Scattering Amplitudes

LECTURE 2

Jaroslav Trnka

Walter Burke Institute for Theoretical Physics, Caltech

Center for Quantum Mathematics and Physics (QMAP), UC Davis

Asian winter school, Okinawa, January 2016

Review of Lecture 1

Spinor helicity variables

- Rewrite the four component momentum

$$p_1^\mu = \sigma^\mu_{a\dot{a}} \lambda_{1a}\widetilde{\lambda}_{1\dot{a}}$$

- Little group scaling

$$\lambda \to t\lambda \qquad p \to p$$
$$\widetilde{\lambda} \to \frac{1}{t}\widetilde{\lambda}$$

- Invariants

$$\langle 12\rangle \equiv \epsilon_{ab}\lambda_{1a}\lambda_{2b} \qquad [12] \equiv \epsilon_{\dot{a}\dot{b}}\widetilde{\lambda}_{1\dot{a}}\widetilde{\lambda}_{2\dot{b}}$$

$$s_{12} = \langle 12\rangle[12]$$

Three point amplitudes

- Three point kinematics

$$p_1^2 = p_2^2 = p_3^2 = 0 \qquad\qquad p_1 + p_2 + p_3 = 0$$

- Two solutions:

$$\langle 12\rangle = \langle 23\rangle = \langle 13\rangle = 0 \qquad\qquad [12] = [23] = [13] = 0$$

$$\lambda_1 \sim \lambda_2 \sim \lambda_3 \qquad\qquad \widetilde{\lambda}_1 \sim \widetilde{\lambda}_2 \sim \widetilde{\lambda}_3$$

$(- - +)$ ↓ No solution for real momenta ↓ $(+ + -)$

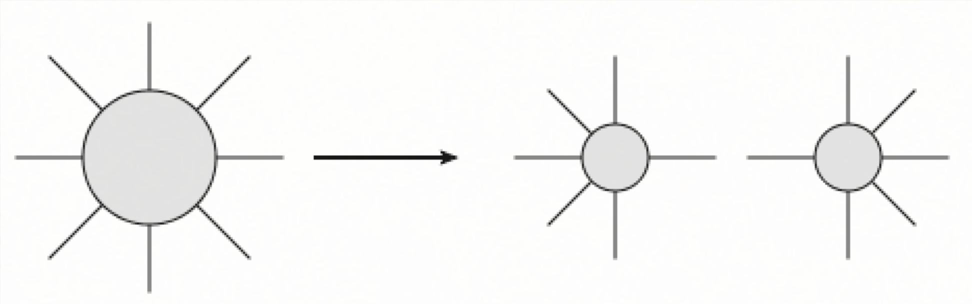
$$\frac{[ab]^4}{[12][23][31]}$$

Yang-Mills amplitudes

$$\frac{\langle a\, b\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}$$

Tree-level amplitudes

- Locality and unitarity



$$\mathcal{M} \xrightarrow[P^2=0]{} \mathcal{M}_L \frac{1}{P^2} \mathcal{M}_R$$

- On-shell constructibility: amplitude fixed by poles

- Color decomposition

$$\mathcal{M} = \sum_{\sigma} \mathrm{Tr}(T^{\sigma_1} T^{\sigma_2} T^{\sigma_3} \ldots T^{\sigma_n})\, A(123\ldots n)$$

Recursion relations

Tree level amplitudes

- Tree-level amplitude is a rational function of kinematics

$$A = \sum (\text{Feyn. diag}) = \frac{N}{\prod_j P_j^2}$$

momenta
polarization vectors

Feynman propagators

$$P_j = \sum_k p_k$$

- Only poles, no branch cuts

- Gauge invariant object: use spinor helicity variables

Momentum shift

- Let us shift two external momenta

$$\lambda_1 \to \lambda_1 - z\lambda_2 \qquad \widetilde{\lambda}_1 \to \widetilde{\lambda}_1$$

$$\lambda_2 \to \lambda_2 \qquad \widetilde{\lambda}_2 \to \widetilde{\lambda}_2 + z\widetilde{\lambda}_1$$

- Momentum is conserved, stays on-shell

$$(\lambda_1 - z\lambda_2)\widetilde{\lambda}_1 + \lambda_2(\widetilde{\lambda}_2 + z\widetilde{\lambda}_1) = \lambda_1\widetilde{\lambda}_1 + \lambda_2\widetilde{\lambda}_2$$

- This corresponds to shifting

$$p_1, p_2, \epsilon_1, \epsilon_2$$

Shifted amplitude

- On-shell tree-level amplitude with shifted kinematics

$$A_n(z) = A(\hat{p}_1(z), \hat{p}_2(z), p_3, \ldots, p_n)$$

- Analytic structure

$$A_n(z) = \frac{N(z)}{\prod_j P_j(z)^2}$$

- Location of poles:

$$P_j(z) = P_j - z\lambda_2\widetilde{\lambda}_1 \quad \text{if} \quad p_1 \in P_j$$

$$P_j(z) = P_j + z\lambda_2\widetilde{\lambda}_1 \quad \text{if} \quad p_2 \in P_j$$

$$P_j(z) = P_j \qquad \text{otherwise}$$

Shifted amplitude

- On the pole if $p_1 \in P_j$

$$P_j(z)^2 = P_j^2 - 2z\langle 1|P_j|2] = 0$$

$$z = \frac{P_j^2}{2\langle 1|P_j|2]} \equiv z_j$$

- Shifted amplitude:

$$A_n(z) = \frac{N(z)}{\prod_j P_j(z)^2}$$

location of poles

Residue theorem

- Shifted amplitude $A_n(z) = \dfrac{N(z)}{\prod_k (z - z_k)}$
- Let us consider the contour integral

$$\int \frac{dz}{z} A_n(z) = 0$$

No pole at $z \to \infty$

- Original amplitude $A_n = A_n(z = 0)$

Residue at

$z = 0$

- Residue theorem: $A_n + \sum_k \text{Res}\left(\frac{A_n(z)}{z}\right)\Bigg|_{z=z_k} = 0$

Residue theorem

$$\boxed{A_n = -\sum_k \mathrm{Res}\left(\frac{A_n(z)}{z}\right)\Bigg|_{z=z_k}}$$

Residue on the pole $P_j(z)^2 = 0$

- Unitarity of shifted tree-level amplitude

$$A_n(z) \xrightarrow[P_j(z)^2=0]{} A_L(z)\frac{1}{P_j(z)^2}A_R(z)$$

Residue theorem

$$\boxed{A_n = -\sum_k \mathrm{Res}\left(\frac{A_n(z)}{z}\right)\Bigg|_{z=z_k}}$$

Residue on the pole $P_j(z)^2 = 2\langle 1|P_j|2](z_j - z) = 0$

- Unitarity of shifted tree-level amplitude $\quad z_j = \dfrac{P_j^2}{2\langle 1|P_j|2]}$

$$A_n(z) \xrightarrow[z=z_j]{} A_L(z_j)\frac{1}{2\langle 1|P_j|2]}A_R(z_j)$$

Residue theorem

$$\boxed{A_n = -\sum_k \mathrm{Res}\left(\frac{A_n(z)}{z}\right)\bigg|_{z=z_k}}$$

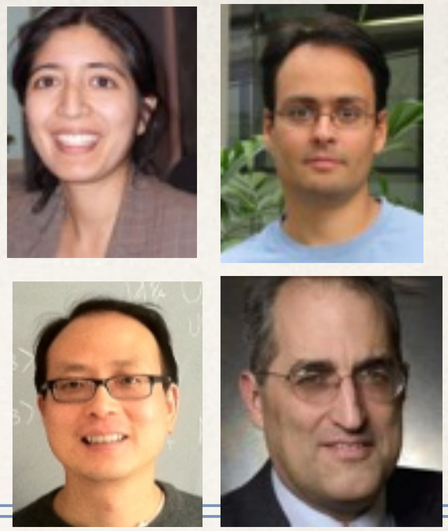
$$A_L(z_j)\frac{1}{2\langle 1|P_j|2]}A_R(z_j) \times \frac{2\langle 1|P_j|2]}{P_j^2} = A_L(z_j)\frac{1}{P_j^2}A_R(z_j)$$

<u>Final formula</u>

$$\boxed{A_n = -\sum_j A_L(z_j)\frac{1}{P_j^2}A_R(z_j)} \qquad z_j = \frac{P_j^2}{2\langle 1|P_j|2]}$$

