# Non-perturbative Refined Topological String from Exact Quantization Condition

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- twisting world sheet Lorentz symmetry with supersymmetry, we would have 2 different model called A-model and B-model.
- They dual to each other by mirror symmetry.

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- Refine means there are two parameter $\epsilon_1, \epsilon_2$, other than one string coupling constant $g_s$

- Due to recent works on ABJM theory, a non-perturbative completion of topological string is suggested. [Hatsuda, Marino, Moriyama, Okuyama, arXiv:1306.1734]
- From ABJM theory, Kallen and Marino ('13) suggest a non-perturbative quantisation condition of a spectral curve

$$-1 + e^x + e^p + z_1 e^{-x} + z_2 e^{-p} = 0$$

- It'is a quantum curve of B-model curve of topological string, for toric cases. Quantum means

$$[x, p] = i\hbar$$

# Related works

- perturbative study [Aganagic, Cheng , Dijkgraaf, Krefl, Vafa, arXiv:1105.0630]

$$Vol_p = 2\pi\hbar(n + 1/2)$$

- numerical study(non-perturbative ) [Huang, Wang, arXiv:1406.6178]
- First gives correct analytic derivation, but the quantisation condition is complicate. Difficult to generalise [Grassi , Hatsuda , Marino, arXiv:1410.3382]

$$Vol = \Omega_p + \Omega_{np} + \lambda = 2\pi\hbar(n + 1/2)$$

where $\lambda$ is infinite correction

- We find a new formula do not need any corrections[XW , Zhang, Huang arXiv:1505.05630]

$$Vol = \mathrm{classical} + \hbar\, f_{NS}(\tilde{t}, \hbar) + \hbar\, f_{NS}(\frac{2\pi\tilde{t}}{\hbar}, \frac{4\pi^2}{\hbar}) = 2\pi\hbar(n + \frac{1}{2})$$

- Our result is generalised to high genus[1] cases by Hatsuda and Marino (relativistic Toda,1511), and Franco, Hatsuda, Marino (Cluster Integrable System,1512).

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- Our result is generalised to high genus[1] cases by Hatsuda and Marino (relativistic Toda,1511), and Franco, Hatsuda, Marino (Cluster Integrable System,1512).
- Recently, we find that, from the brane probe method by Aganagic, Cheng etc. The form of topological string have to be modified. Exactly the same modification as in the paper of Lockhart and Vafa ('12)

$$\begin{aligned} Z^{LV}(\mathbf{T}, \tau_1, \tau_2) = & Z_{ref}(\mathbf{T}, \tau_1 + 1, \tau_2) \\ & \times Z_{ref}(\frac{\mathbf{T}}{\tau_1}, \frac{1}{\tau_1}, \frac{\tau_2}{\tau_1} + 1) Z_{ref}(\frac{\mathbf{T}}{\tau_2}, \frac{\tau_1}{\tau_2} + 1, \frac{1}{\tau_2}). \end{aligned} \tag{1}$$

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$$Z^{LV}(\mathbf{T},\tau_1,\tau_2) = Z_{ref}(\mathbf{T},\tau_1+1,\tau_2) \times Z_{ref}\left(\frac{\mathbf{T}}{\tau_1},\frac{1}{\tau_1},\frac{\tau_2}{\tau_1}+1\right) Z_{ref}\left(\frac{\mathbf{T}}{\tau_2},\frac{\tau_1}{\tau_2}+1,\frac{1}{\tau_2}\right). \tag{1}$$

- To restore the normal Gopakumar Vafa formula, one also must shift the Kahler class parameter by a **B** field.

$$\mathbf{t} \to \mathbf{T} = \mathbf{t} + i\pi\mathbf{B}$$

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- Finally, the quantisation condition is [2]

$$
\begin{aligned}
Vol_i =& \frac{\partial}{\partial a_i}\epsilon_1\epsilon_2 F^{(LV,np)}(\mathbf{t} + i\pi\mathbf{B}, \epsilon_1 \to i\hbar, \epsilon_1 \to 0) \\
=& 2\pi h(n_i + 1/2)
\end{aligned} \tag{2}
$$

[2] $a_i = \sum_j C_{ij} t_j$