = \sum_{\text{bosonic modes}} \frac{E}{2} - \sum_{\text{fermionic modes}} \frac{E}{2}$$

- regularize/renormalize the infinite sum: symmetries of the problem constrain it.

$$\epsilon_0 = \lim_{\beta'\to 0} \mathrm{tr}\left[(-1)^F \frac{E}{2} e^{-\beta' E}\right]$$

- In the index, these are constrained by different symmetries: Maximal SUSY

$$(\epsilon_0)_{\text{index}} \equiv \mathrm{tr}\left[(-1)^F \frac{E-R_1}{2}\right] = \sum_{\text{bosonic modes}} \frac{E-R_1}{2} - \sum_{\text{fermionic modes}} \frac{E-R_1}{2} \qquad (\epsilon_0)_{\text{index}} = \lim_{\beta'\to 0} \mathrm{tr}\left[(-1)^F \frac{E-R_1}{2} e^{-\beta'(E-R_1)}\right]$$

- "supersymmetric Casimir energy": morally, it is also a kind of measure of d.o.f.

- Indeed, it sees N^3.

- Question: SUSY Casimir energy from gravity dual?

- "SUSY Casimir energy" = chiral anomalies in Ω-background

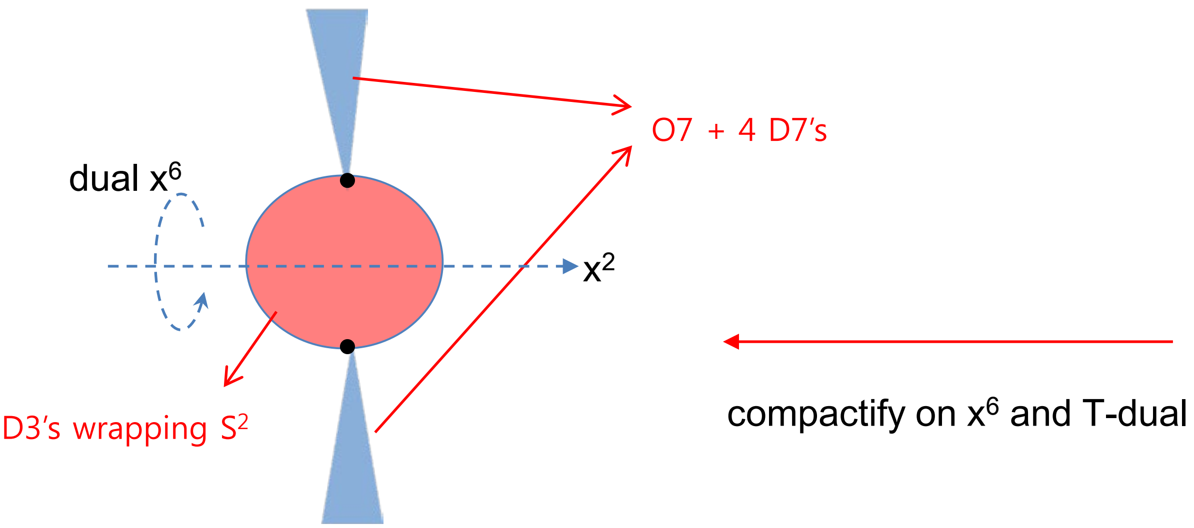
  [Bobev, Bullimore, Hee-Cheol Kim] 2015

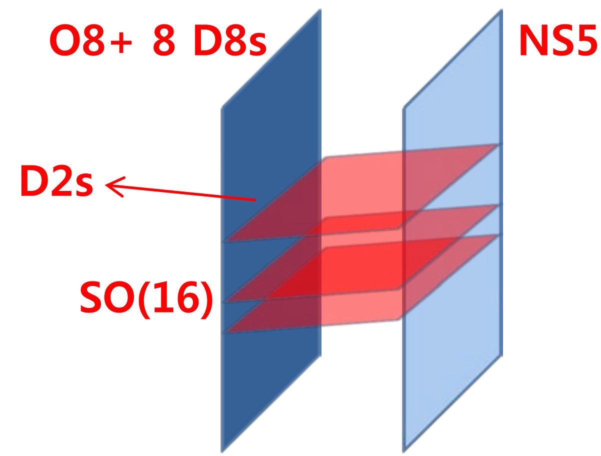
The (1,0) CFT for E-strings

- This CFT is important for many reasons.

- Either from M5-M9, or by F-theory on $B_4 = O(-1) \to P^1$.

