5d and 6d superconformal field theories

(lecture 1/4)

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[Remarks]

I was asked to lecture on “exact results in supersymmetric theories”.

This often generally refers to exactly computable observables in SUSY theories.

Has quite a long history, from early 90s. Recently, it became richer & more powerful:

- SUSY QFTs on curved spaces
- better computational strategies
- new interpretations, deeper inter-relations

Developments happening…

- in various spacetime dimensions (e.g. QFT in d = 1,2,3,4,5,6)
- with various observables

Technically, cannot review them all. Also, it may not be that useful.

So I’ll talk about a particular physical subject in which these methods are very useful.

“5d and 6d superconformal field theories” ~ “higher dimensional SCFTs”

Plan

- Lecture 1 (today): overview & basic strategy

- 5d/6d SCFTs from string theory
- string theory settings, effective field theories, solitons & SUSY observables

- Lecture 2: 5d SCFTs

- Nekrasov's partition functions & related objects
- superconformal indices, symmetry enhancements, dualities
- $Z[S^5]$ & $N^{5/2}$

- Lecture 3: Some simple 6d SCFTs

- 6d (2,0) theories & E-string theories
- circle compactified CFTs, 5d SYM & solitons
- 6d superconformal index for (2,0): W-algebra, N^3, etc.

- Lecture 4: 6d (1,0) SCFTs & self-dual strings

- F-theory constructions & simple models: "atomic" SCFTs
- self-dual strings & their new gauge theory descriptions

Higher dimensional QFTs

- In conventional Lagrangian QFT, consistent interacting QFT in $d > 4$ is unfamiliar.

- A genuine "prediction" of string theory on QFT.
- Reflects our fundamental ignorance on QFT, especially at strong coupling.

- The most mysterious part of the string dualities found in '90s.
- Many primitive conjectures/speculations were left behind: lack of technical tools

- Now we can study them, very precisely, and quite flexibly.
- Not just 5d/6d CFTs, but more broadly, nonperturbative aspects of string theory.

Higher dimensional QFTs

- Isolated, without tunable coupling. Don't know Lagrangian descriptions.

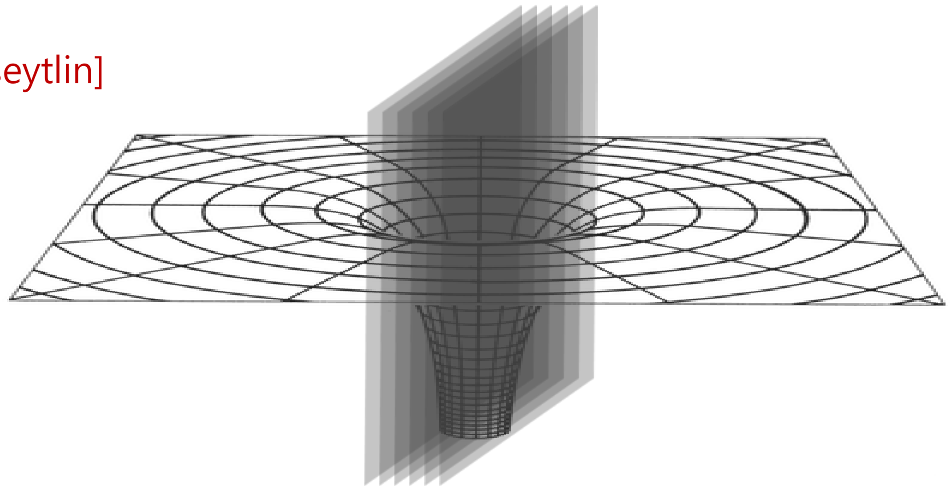
- Contains various novel objects.
- 6d CFTs: tensionless strings

- Many have non-conformal deformations, after which one finds Yang-Mills EFT.
- Back at the UV fixed points, # of light d.o.f. are much larger than N^2.

$N \gg 1$ M5's of M-theory make AdS_7:

black 5-branes at temperature T [Klebanov,Tseytlin]

$$\frac{S_{BH}}{(\text{volume})_5} \sim N^3 T^5$$



$N \gg 1$ D4's of massive IIA make AdS_6:

black 4-branes at temperature T

$$\frac{S_{BH}}{(\text{volume})_4} \sim N^{\frac{5}{2}} T^4$$

Discovery from string theory

- String theory contains much more than QFTs.
- Should take suitable decoupling limits: (higher dim'l) gravity decoupled

- In the low E limit, the Newton constant $G \sim \kappa^2$ is "small"
- The sector which defines SCFT decouples from gravity in the $\kappa \to 0$ limit with fixed E.