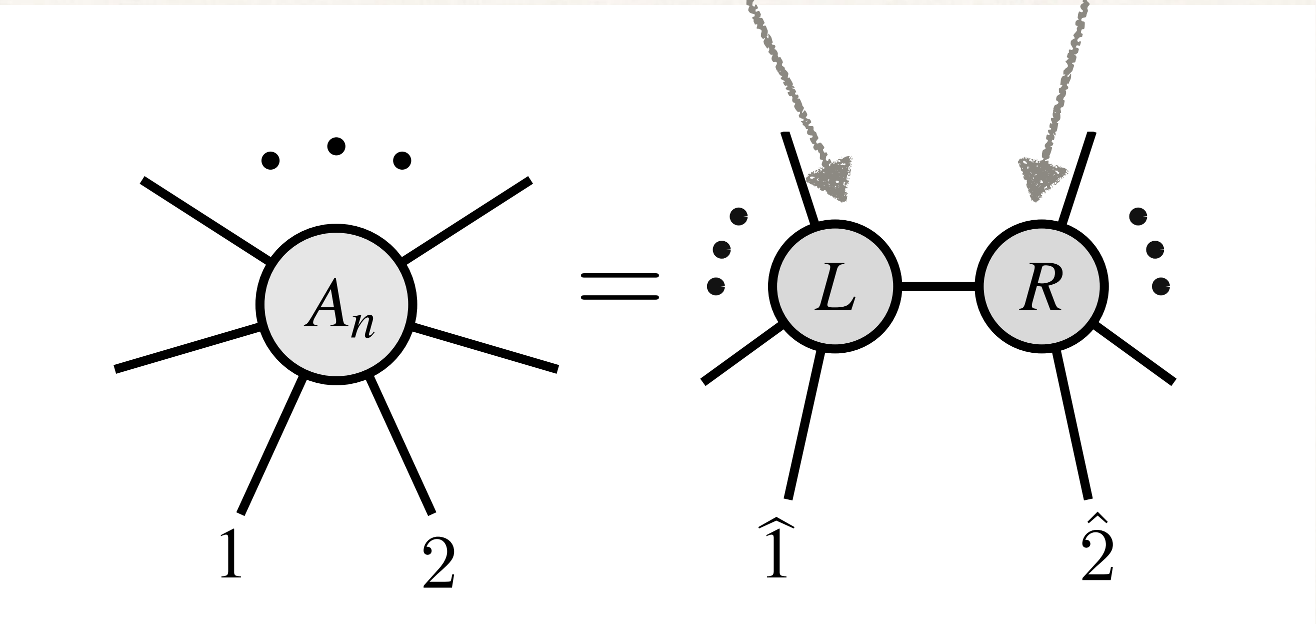
BCFW recursion relations

(Britto, Cachazo, Feng, Witten, 2005)



$$\boxed{A_n = -\sum_j A_L(z_j)\frac{1}{P_j^2}A_R(z_j)}$$

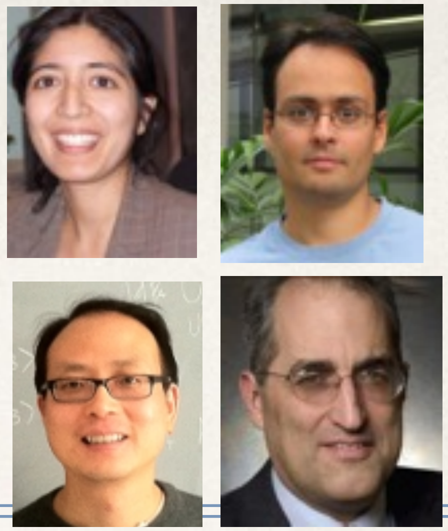
$$z_j = \frac{P_j^2}{2\langle 1|P_j|2]}$$



Chosen such that internal line is on-shell

Sum over all distributions of legs keeping 1,2 on different sides

BCFW recursion relations

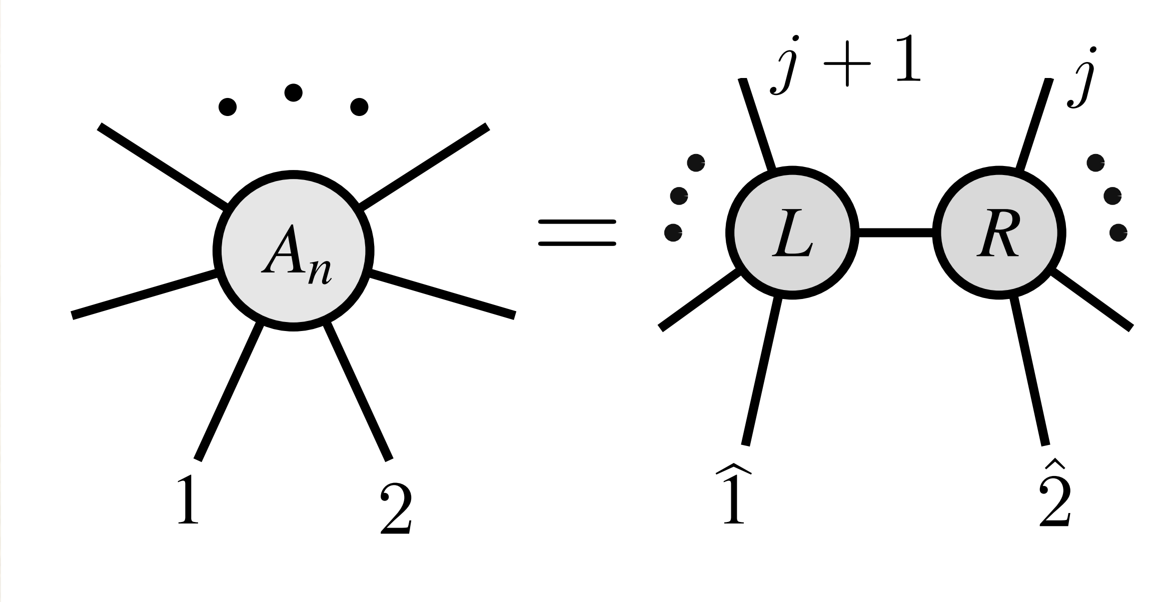


(Britto, Cachazo, Feng, Witten, 2005)

$$\boxed{A_n = -\sum_j A_L(z_j)\frac{1}{P_j^2}A_R(z_j)}$$

$$z_j = \frac{P_j^2}{2\langle 1|P_j|2]}$$

For ordered amplitudes $A(123\ldots n)$



Also sum over helicities of internal particle

Sum over all $j = 3, 4, \ldots n-1$

Comment on applicability

- The crucial property is $A_n(z) \to 0$ for $z \to \infty$
- In Yang-Mills theory this is satisfied if

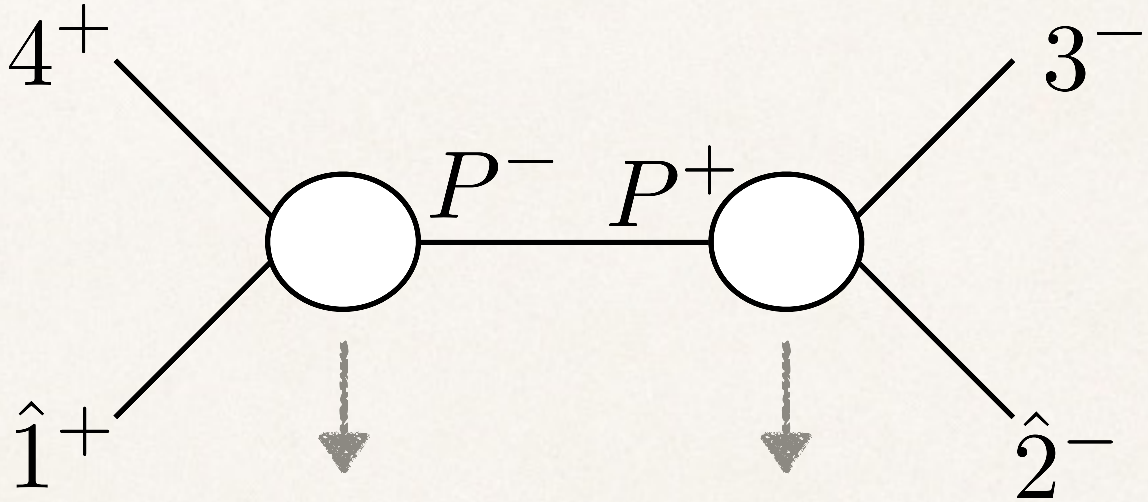
$$\lambda_1 \to \lambda_1 - z\lambda_2 \longleftarrow \text{Helicity +}$$

$$\widetilde{\lambda}_2 \to \widetilde{\lambda}_2 + z\widetilde{\lambda}_1 \longleftarrow \text{Helicity -}$$

- Same is true for Einstein gravity, and many others
- This means that amplitudes in these theories are fully specified by residues on their poles

Example 1: 4pt amplitude

- Let us consider amplitude of gluons $A_4(1^+2^-3^-4^+)$



Only one term contributes

$$\frac{[\hat{1}4]^3}{[\hat{1}P][4P]} \quad \frac{1}{s_{23}} \quad \frac{\langle\hat{2}3\rangle^3}{\langle\hat{2}P\rangle\langle 3P\rangle}$$

$$\hat{\lambda}_1 = \lambda_1 - z\lambda_2$$

$$\hat{\tilde{\lambda}}_2 = \tilde{\lambda}_2 + z\tilde{\lambda}_1$$

z takes the value when P is on-shell momentum

Example 1: 4pt amplitude

- Let us consider amplitude of gluons $A_4(1^+2^-3^-4^+)$

$$P^2 = \langle \hat{1}4\rangle[14] = 0$$

$$\downarrow$$

$$\langle \hat{1}4\rangle = \langle 14\rangle - z\langle 24\rangle = 0 \quad \rightarrow \quad z = \frac{\langle 14\rangle}{\langle 24\rangle}$$