	0	1	2	3	4	5	6	7	8	9
NS5	•	•		•	•	•	•			
D8-O8	•	•		•	•	•	•	•	•	•
D2	•	•	•							



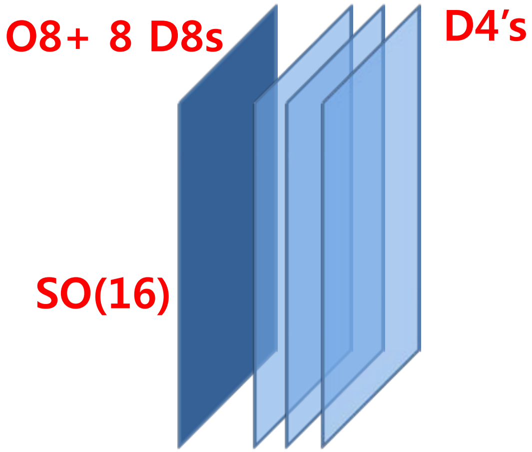


- Its compactification on S1 or T2 yield the 5d/4d Minahan-Nemeschansky CFTs
- non-perturbative sector of E8 x E8 heterotic strings (E-string ~ fractional heterotic string)
- Plays key roles of recent F-theory “atomic” classification: this is the “glue”

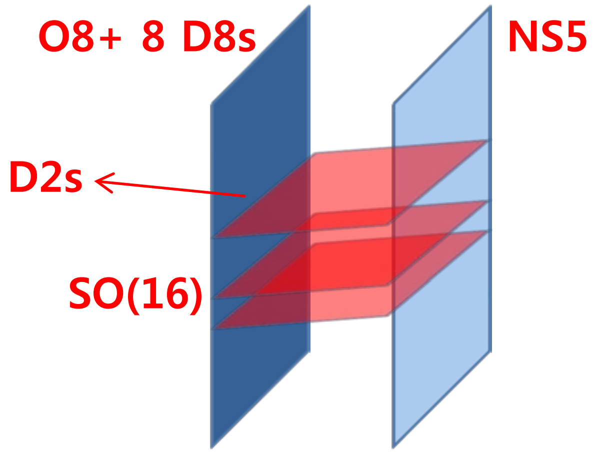
Descriptions of E-strings

- Wilson line S1 reduction: 5d SYM & instantons

$$\mathbf{248} \to \mathbf{120} + \mathbf{128} \qquad RA_9 = (0,0,0,0,0,0,0,2\pi)$$



- 2d gauge theories for the E-strings



- Both can be used to compute Z[R4 x T2]:
- Seiberg-Witten data of the Minahan-Nemeschansky SCFT
- Can be used in curved space partition function (not studied in detail yet)

Instanton partition function

- Results: One obtains Z = $Z_{QFT}\ Z_{extra}$, where Z_{extra} consists of two factors
- 1st decoupled factor from D0's + 8 D8's + O8: makes the spectrum of 10d E8 SYM
- 2nd decoupled factor from D0's in bulk: SUGRA continuum modes
- Zextra is the index for the D0 + "N_f = 8 D8" + O8 system

$$Z_{N_f=8} = \mathrm{PE}\Bigg[\boxed{-\frac{(t+t^3)(u+u^{-1}+v+v^{-1})}{2(1-tu)(1-t/u)(1-tv)(1-t/v)}\frac{q^2}{1-q^2}} \\ \boxed{-\frac{t^2}{(1-tu)(1-t/u)(1-tv)(1-t/v)}\left(\chi(y_i)^{SO(16)}_{\mathbf{120}}\frac{q^2}{1-q^2}+\chi(y_i)^{SO(16)}_{\mathbf{128}}\frac{q}{1-q^2}\right)}\Bigg]$$

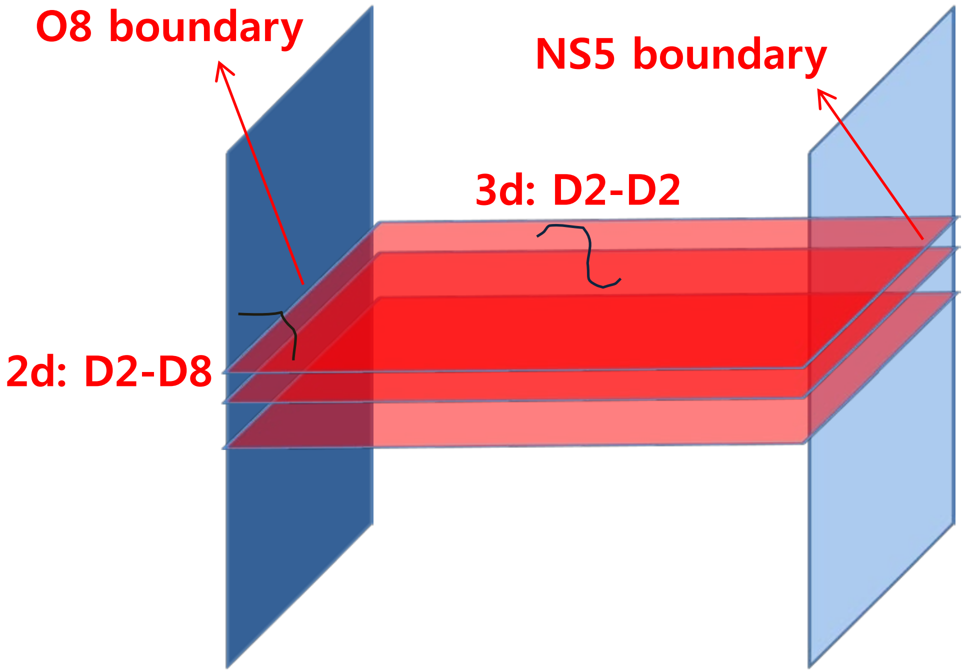
contribution from the continuum of 11d SUGRA on R^{8+1} x R^+ x S^1