- For instance, this is realized by taking branes in flat spacetime
- Or by doing certain "compactifications" with singularities which support localized d.o.f.

- All 5d/6d CFTs are "discovered" this way in mid 90's, and also in recent years.
- However, we are not given useful formulations to study them.

- SUSY observables helped a lot to get progress, especially in the contexts of

- Studying the dynamics of "solitons" in effective field theories & extracting CFT info
- curved space partition functions

6d SCFTs

- Superconformal field theories in higher dimensions (d=3,4,5,6) [Nahm]

d	symmetry		bosonic subgroup	$\mathcal{N}$ (extended SUSY)
3	$OSp(\mathcal{N}\|4)$	$\supset$	$SO(3,2)\times SO(\mathcal{N})$	$1,2,3,4,5,6,8$
4	$SU(2,2\|\mathcal{N})$	$\supset$	$SO(4,2)\times SU(\mathcal{N})\times U(1)$	$1,2$
4	$PSU(2,2\|4)$	$\supset$	$SO(4,2)\times SU(4)$	4
5	$F(4)$	$\supset$	$SO(5,2)\times SU(2)$	1
6	$OSp(8^*\|2\mathcal{N})$	$\supset$	$SO(6,2)\times Sp(2\mathcal{N})$	$1,2$

- Possible superconformal symmetries in 6d

$$Q^i_\alpha,\ S^\alpha_i \text{ with } (i=1,\cdots,2\mathcal{N};\ \alpha=1,\cdots,4,\text{ chiral})$$

$$\overline{Q}^\beta_j = Q^i_\alpha C^{\alpha\beta}\Omega_{ij}\ ,\ \overline{S}^J_\beta = S^\alpha_i C_{\alpha\beta}\Omega^{ij}$$

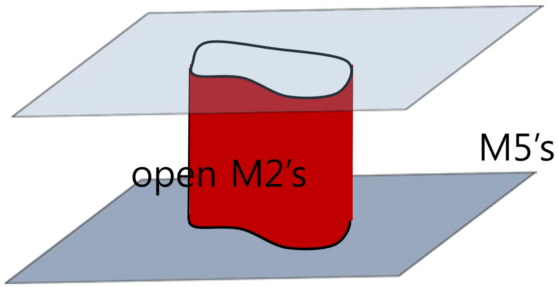
$$\{Q^i_\alpha, S^\beta_j\} = \delta^i_j\delta^\beta_\alpha(-iD) - \delta^i_j J_\alpha{}^\beta - 4\delta^\beta_\alpha R_i{}^j$$

- Unlike familiar gauge theories, 6d SCFTs will always contain
- self-dual tensor multiplets $B_{\mu\nu}$ with $H=dB=\star dB\ ,\ \ \Psi^A\ ,\ \ \Phi$
- self-dual strings which couple to them

6d SCFTs from branes in "flat" space

- The 6d (2,0) theory from N M5-branes: (A_{N-1} or D_N types)

← Lecture 3



probing R^5 or R^5/Z_2 orbifold

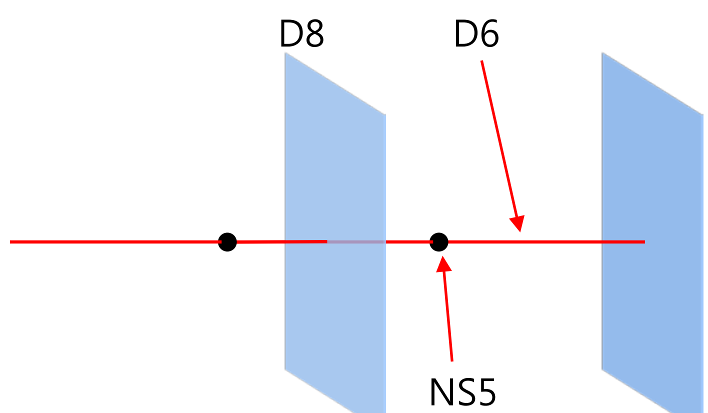
- Or, make a transverse S^1 compactifications to N NS5-branes