We can now rewrite

Shouten identity

$$\hat{\lambda}_1 = \lambda_1 - z\lambda_2 = \lambda_1 - \frac{\langle 14\rangle}{\langle 24\rangle}\lambda_2 \overset{\downarrow}{=} \frac{\langle 12\rangle}{\langle 24\rangle}\lambda_4$$

$$\widetilde{\lambda}_2 = \widetilde{\lambda}_2 + z\widetilde{\lambda}_1 = \frac{[12]}{[13]}\widetilde{\lambda}_3 \quad \longleftarrow$$

Use of momentum conservation

Example 1: 4pt amplitude

- Let us consider amplitude of gluons $A_4(1^+2^-3^-4^+)$

$$\hat{\lambda}_1 = \frac{\langle 12\rangle}{\langle 24\rangle}\lambda_4$$

Calculate on-shell momentum P

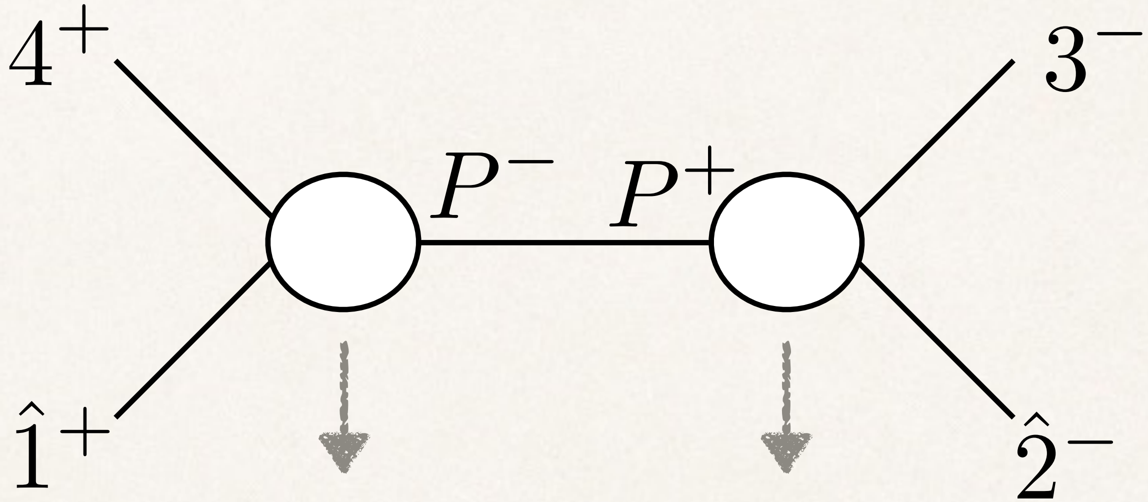
$$P = \hat{\lambda}_1\widetilde{\lambda}_1 + \lambda_4\widetilde{\lambda}_4 = \lambda_4\left(\frac{\langle 12\rangle}{\langle 24\rangle}\widetilde{\lambda}_1 + \widetilde{\lambda}_4\right)$$

$$\downarrow \qquad\qquad \downarrow$$

$$\lambda_P = \lambda_4 \qquad \widetilde{\lambda}_P = \frac{\langle 23\rangle}{\langle 24\rangle}\widetilde{\lambda}_3$$

Example 1: 4pt amplitude

- Let us consider amplitude of gluons $A_4(1^+2^-3^-4^+)$



$$\frac{[\hat{1}4]^3}{[\hat{1}P][4P]}\quad\frac{1}{s_{23}}\quad\frac{\langle\hat{2}3\rangle^3}{\langle\hat{2}P\rangle\langle 3P\rangle}$$

$$\hat{\lambda}_1=\frac{\langle 12\rangle}{\langle 24\rangle}\lambda_4$$

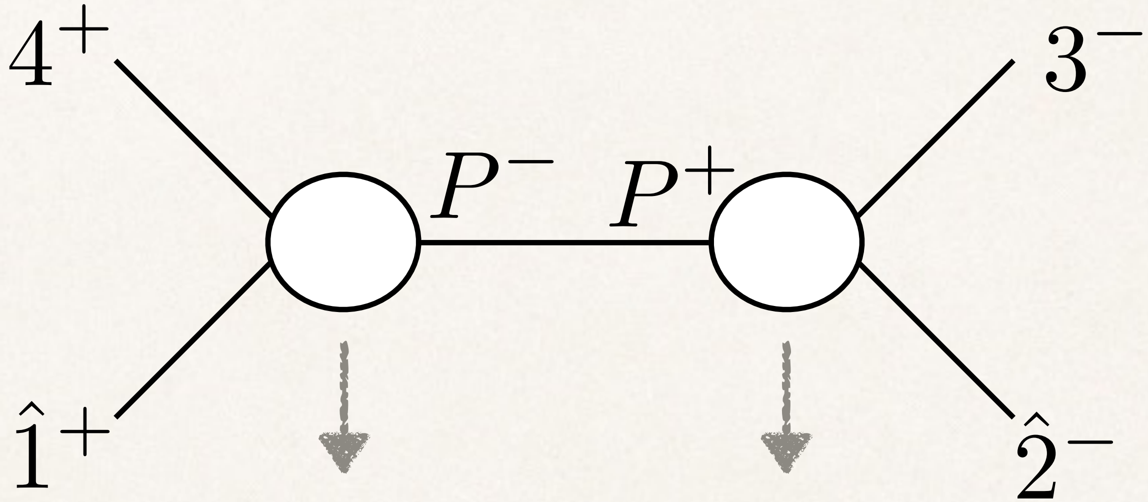
$$\hat{\tilde{\lambda}}_2=\frac{[12]}{[13]}\tilde{\lambda}_3$$

$$\lambda_P=\lambda_4$$

$$\tilde{\lambda}_P=\frac{\langle 23\rangle}{\langle 24\rangle}\tilde{\lambda}_3$$

Example 1: 4pt amplitude

- Let us consider amplitude of gluons $A_4(1^+2^-3^-4^+)$



$$\frac{[\hat{1}4]^3}{[\hat{1}P][4P]}\ \frac{1}{s_{23}}\ \frac{\langle\hat{2}3\rangle^3}{\langle\hat{2}P\rangle\langle 3P\rangle}$$

$$\boxed{=\frac{\langle 23\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}}$$

$$\hat{\lambda}_1=\frac{\langle 12\rangle}{\langle 24\rangle}\lambda_4$$

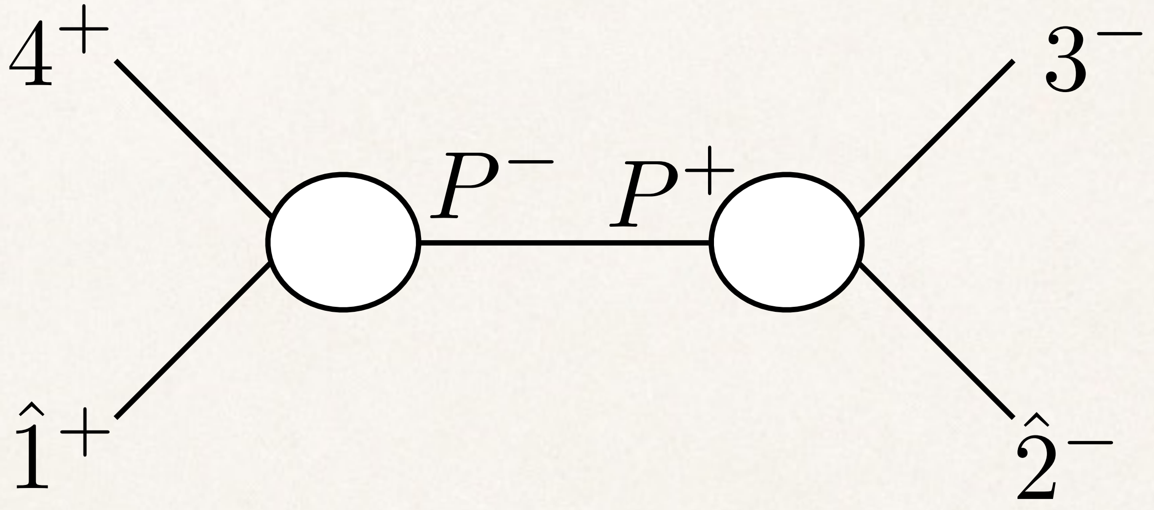
$$\hat{\tilde{\lambda}}_2=\frac{[12]}{[13]}\tilde{\lambda}_3$$