Combines with 9d SO(16) SYM on D8-O8 to be the index of circle-compactified 10d SYM on M9, with E_8 gauge group

- Even the decoupled sector is interesting: supports "type I' – HE duality"
- The QFT part is the partition function: Again, is this a 6d observable?

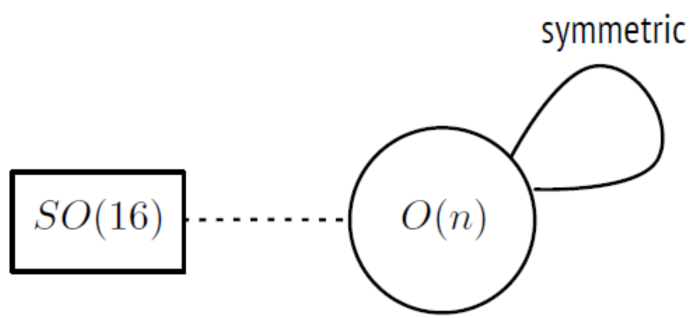
E-string gauge theories in 2d

- The 2d QFT:



- Field content:

$$\begin{array}{rcl} \text{vector} & : & O(n)\ \text{antisymmetric} \quad (A_\mu,\ \lambda_+^{\dot\alpha A}) \\ \text{hyper} & : & O(n)\ \text{symmetric} \quad (\varphi_{\alpha\dot\beta},\ \lambda_-^{\alpha A}) \\ \text{hyper} + \text{Fermi} & : & O(n)\ \text{fundamental} \quad (q_{\dot\alpha},\ \psi_-^A),\ \psi_+^A \\ \text{Fermi} & : & SO(16)\times O(n)\ \text{bifundamental} \quad \Psi_l \end{array}$$



The E-string partition functions

- The elliptic genus: similar contour integral formula + discrete O(n) holomonies
- Example: one E-string, O(1) ~ Z_2 gauge symmetry

- 4 discrete holonomies on two circle factors of T^2.

$$(1,1),\ (1,-1),\ (-1,1),\ (-1,-1)$$

- Result:

$$Z_1 = \sum_{n=1}^{4} \frac{Z_1^{(n)}}{2} = -\frac{\eta^2}{\theta_1(\epsilon_1)\theta_1(\epsilon_2)} \cdot \boxed{\frac{\Theta(q,m_l)}{\eta^8}}$$

$$\Theta(\tau,m_l) = \frac{1}{2}\sum_{n=1}^{4}\prod_{l=1}^{8}\theta_n(\tau,m_l)$$

partition function of E_8 root lattice:
E_8 symmetry enhancement

- Agrees with expectations: one E8 current algebra on left-mover, “half of heterotic string”

[Ganor, Hanany] [Klemm, Mayr, Vafa] (95-96)

- Checked that 5d instanton partition function is a 6d observable

$$Z^{\mathbb{R}^4\times T^2}(q,v,y) = Z_{\rm NS5}(q)\sum_{n=0}^{\infty} Z_n^{T^2}(q,y_{1\sim 7},y_8 q^{-2})e^{-n\lambda} = Z^{\mathbb{R}^4\times S^1}(q,v,y) = Z_{\rm pert}(v)\sum_{k=0}^{\infty} q^k Z_k(v,y)$$

Conclusions and remarks

- Studied 6d CFTs' solitons & curved space partition functions
- Explores many features of 6d SCFTs, M-theory: chiral algebra, N^3, dualities, …

- The approach using 2d GLSM: new 2d CFTs, w/ (0,4) SUSY
- The 2d QFTs for self-dual strings, for various 6d (1,0) CFTs, show very novel features: implications on both GLSM, instanton calculus, etc. (tomorrow)
- D3 + 7-branes wrapped on spheres (also with 7-brane "punctures")