- M5 or NS5 host self-dual tensor multiplets: brane separation is scalar VEV

$$B_{\mu\nu} \text{ with } H = dB = \star dB \ , \quad \Psi^A \ , \quad \Phi$$

- 6d (1,0) CFTs: NS5, D6, D8, (+ O8,O6)
- contains tensor + vector + hyper multiplets

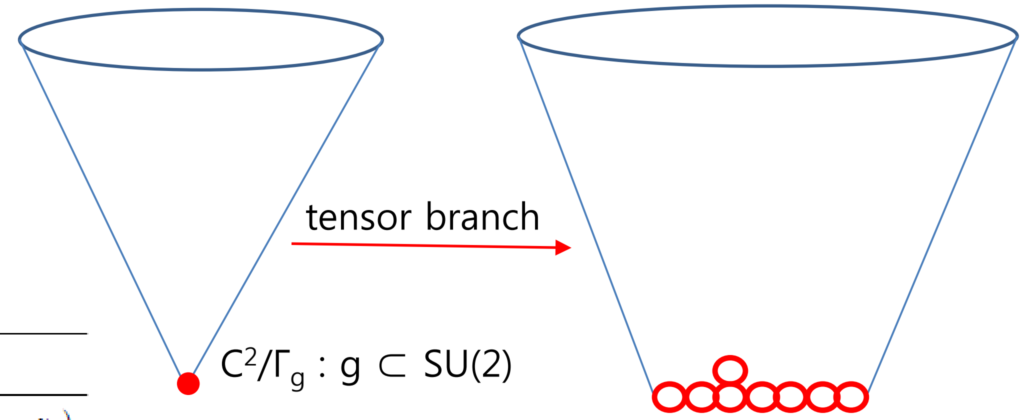
[Brunner, Karch] [Hanany, Zaffaroni] 1997



6d SCFTs from F-theory

- 6d (2,0) theory of ADE types:

  [Witten] '95



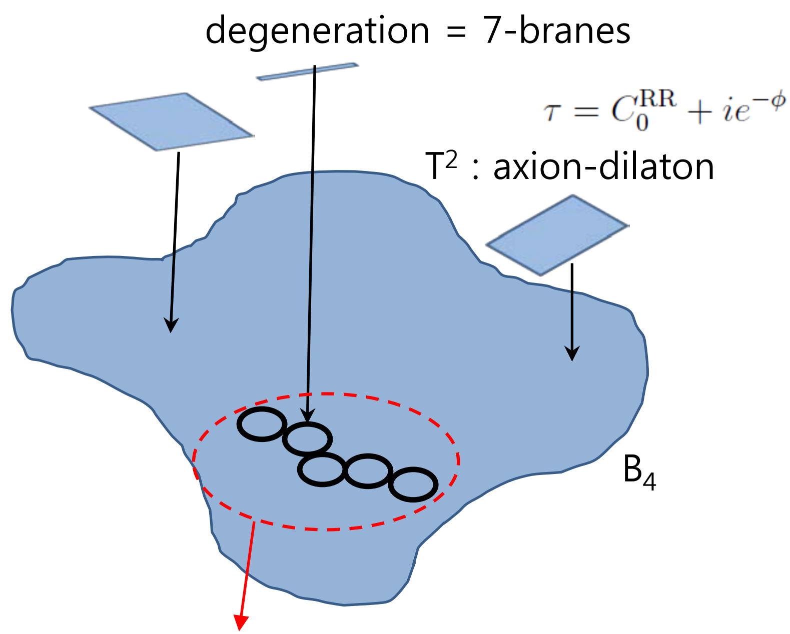
A_n	$(z_1, z_2) \to \left(e^{\frac{2\pi i}{n+1}} z_1, e^{-\frac{2\pi i}{n+1}} z_2\right)$
D_n	$(z_1, z_2) \to \left(e^{\frac{\pi i}{n-2}} z_1, e^{-\frac{\pi i}{n-2}} z_2\right)$, $(z_1, z_2) \to (z_2, -z_1)$
$E_6,\ E_7,\ E_8$	more involved...

- Wrapped D3's = self-dual strings

- (1,0) SCFTs from F-theory on singular CY_3

- Brane models are dualized to F-theory.
- F-theory models are broader: (p,q) 7-branes
- Leads to new subtle models in M-theory.