$$\lambda_P=\lambda_4$$

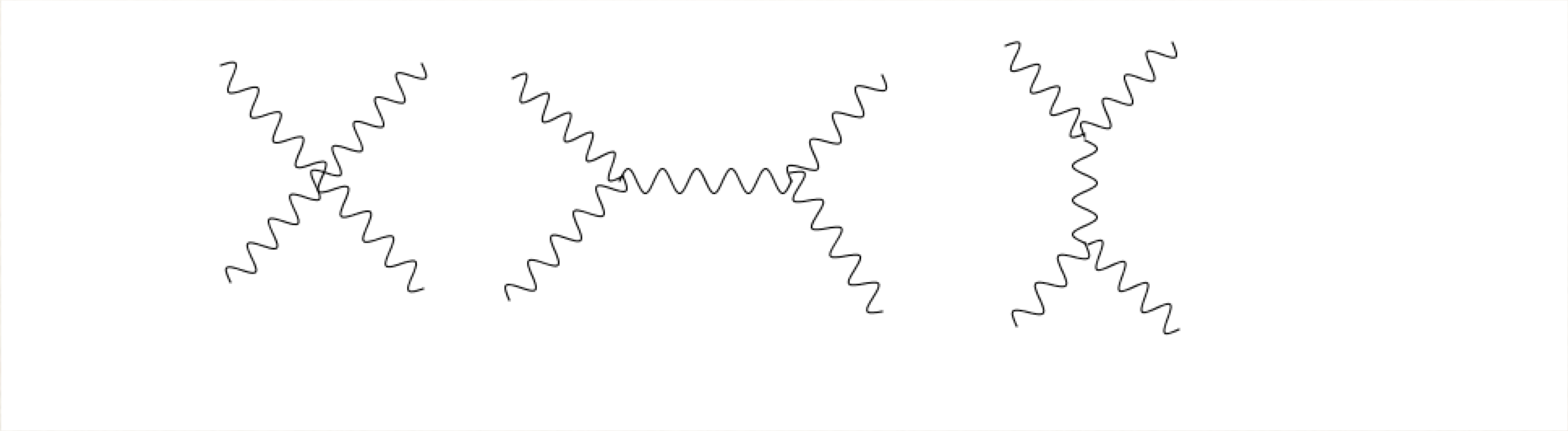
$$\tilde{\lambda}_P=\frac{\langle 23\rangle}{\langle 24\rangle}\tilde{\lambda}_3$$

Example 1: 4pt amplitude

- Let us consider amplitude of gluons $A_4(1^+2^-3^-4^+)$

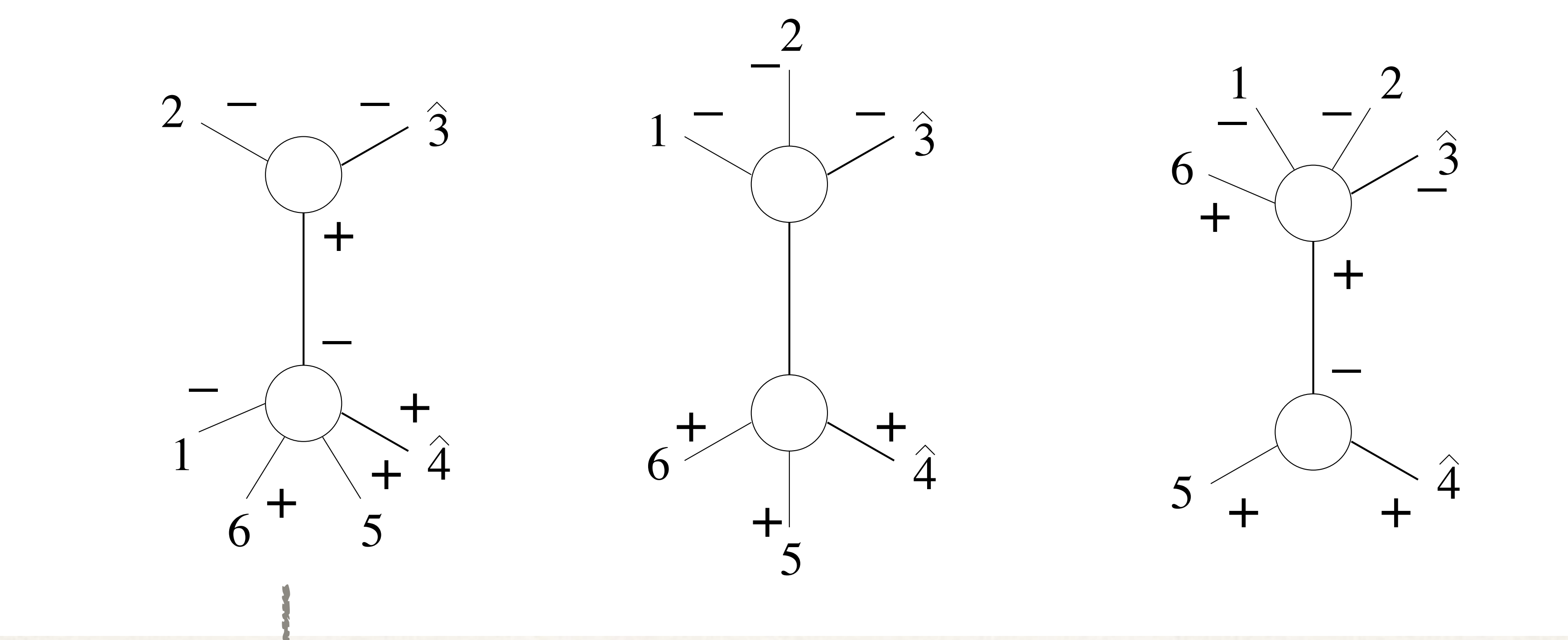


One gauge invariant object equivalent to three Feynman diagrams



Example 2: 6pt amplitude

- Let us consider $A_6(1^-2^-3^-4^+5^+6^+)$ and shift legs 3,4



vs

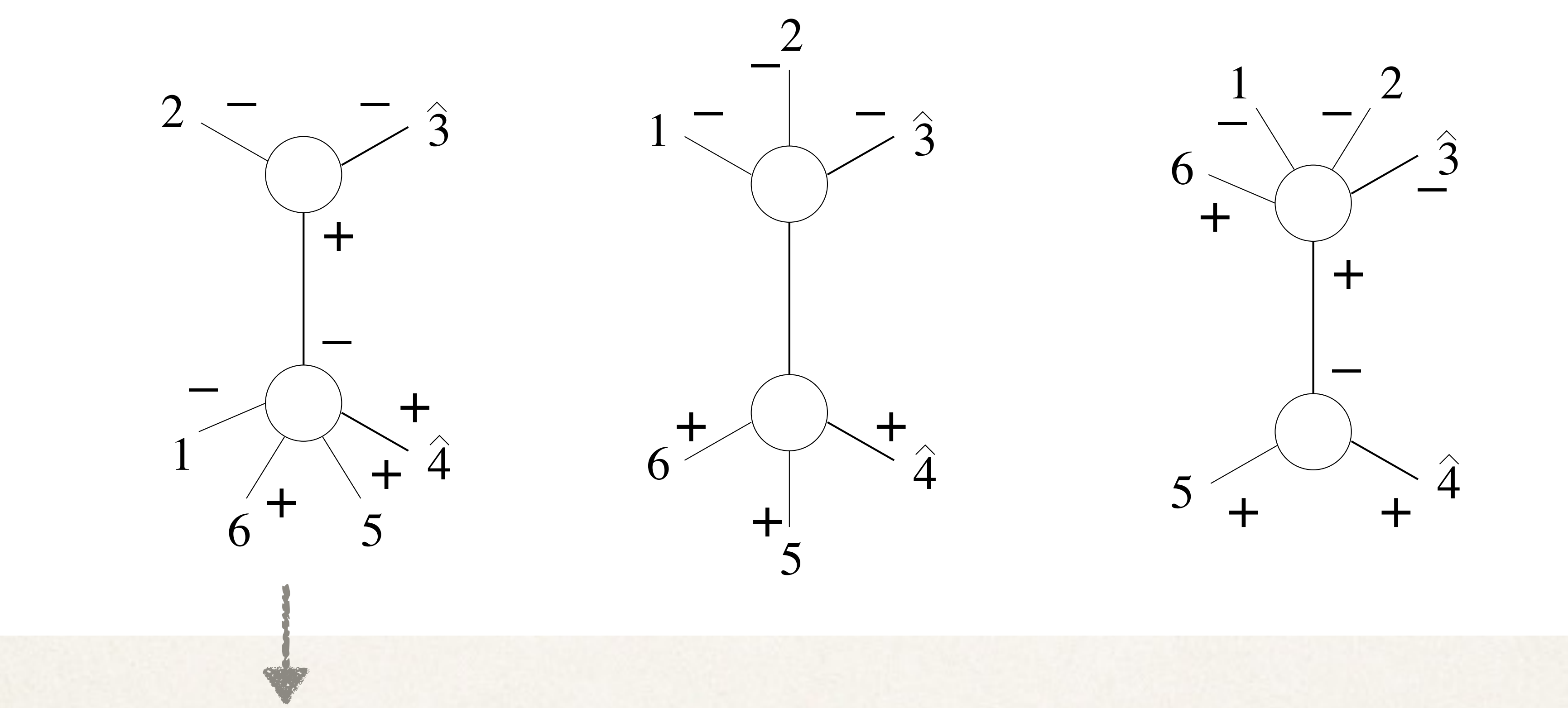
220 Feynman diagrams

$$\frac{\langle 1|2+3|4]^3}{[23][34]\langle 56\rangle\langle 61\rangle s_{234}\langle 5|3+4|2]}$$

$$\langle 1|2+3|4] = \langle 12\rangle[24] + \langle 13\rangle[34]$$

Example 2: 6pt amplitude

- Let us consider $A_6(1^-2^-3^-4^+5^+6^+)$ and shift legs 3,4



$$\frac{\langle 1|2+3|4]^3}{[23][34]\langle 56\rangle\langle 61\rangle s_{234}\boxed{\langle 5|3+4|2]}}$$

