On Higgs Branch Localization of Seiberg-Witten Theories on Ellipsoid

Tsung-Hsuan Tsai

Supervised by Prof. Heng-Yu Chen

National Taiwan University

arXiv:1506.04390

OIST, Jan 9, 2016

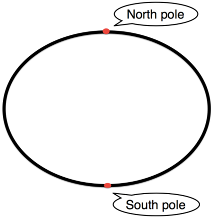

Figure : $S^4_{b^2}$ with north pole at $\rho = 0$ and south at pole $\rho = \pi$

$$S^4_{b^2}: \quad \frac{x_0^2}{r^2} + \frac{x_1^2 + x_2^2}{\ell^2} + \frac{x_3^2 + x_4^2}{\widetilde{\ell}^2} = 1$$

Basing on localization principle, with a appropriate deformed term,

$$\mathcal{I}_{\text{vec.}} = \mathrm{Tr}\left[(\mathbf{Q}\lambda_{\alpha A})^\dagger(\mathbf{Q}\lambda_{\alpha A}) + (\mathbf{Q}\bar{\lambda}^{\dot{\alpha}}{}_A)^\dagger(\mathbf{Q}\bar{\lambda}^{\dot{\alpha}}{}_A)\right].$$

Hama and Hosomichi have performed the exact result of path integral

$$\mathcal{Z}^{\text{SQCD}}_{S^4_{b^2}} =$$

$$\int \prod_{\mathrm{a}=1}^{N_c} \frac{d\hat{a}_{\mathrm{a}}}{N_c!}\, e^{-\frac{8\pi^2}{g^2_{\text{YM}}}\sum_{\mathrm{a}=1}^{N_c}\hat{a}^2_{\mathrm{a}} + 16 i\pi^2\hat{\zeta}\sum_{\mathrm{a}=1}^{N_c}\hat{a}_{\mathrm{a}}} \frac{\prod_{\mathrm{a}\neq\mathrm{b}=1}^{N_c}\Upsilon(i(\hat{a}_{\mathrm{a}} - \hat{a}_{\mathrm{b}}))}{\prod_{\mathrm{a}=1}^{N_c}\prod_{I=1}^{N_f}\Upsilon\left(i(\hat{a}_{\mathrm{a}} + \hat{\mu}_I) + \frac{Q}{2}\right)} |\mathcal{Z}_{\text{inst}}(\vec{\hat{a}}, \vec{\mu}, q)|^2$$

Higgs Branch Localization

Another deformed terms are

$$\mathcal{I}_{\text{Higgs}} = 2\mathbf{Q}\mathrm{Tr}\left[(\lambda_{\alpha A})^\dagger H_{\alpha A} + (\bar{\lambda}^{\dot{\alpha}}{}_A)^\dagger \bar{H}^{\dot{\alpha}}{}_A\right], \mathcal{I}_{\text{hyp.}} = \frac{1}{4}\mathrm{Tr}[(\mathbf{Q}\psi_{\alpha\hat{\imath}})^\dagger(\mathbf{Q}\psi_{\alpha\hat{\imath}}) + (\mathbf{Q}\bar{\psi}^{\dot{\alpha}}{}_{\hat{\imath}})^\dagger(\mathbf{Q}\bar{\psi}^{\dot{\alpha}}{}_{\hat{\imath}})]$$

Saddle point equation is

$$\mathcal{I}_{\text{vec.}} + \mathcal{I}_{\text{Higgs}} + \mathcal{I}_{\text{hyp.}}\,|_{\text{Bose}} = 0 \Longrightarrow \text{BPS solitons on sub-manifold}$$

The vortex equations are described by

$$\begin{aligned}
&(F_{32} + F_{41}) = \cos\theta H(q), \quad (F_{31} + F_{42}) = \sin\theta H(q), \quad (F_{12} + F_{34}) = 0,\\
&(F_{32} - F_{41}) = \cos\theta H(q), \quad (F_{31} - F_{42}) = \sin\theta H(q), \quad (F_{12} - F_{34}) = 0.\\
&D_n q_{1\hat{\imath}} = 0, \quad H(q) = (q_{\hat{\imath}A})^\dagger q_{\hat{\imath}A} - \zeta
\end{aligned}$$

Near the north pole(or south pole), the above set of equations describes the generalization instanton-vortex (anti-instanton-vortex) mixture configurations in 4d Ω-background with arbitrary $\epsilon_{1,2}$.