(conformal matters [Del Zotto, Heckman, Tomasiello, Vafa])



CFT supported on singularity of collapsed 2-cycles: volume of 2-cycles = tensor multiplet scalars

6d SCFTs from F-theory

- 6d (1,0): classification of "non-compact B_4 with singularity" in tensor branch
  [Heckman,Morrison,Vafa] 2013 [Heckman, Morrison, Rudelius, Vafa] 2015
- Resolved singularities: Intersecting P^1's. Should study models w/ 1d tensor branches.

- Important examples: [Morrison,Vafa] [Witten] 1996

- B_4 = "Hirzebruch surfaces" F_n: a P^1_f fibered over P^1_b w/ $P^1_b \cap P^1_b = -n$

  [originally found as F-theory duals of E_8 x E_8 heterotic on K3: n = 0,1, … , 12 allowed

$$\int_{K3} dH = \frac{\alpha'}{4}\int_{K3}(\mathrm{tr}R_{K3}\wedge R_{K3} - \mathrm{tr}(F\wedge F)_{E_8\times E_8}) \sim 24 - k_1 - k_2 \qquad (k_1,k_2) = (12-n, 12+n)$$
]

- noncompact limit O(-n) → P^1: Play the roles of "atoms" in F-theory models of 6d SCFTs

- Various gauge symmetries allowed

n	1	2	3	4	5	6	7	8	12
gauge symmetry	-	-	$SU(3)$	$SO(8)$	F_4	E_6	E_7	E_7	E_8
global symmetry	E_8	-	-	-	-	-	-	-	-
matters	-	-	-	-	-	-	$\frac{1}{2}\mathbf{56}$	-	-

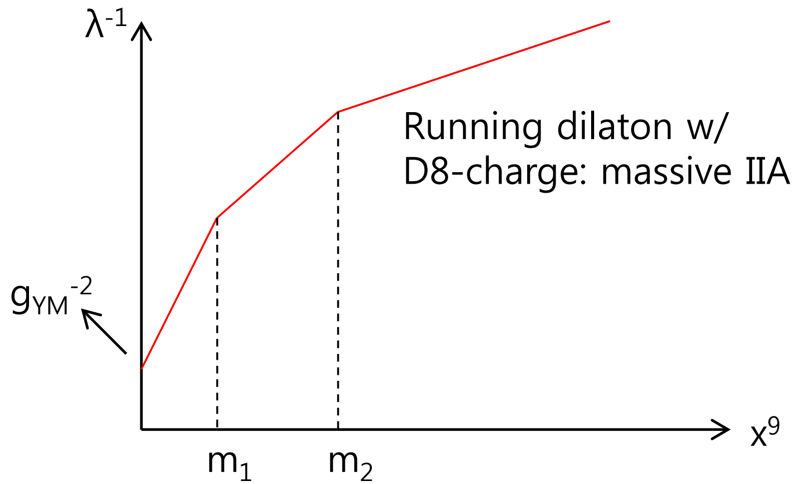
← Lecture 4

5d SCFTs from string theory

- N D4's probing an O8 and $N_f \le 7$ D8's

[Seiberg] 1996

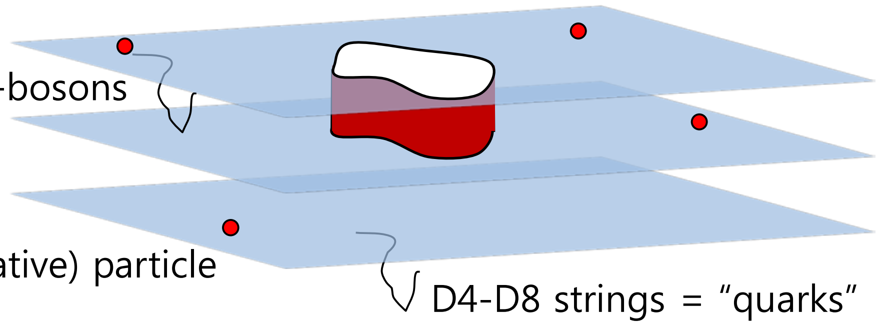
	0	1	2	3	4	5	6	7	8	9
D4	•	•	•	•	•					v
D8/O8	•	•	•	•	•	•	•	•	•	m_i



- "$1/g_{YM} = 0$, $m_i = 0$, $v=0$" : 5d SCFT on D4's.
- $1/g_{YM}$ (or m_i): relevant deformation, SYM description

← **Lecture 2**

- Particles & strings:



- Richer examples from M-theory on CY_3, or 5-brane webs (optionally w/ D7, O7, O5, …)

6d SYM from 6d CFT

- tensor branch: 6d Yang-Mills + tensors (+ hypers)

$$S_{\rm bos} = \int d^6x \left[-\frac{1}{6}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi\right] + c\int d^6x \left[-\frac{1}{4}\Phi{\rm tr}(F_{\mu\nu}F^{\mu\nu})\right] + c\int B\wedge{\rm tr}(F\wedge F)$$

$$H = dB + c{\rm tr}\left(AdA - \frac{2i}{3}A^3\right)$$

- broken scale invariance by VEV, so the above EFT formally respects it.
- self-duality by hand (like IIB SUGRA)
- classical gauge anomaly:

$$\delta\left[\sqrt{c}\int B\wedge{\rm tr}(F\wedge F)\right] = -c\int{\rm tr}(\epsilon F)\wedge{\rm tr}(F\wedge F)$$

$$\text{contributing a term} \sim c\,{\rm tr}(F^2)^2 \qquad \delta A_\mu = D_\mu\epsilon\ ,\quad \delta B_{\mu\nu} = -\sqrt{c}\,{\rm tr}(\epsilon F_{\mu\nu})$$

- Studying tensor branch itself is interesting and important:
- strings become tensionful: analogous to W-bosons of gauge theories in Coulomb branch
- BPS observables in tensor branch are useful in many ways

5d SYM from 5d/6d CFTs

- S^1 compactification of 6d CFTs: dualize tensor to 5d vectors. non-Abelian 5d SYM

- A typical example is the 5d maximal SYM from (2,0) theory: M5 → D4
- Non-renormalizable. Cannot expect it to be consistent descriptions of 6d QFT.

- Careful study of soliton sector reveals very rich information on 6d QFT.
- SUSY observables: Even here, should expect to put in extra UV completion somewhere.

- More subtle S1 compactifications of 6d (1,0) lead to interesting 5d SYM with N=1 SUSY

- Relevant deformations of 5d CFTs: Yang-Mills theories

$$\mathcal{L} \leftarrow -\frac{1}{4g_{YM}^2}\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + (\text{other relevant deformations})$$

- Should be viewed on equal footing w/ other relevant deformations, e.g. quark masses.
- Plays very important roles in UV symmetry enhancements: solitonic particles in SYM

Solitons in effective field theories

- Co-dimension 4 solitons play important roles in EFT
- Yang-Mills instantons: (or anti-instantons) finite action solutions localized in R^4

$$F_{\mu\nu} = \star_4 F_{\mu\nu} \qquad\qquad k \equiv \frac{1}{8\pi^2}\int \mathrm{tr}\,(F \wedge F) \in \mathbb{Z}$$

- In 4d Euclidean Yang-Mills, these describe semi-classical tunneling.
- In 5d/6d Yang-Mills, they are stationary particle-like or string-like solitons.
- SUSY (e.g. in 5d SYM): 2d N=(0,4) or their 1d reductions

$$\{Q^A_M, Q^B_N\} = P_\mu(\Gamma^\mu C)_{MN}\epsilon^{AB} + i\frac{4\pi^2 k}{g^2_{YM}}C_{MN}\epsilon^{AB} + i\mathrm{tr}(v\Pi)C_{MN}\epsilon^{AB}$$