Spurious pole

Remark on BCFW

- Extremely efficient (3 vs 220 for 6pt, 20 vs 34300 for 8pt)
- Terms in BCFW recursion relations
    - Gauge invariant
    - Spurious poles
- Amplitude = sum of these terms dictated by unitarity
- Note: not all factorization channels are present

  when 1,2 are on the same side

Unitarity methods

One-loop amplitudes

- Sum of Feynman diagrams

$$\mathcal{M}^{1-loop} = \sum_j \int d\mathcal{I}_j \quad \text{where} \quad d\mathcal{I}_j = d^4\ell \, \mathcal{I}_j$$

- Re-express as basis of canonical integrals

Rational

$$\mathcal{M}^{1-loop} = \sum_j a_j \int d\mathcal{I}_j^{(4)} + \sum_j b_j \int d\mathcal{I}_j^{(3)} + \sum_j c_j \int d\mathcal{I}_j^{(2)} + \mathcal{R}$$

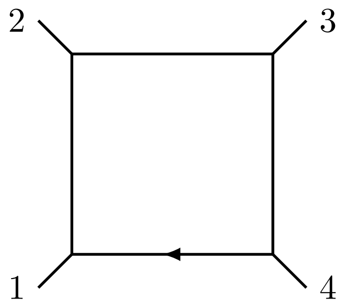
Box

Triangle

Bubble

One loop amplitudes

- Box integral



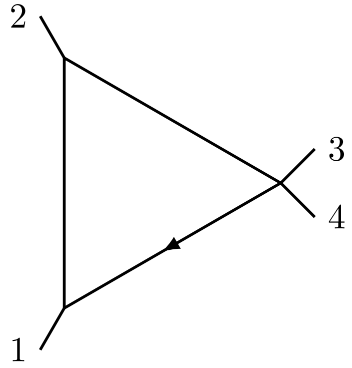
$$I = \frac{d^4\ell \; st}{\ell^2(\ell+k_1)^2(\ell+k_1+k_2)^2(\ell-k_4)^2}$$

Tadpoles and other integrals

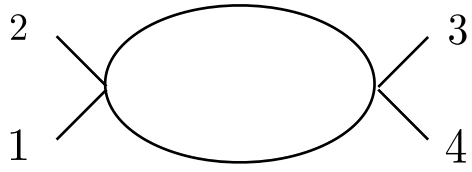
↓

Vanish in dim reg

- Triangle and box integrals



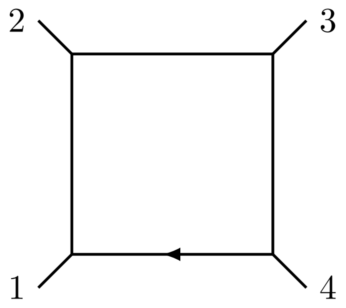
$$I = \frac{d^4\ell \; s}{\ell^2(\ell+k_1)^2(\ell+k_1+k_2)^2}$$



$$I = \frac{d^4\ell \, s}{\ell^2(\ell+k_1+k_2)^2}$$

One loop amplitudes

- Box integral



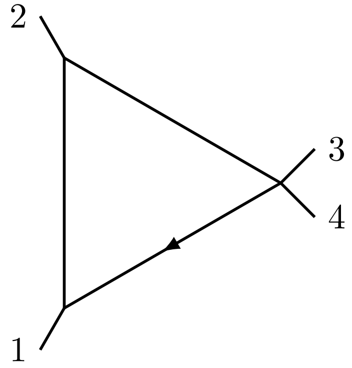
$$I = \frac{d^4\ell \;\, st}{\ell^2(\ell+k_1)^2(\ell+k_1+k_2)^2(\ell-k_4)^2}$$

Tadpoles and
other integrals

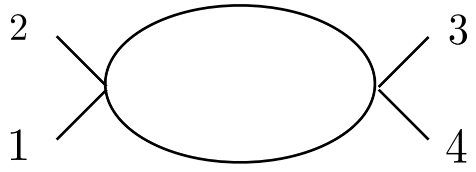
↓

Vanish in dim reg

- Triangle and box integrals



$$I = \frac{d^4\ell \;\, s}{\ell^2(\ell+k_1)^2(\ell+k_1+k_2)^2}$$



$$I = \frac{d^4\ell\, s}{\ell^2(\ell+k_1+k_2)^2}$$

UV divergent

(Super) Yang Mills amplitudes

- One-loop expansion in SYM theory

$$\mathcal{M} = \sum \text{Boxes} + \sum \text{Triangle} + \sum \text{Bubble} + \text{Rational}$$

Pure Yang-Mills

(Super) Yang Mills amplitudes

- One-loop expansion in SYM theory

$$\mathcal{M} = \sum \text{Boxes} + \sum \text{Triangle} + \sum \text{Bubble} + \xcancel{\text{Rational}}$$

N=1 and N=2 Super Yang-Mills

(Super) Yang Mills amplitudes

- One-loop expansion in SYM theory

$$\mathcal{M} = \sum \mathrm{Boxes} + \sum \xcancel{\mathrm{Triangle}} + \sum \xcancel{\mathrm{Bubble}} + \xcancel{\mathrm{Rational}}$$

N=4 Super Yang-Mills

- Note that it is UV finite at 1-loop, but also all loops

One loop expansion

- One-loop expansion

$$\mathcal{M} = \sum_j a_j \mathrm{Boxes}_j + \sum_j b_j \mathrm{Triangle}_j + \sum_j c_j \mathrm{Bubble}_j + \mathrm{Rational}$$

One loop expansion

- One-loop expansion

$$\mathcal{M} = \sum_j a_j \text{Boxes}_j + \sum_j b_j \text{Triangle}_j + \sum_j c_j \text{Bubble}_j + \text{Rational}$$

How to calculate
these coefficients?

How to calculate
this function?

One loop expansion

- One-loop expansion

$$\mathcal{M} = \sum_j a_j \text{Boxes}_j + \sum_j b_j \text{Triangle}_j + \sum_j c_j \text{Bubble}_j + \text{Rational}$$

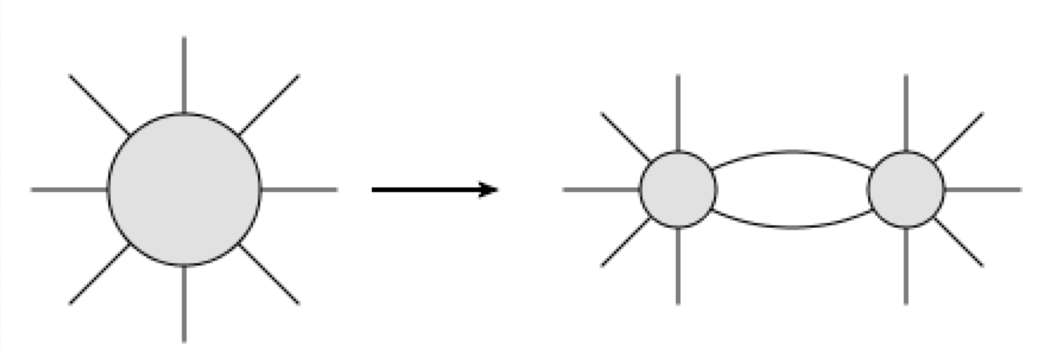
How to calculate
these coefficients?

How to calculate
this function?