From the geometry structure and known 2d/4d duality on $\mathbb{R}^4$, we conjecture that two vortex world volume theories are $\mathcal{N} = (2, 2)$ SQCDA theories on S_b^2 and $S_{b^{-1}}^2$ respectively.

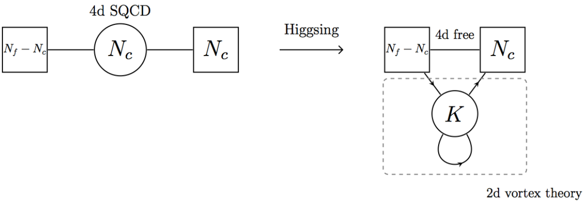

Figure : Higgsing of 4d $\mathcal{N} = 2$ SQCD into 2d $\mathcal{N} = (2, 2)$ SQCDA coupled to free 4d hypermultiplets.

Comparing with S_b^2 partition function of $\mathcal{N}=(2,2)$ theory

We integrate the $S^4_{b^2}$ partition function of $\mathcal{N}=2$ SQCD with $U(N_c)$ gauge group and N_f fundamental hypermultiplets by residue theorem. The pole is

$$\hat{a}_{\mathrm{a}} = -\left(\hat{\mu}_{I_{\mathrm{a}}} - i\frac{Q}{2}\right) + i\left(m_{\mathrm{a}} b + \frac{n_{\mathrm{a}}}{b}\right), \quad I_{\mathrm{a}} \in \{I\}, \quad m_{\mathrm{a}}, n_{\mathrm{a}} = 0,1,2,3,\ldots$$

$$\mathcal{Z}^{\mathrm{SQCD}}_{S^4_{b^2}} = \left(\sum_{m_{\mathrm{a}}=0}^{\infty}\sum_{n_{\mathrm{a}}=0}^{\infty} \mathbf{Z}^{\mathrm{class.}}_{b}\mathbf{Z}^{\mathrm{class.}}_{b^{-1}}\mathbf{Z}^{1\ \mathrm{loop}}_{\{\hat{m}_{\mathrm{a}}\}b}\mathbf{Z}^{1\ \mathrm{loop}}_{\{\hat{m}_{\mathrm{a}}\}b^{-1}}\left|\mathbb{Z}^{\mathrm{inst.}}_{\{m_{\mathrm{a}},n_{\mathrm{a}}\}}\right|^2\mathbb{Z}^{\mathrm{cross}}_{\{m_{\mathrm{a}},n_{\mathrm{a}}\}}\right)\times\mathbb{Z}^{\mathrm{4d\ free}}$$

$$\mathcal{Z}^{\mathrm{SQCDA}}_{S^2_b} = \sum_{\{\hat{m}_{\mathrm{a}}\}} \mathbf{Z}^{\mathrm{class.}}_{b}\mathbf{Z}^{1\ \mathrm{loop}}_{\{\hat{m}_{\mathrm{a}}\}b}\left|\mathbf{Z}^{\mathrm{vort.}}_{\{\hat{m}_{\mathrm{a}}\}b}(\mathrm{z})\right|^2.$$

Comparing both partition functions, the various parameters match as follow

Classical part $\quad \dfrac{4\pi}{g^2_{\mathrm{YM}}} = \mathbf{r}, \quad m_{\mathrm{a}} = \hat{m}_{\mathrm{a}}, \quad b^2 = i\mathrm{M}_{\mathbf{X}}, \quad ib\hat{\mu}_{I_{\mathrm{a}}} + b^2 + \dfrac{1}{2} = i\mathrm{M}_{\mathrm{a}}, \quad \mathrm{a} = 1,\ldots,N_c$

Perturbative part $\quad -ib\hat{\mu}_j - \dfrac{1}{2} = i\mathrm{M}_{\mathbf{j}}, \quad j \in \{I\}/\{I_{\mathrm{a}}\}, \quad \mathbf{j} = 1,\ldots,N_f - N_c.$

Non-perturbative part in some limit $\quad Y_{\mathrm{a}(m_{\mathrm{a}}-r)} = k_{\mathrm{a}r}, \quad \theta = \theta + (K-1)\pi,$