$$M = \frac{4\pi^2 k}{g^2_{YM}} + \mathrm{tr}(v\Pi)$$

Q^A_α, $\bar{Q}^A_{\dot\alpha}$

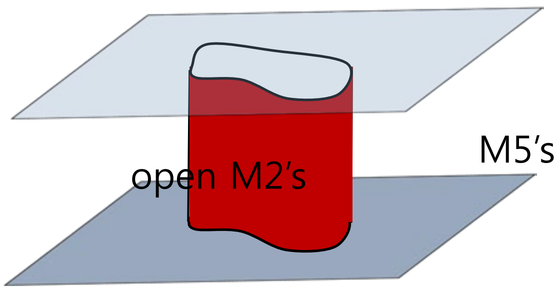
broken preserved

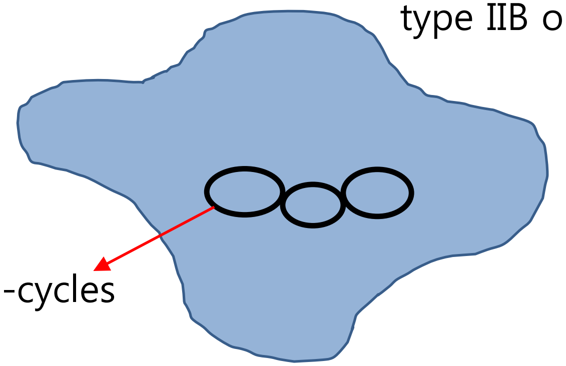
- They reflect the UV QFT physics in the IR EFT.
- Even after (massive) deformations towards SYM, “heavy” d.o.f. are visible as solitons.
- Analogous to the Skyrmions in the Skyme model: baryons in meson’s EFT

- To extract out the UV physics from the EFT, studying them is very important.

6d self-dual strings = instanton strings

- If 6d SCFT has gauge symmetry, the self-dual strings are solitons in 6d SYM.
  - tensionless strings couple to: $B_{\mu\nu}$ with $H = dB = \star dB$, Ψ^A , Φ
  - tensionful in tensor branch.





- Self-dual strings are solitons in 6d SYM: instanton strings in 6d SYM

$$F_{\mu\nu} = \star_4 F_{\mu\nu} \qquad k \equiv \frac{1}{8\pi^2}\int \mathrm{tr}\,(F\wedge F) \in \mathbb{Z} \qquad S \leftarrow \int B \wedge \mathrm{tr}(F\wedge F)$$

  - In 6d, we only know low E effective theory (in tensor branch).
  - We often know exact 2d QFTs on the strings: subsector of 6d QFT from 2d Lagrangian
  - Exist "ADHM gauge theory description" for instantons [Atiyah, Drinfeld, Hitchin, Manin] (1978)

5d instantons = KK modes

- Coupling to gravi-photon: U(1) Kaluza-Klein gauge fields

- 5d MSYM in type IIA:

$$S \leftarrow \int C_1^{RR} \wedge \mathrm{tr}(F \wedge F)$$

- Yang-Mills instantons are Kaluza-Klein modes of S^1 compactified 6d CFT
- At least in certain BPS observables, they allows us to reconstruct 6d physics.

- Also true in certain 6d (1,0) CFTs: example of E8 (1,0) SCFT & 5d SYM will be discussed.

- Leads to the 6d versions of the M-theory's DLCQ descriptions via D0-branes

  [Aharony, Berkooz, Seiberg]

- Often related to the 5d descriptions of 6d CFTs in various settings:

  [Douglas] [Lambert, Papageorgakis, Schmidt-Sommerfeld] [H.-C.Kim, SK, Koh, K.Lee, S.Lee]

  [Hee-Cheol Kim, SK] [Kallen, Qiu, Minahan, Zabzine] [Lockhart, Vafa] [H.-C.Kim, J.Kim, SK]

  [Cordova, Jafferis] [S.Lee, Yamazaki] ……

5d instantons = nonperturbative particles

- Massive particles after massive deformations of 5d CFT

$$\{Q_M^A, Q_N^B\} = P_\mu(\Gamma^\mu C)_{MN}\epsilon^{AB} + i\frac{4\pi^2 k}{g_{YM}^2}C_{MN}\epsilon^{AB} + i\mathrm{tr}(v\Pi)C_{MN}\epsilon^{AB}$$

$$M = \frac{4\pi^2 k}{g_{YM}^2} + \mathrm{tr}(v\Pi)$$

- At low energy, $E << g_{YM}^2$, they are heavy “solitons”

- In UV, these particles become light. They play important roles in clarifying the UV physics.

- E.g. crucial for making the UV symmetry enhancement to work: “instanton operators”
- analogous to the roles of magnetic monopole operators in IR symmetry enhancements

1d/2d QFTs on solitons

- 2d SCFT or 1d superconformal quantum mechanics on soliton worldvolume

$$S_{QM} = \int dt \left[g_{MN}(X)\dot{X}^M\dot{X}^N + \cdots \right]$$

$$S_{2d} = \int d^2x \left[- g_{MN}(X)\partial_\mu X^M \partial^\mu X^N + \cdots \right]$$

coordinates of $4c_2k$ dimensional instanton moduli space

- The instanton moduli space has small instanton singularities
- E.g. single U(N) instanton: (conical singularity at the tip of the cone: $\lambda = 0$)

$$ds^2 = g_{MN}(X)dX^M dX^N = ds^2(\mathbb{R}^4) + d\lambda^2 + \lambda^2\left[ds^2(S^3/\mathbb{Z}_2) + ds^2(\mathcal{M}_{4N-8})\right]$$

center-of-mass; **instanton "size"**; $SU(2)$ orientation; $\frac{SU(N)}{SU(2)\times U(N-2)}$