Unitarity methods

One loop unitarity

- Analogue of tree-level unitarity at one-loop



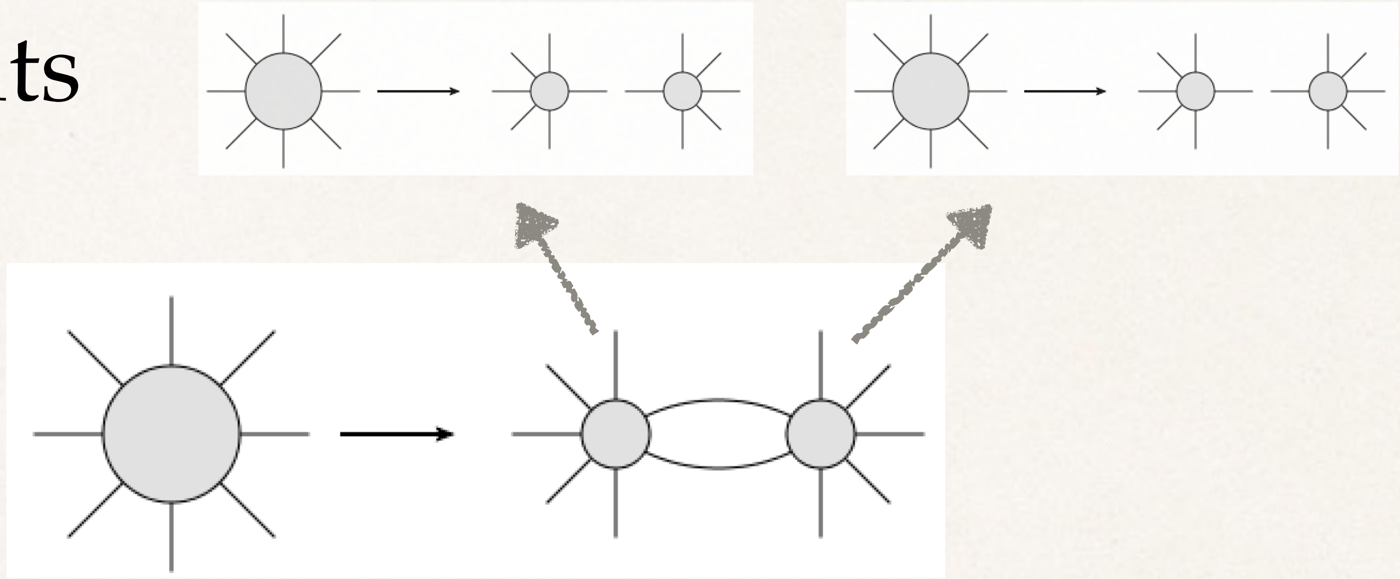
$$\mathcal{M}^{1-loop} \xrightarrow[\ell^2=(\ell+Q)^2=0]{} \mathcal{M}_L^{tree} \frac{1}{\ell^2(\ell+Q)^2} \mathcal{M}_R^{tree}$$

Unitarity cut

- In general $\text{Cut} \leftrightarrow \ell^2 = 0$

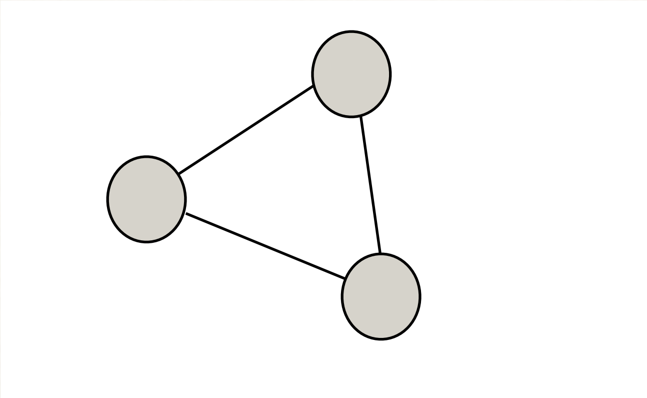
One-loop unitarity

- Higher cuts



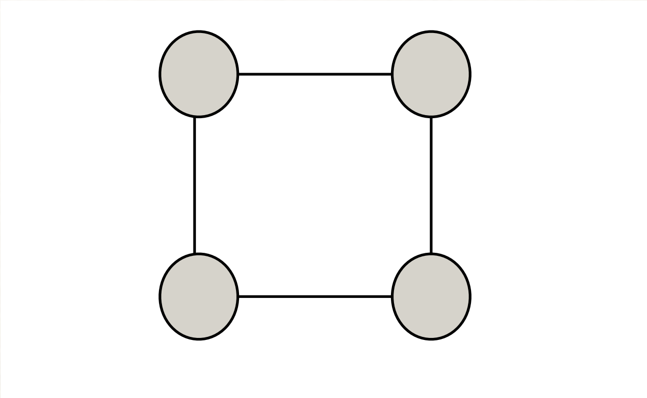
Triple cut

$$\ell^2 = (\ell + Q_1)^2 = (\ell + Q_2)^2 = 0$$



Quadruple cut

$$\ell^2 = (\ell + Q_1)^2 = (\ell + Q_2)^2 = (\ell + Q_3)^2 = 0$$



Fixing coefficients

- Perform cut on both side of equation

$$\mathcal{M} = \sum_j a_j \mathrm{Boxes}_j + \sum_j b_j \mathrm{Triangle}_j + \sum_j c_j \mathrm{Bubble}_j + \mathrm{Rational}$$

Product of trees   Linear combination of coefficients

- Example: Quadruple cut - only one box contributes

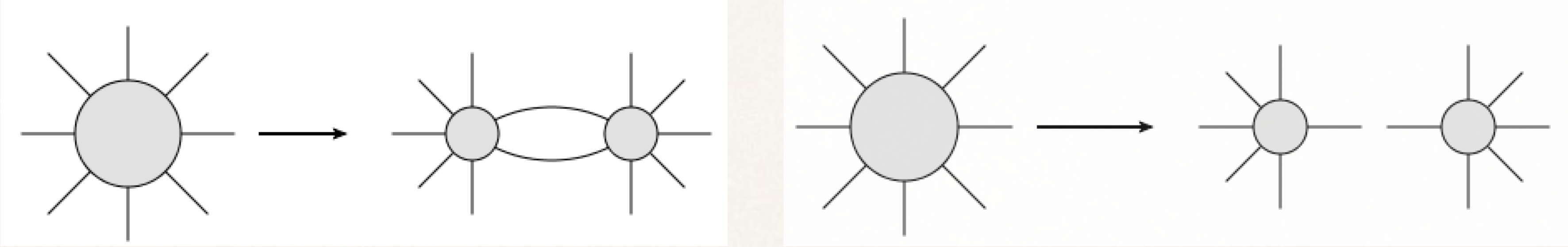
$$\mathcal{M}_1^{tree} \mathcal{M}_2^{tree} \mathcal{M}_3^{tree} \mathcal{M}_4^{tree} = a_j$$

- All coefficients a_j, b_j, c_j can be obtained

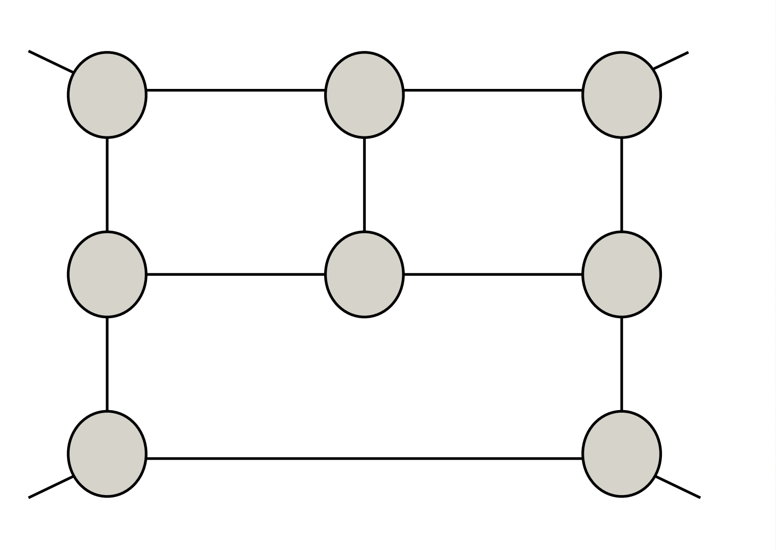
Unitarity methods

(Bern, Dixon, Kosower)

- We can iterate both types of cuts



- Stop when all propagators are cut: maximal cut



Product of 3pt on-shell amplitudes

Which 3pt amplitude?

Default answer: sum over all possibilities

Wait until next lecture for more….