- incomplete QFT: reflects UV incompleteness of the 5d/6d SYM descriptions
- We don't know how to UV complete 5d/6d SYM. Can often do so in 1d/2d.
- Exact QFT descriptions for subsectors of 5d/6d CFTs

- Can compute building blocks of interesting CFT observables (inspired by EFT)

Gauge theories on 5d solitons

- UV completion in 1d: ADHM quantum mechanics (GLSM):

- Discovered originally as an ansatz which solves self-duality eqns

$$A_\mu = i v^\dagger \partial v \quad (v_{(N+2k)\times N},\ v^\dagger v = \mathbf{1}_{N\times N}) \qquad U^\dagger v = 0\ ,\quad U_{(N+2k)\times 2k} = \begin{pmatrix} \bar{q}_{N\times 2k} \\ (a_{\alpha\dot\beta})_{k\times k} - x_{\alpha\dot\beta}\otimes \mathbf{1}_{k\times k}\end{pmatrix}$$

$$D^I \equiv q_{\dot\alpha}(\tau^I)^{\dot\alpha}{}_{\dot\beta}\bar{q}^{\dot\beta} + (\tau^I)^{\dot\alpha}{}_{\dot\beta}[a_{\alpha\dot\alpha}, a^{\alpha\dot\beta}] = 0$$

- Can promote the matrices into fields of QM, UV-uplifting the nonlinear sigma model:

5d gauge group: $G_N = U(N), SO(N), Sp(N)$

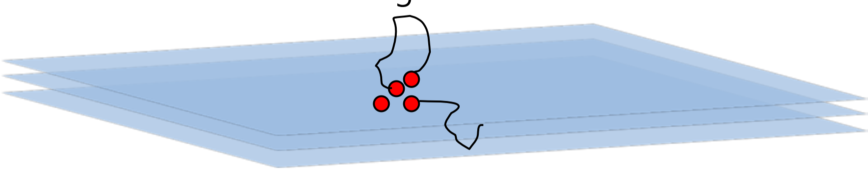
QM gauge group: $\hat{G}_k = U(k), Sp(k), O(k)$

QM vector multiplet: $\varphi, A_t, \bar\lambda^A_{\dot\alpha}$ ($\hat{G}_k$ adjoint)

$\hat{G}_k$ adjoint/antisymmetric/symmetric hyper: $a_{\alpha\dot\beta}, \Psi^A_\alpha$

$G_N \times \hat{G}_k$ bi-fundamental hyper: $q_{\dot\alpha}, \psi^A$

Intuition: k D0's & N D4's
(optionally w/ orientifold)



$$L_{\rm QM} = \frac{1}{g_{QM}}{\rm tr}\left[\frac{1}{2}(D_t\varphi)^2 + \frac{1}{2}(D_t a_m)^2 + D_t q_{\dot\alpha} D_t \bar{q}^{\dot\alpha} + \frac{1}{2}[\varphi, a_m]^2 - (\varphi\bar{q}^{\dot\alpha} - \bar{q}^{\dot\alpha} v)(q_{\dot\alpha}\varphi - v q_{\dot\alpha}) - D^{\dot\alpha}{}_{\dot\beta} D^{\dot\beta}{}_{\dot\alpha} + \cdots\right]$$

- Extra d.o.f. from 5d hypermultiplets: see, e.g. [Shadchin] 2005

ADHM quantum mechanics

- Some (0,4) multiplets: (on-shell) $\text{supercharges}: \ Q_{A\dot{\alpha}}$

$$\text{vector}: \ A_t, \ \phi, \ \lambda_{A\dot{\alpha}} \qquad \text{hyper}: \ \varphi_{\dot{\alpha}}, \ \psi_A \qquad \text{twisted hyper}: \ \varphi_A, \ \psi_{\dot{\alpha}} \qquad \text{Fermi}: \ \Psi$$

- UV completion of non-linear sigma model:
- Some UV degrees of freedom are coarse-grained out in IR: e.g. 1d gauginos
- May exist extra d.o.f. in IR (e.g. vector multiplet scalar, twisted hyper)

$$\text{vector}: \ A_t, \ \phi, \ \cancel{\lambda_{A\dot{\alpha}}}$$

$$\text{twisted hyper}: \ \varphi_A, \ \psi_{\dot{\alpha}}$$

- All the extra d.o.f's should decouple in IR
- Even in certain SUSY observables, computable in UV, the decoupled extra d.o.f. leave subtle traces which we should factor out.