Unitarity methods

(Bern, Dixon, Kosower)

- Expansion of the amplitude

$$\mathcal{M}^{\ell-loop} = \sum_j a_j \int d\mathcal{I}_j$$

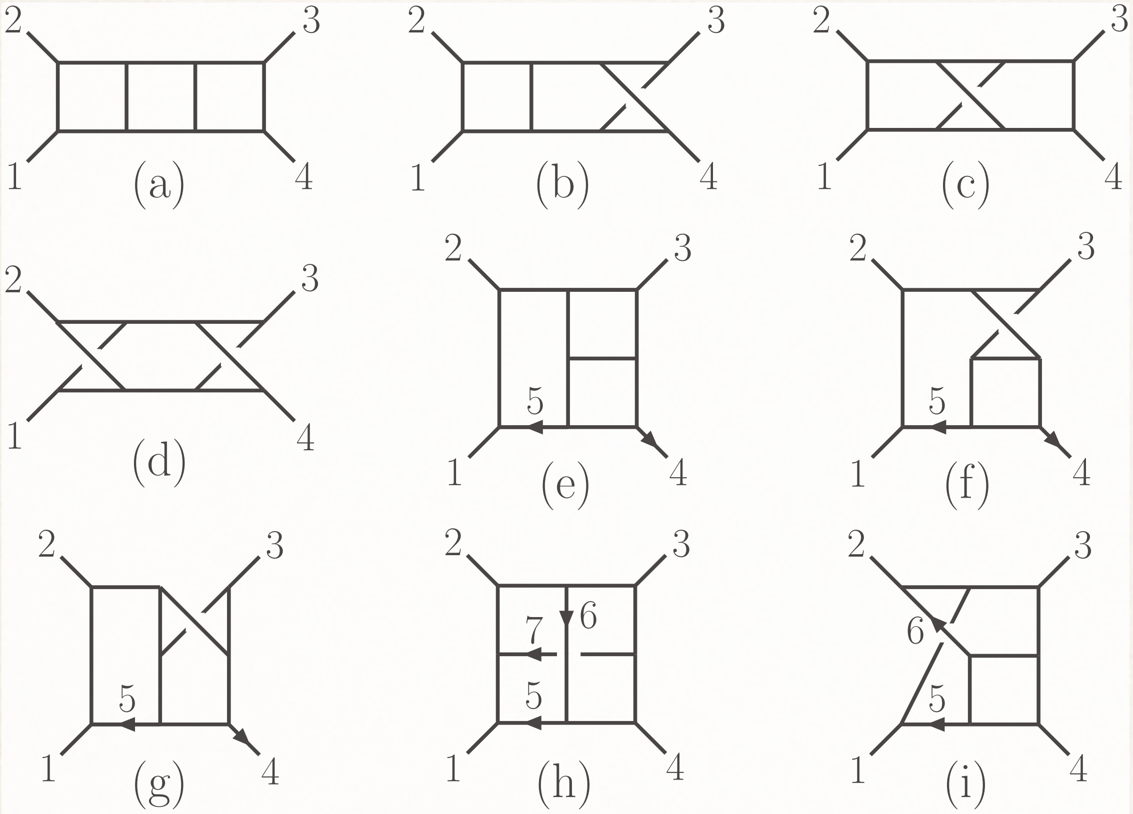
Cuts give product of trees

Linear combinations of coefficients a_j

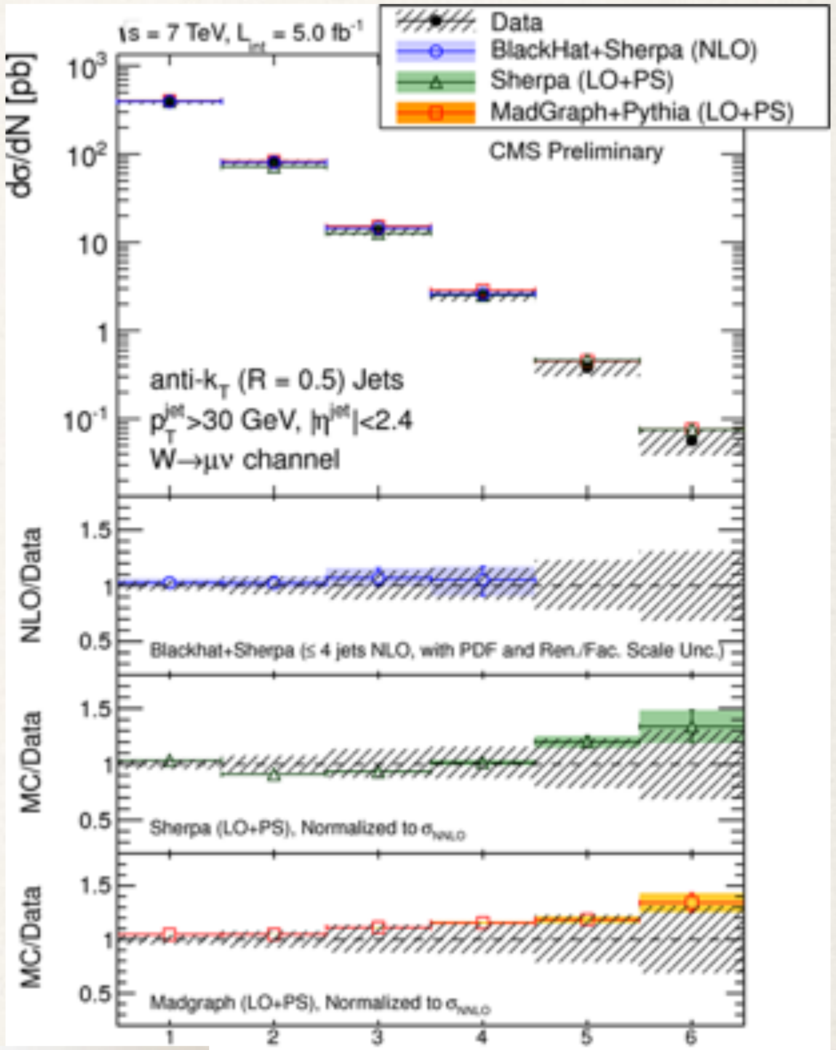
- Very successful method for loop amplitudes in different theories
- Practical problems:
  - Find basis of integrals
  - Solve (long) system of equations

Unitarity methods

- Results in susy theories and QCD



Basis of integrals for 3-loop amplitudes
in N=4 SYM and N=8 SUGRA



Black Hat

Leading singularity

(Cachazo)

- Maximal cut: number of propagators cut $P \leq 3L + n - 3$
- Number of degrees of freedom in loop momenta $4L$
- For example: n=4 we can cut only $3L + 1 < 4L$ times
- Residue depends on remaining degrees of freedom

$$\mathcal{M} \xrightarrow{\text{Maxcut}} F(\alpha_j, p_k) \longleftarrow \text{This function has poles in } \alpha_j$$

$$\Big\downarrow \text{ Calculate residue until all } \alpha_j \text{ are fixed}$$

$G(p_k)$ Leading singularity

On-shell good, off-shell bad

- Feynman diagrams: off-shell objects

Off-shell objects

- Unitarity methods: $\mathrm{Cut}[\mathcal{M}] = \mathrm{Cut}[\text{Basis of integrals}]$

On-shell objects

Locality
Unitarity

- Recursion relations

$$\mathcal{M} \sim \mathcal{M}_L \, \mathcal{M}_R$$

On-shell objects

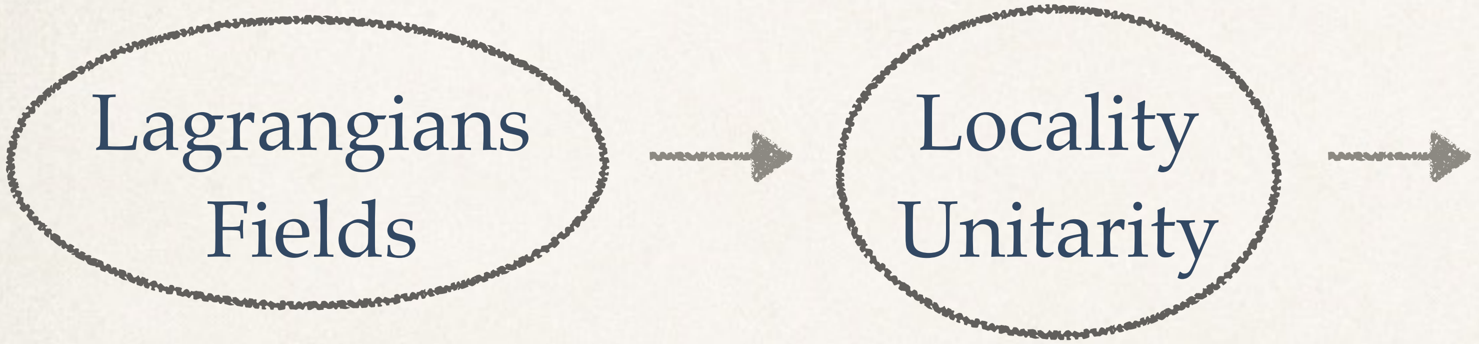
Locality lost
Unitarity

- Next direction: loosing manifest locality and unitarity

Toy model: Planar N=4 SYM

New framework

- Motivation: find different formulation of amplitudes
- Looking for a different starting point

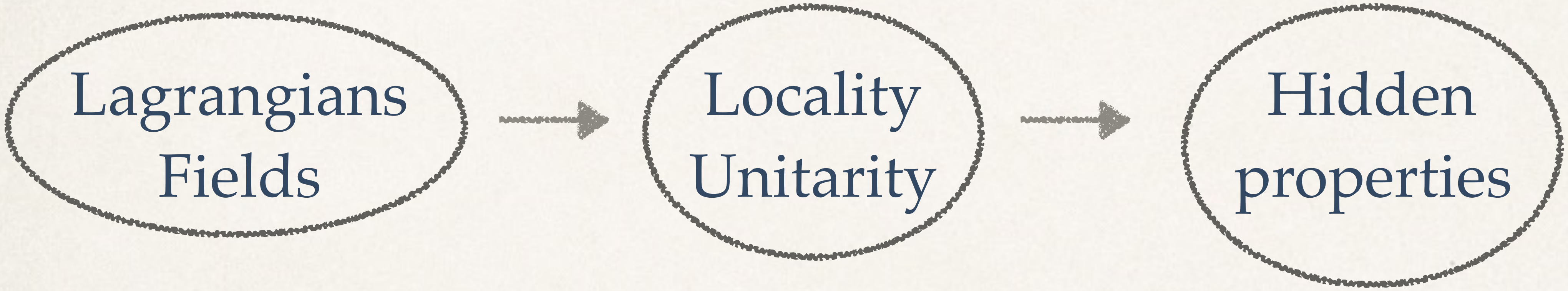


Lagrangians
Fields

Locality
Unitarity

New framework

- Motivation: find different formulation of amplitudes
- Looking for a different starting point