- Understanding these subtleties is crucial to get the correct partition function: e.g. 4d SU(2) w/ N_f = 4, SU(N) N=2*, etc.
- After getting rid of these decoupled factors, we get intrinsic CFT observables.

Gauge theories on 6d soliton strings

- If there is nontrivial 6d gauge groups, self-dual strings = instanton strings

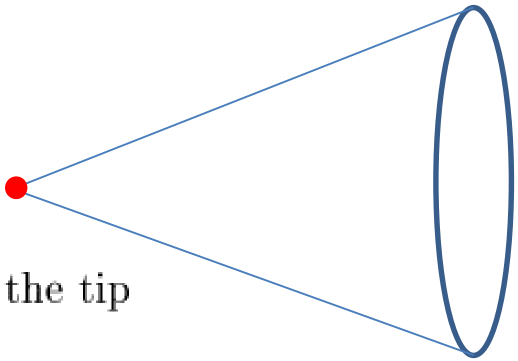
- Need UV completions of 2d NLSM on instanton moduli space

  [Even in case without 6d gauge symmetry, one can just find 2d gauge theories, e.g. from branes, etc.]

- Even for classical G, standard ADHM often goes wrong in 2d. Gauge anomalies.
- Need to cure the anomalies: add more fields to the quiver

- The non-linear sigma model description shouldn't change.
- All the other fields: extra degrees localized at the small instanton singularity

$$V(\phi_{\text{ADHM}}, \phi_{\text{extra}}) \sim V(\phi_{\text{ADHM}}) + V(\phi_{\text{extra}}) + |\phi_{\text{extra}}\phi_{\text{ADHM}}|^2$$



$$S_{\text{2d}} = \int d^2x \left[-g_{MN}(X)\partial_\mu X^M \partial^\mu X^N + \cdots \right]$$

ϕ_{extra} are massless only at the tip

Soliton partition functions & CFT partition functions

- Conjectures inspired by 5d SYM effective field theory descriptions

- $Z[S^4 \times S^1]$: [Hee-Cheol Kim, Sung-Soo Kim, Kimyeong Lee] 2012

$$Z_{S^4\times S^1}[x=e^{-\epsilon_+}, y=e^{-\epsilon_-}, m_i, q] = \int [d\alpha] Z_{\rm Nek}^{\mathbb{R}^4\times S^1}(q,\epsilon_{1,2},m_i,\alpha) Z_{\rm Nek}^{\mathbb{R}^4\times S^1}(q^{-1},\epsilon_{1,2},m_i,\alpha)$$

- $Z[S^5]$ or $Z[S^5 \times S^1]$: [Kallen, Qiu, Zabzine] [H.-C. Kim, SK] [Lockhart, Vafa]

  [H.–C. Kim, Joonho Kim, SK] [Qiu, Zabzine]

$$Z[S^5\times S^1] = \int [d\phi] e^{-\frac{4\pi^2 {\rm tr}(\phi^2)}{\beta\omega_1\omega_2\omega_3}} Z^{\mathbb{R}^4\times T^2}\left(q=e^{-\frac{4\pi^2}{\beta\omega_1}}, \epsilon_1=\frac{\omega_2-\omega_1}{\omega_1}, \epsilon_2=\frac{\omega_3-\omega_1}{\omega_1}, \frac{m_i}{\omega_1}, \frac{\phi}{\omega_1}\right) Z^{\mathbb{R}^4\times T^2}(2) Z^{\mathbb{R}^4\times T^2}(3)$$

- Here one really relies on non-renormalizable QFTs to get the proposals.

- In UV-incomplete SYM, the factors are NLSM partition functions, which is ambiguous.
- We use our 1d/2d UV completions to eliminate such ambiguities, claiming them to be intrinsic 5d/6d CFT partition functions: the final proposal doesn't refer to EFT.
- "guidance from EFT" + "UV completions in subsectors" + perhaps "natural guessworks"