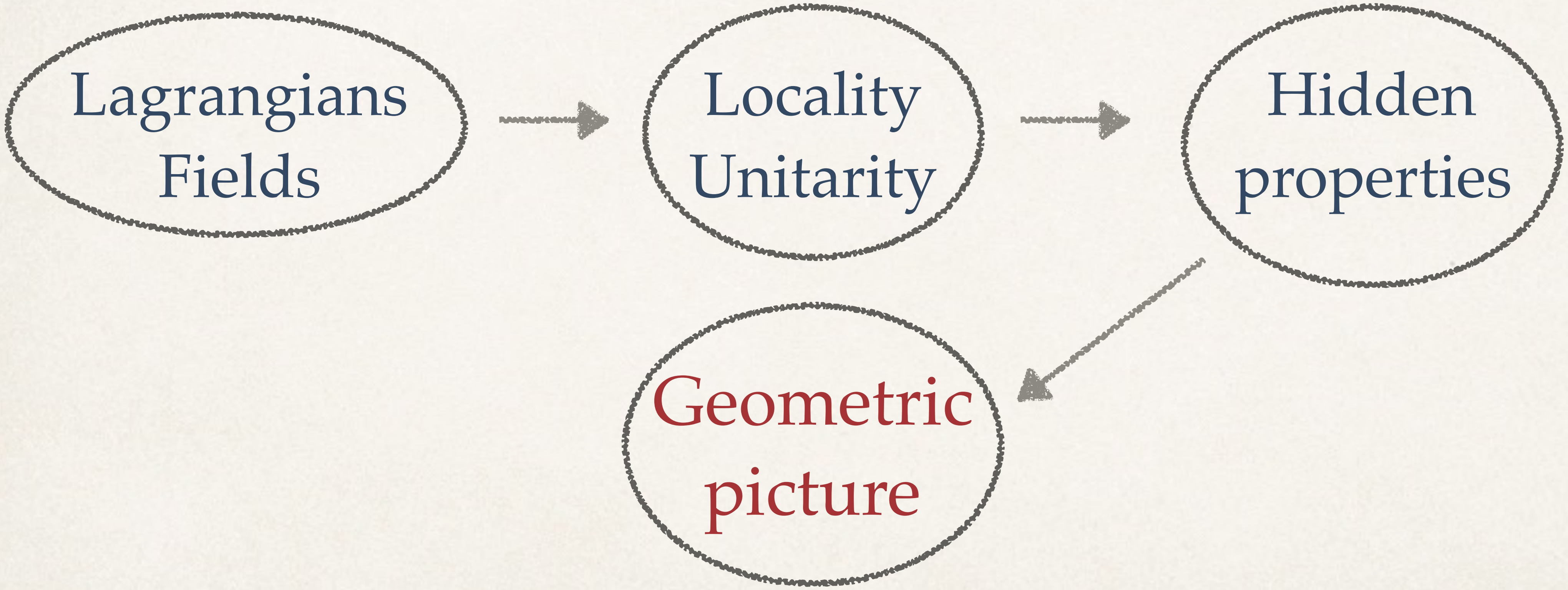
Lagrangians
Fields

Locality
Unitarity

Hidden
properties

New framework

- Motivation: find different formulation of amplitudes
- Looking for a different starting point



Lagrangians
Fields

Locality
Unitarity

Hidden
properties

Geometric
picture

Toy model

- This is a great success; is there a deeper structure?
- Time-proven method: study a toy model first

  Wish list:

  - Four-dimensional interacting theory
  - Close to the real world (QCD) as much as possible
  - Ability to generate plenty of explicit results

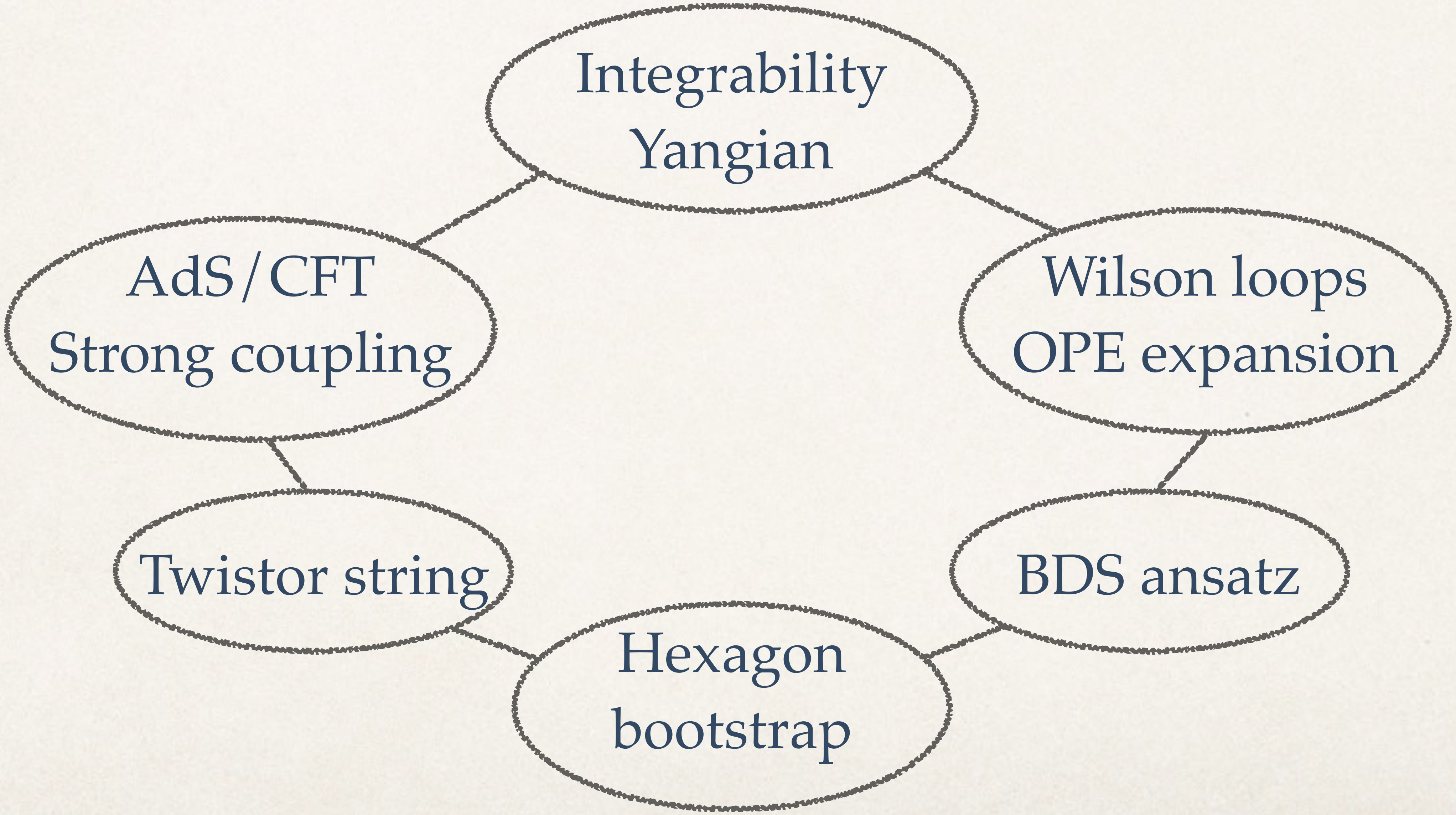
Planar N=4 Super Yang-Mills theory

(Brink-Scherk-Schwarz 1977)

- Conformal, convergent series
- Great toy model for QCD
  - Tree-level amplitudes identical
  - Loop amplitudes simpler, structures similar
  - But, no confinement :(
- Past: new methods for amplitudes originated here

Many faces of the theory

- Useful playground for many theoretical ideas



Integrability
Yangian

AdS/CFT
Strong coupling

Wilson loops
OPE expansion

Twistor string

BDS ansatz

Hexagon
bootstrap

Amplitudes in N=4 SYM

- N=4 superfield

$$\Phi = G_+ + \tilde{\eta}_A \Gamma_A + \frac{1}{2}\tilde{\eta}^A \tilde{\eta}^B S_{AB} + \frac{1}{6}\epsilon_{ABCD}\tilde{\eta}^A \tilde{\eta}^B \tilde{\eta}^C \overline{\Gamma}^D + \frac{1}{24}\epsilon_{ABCD}\tilde{\eta}^A \tilde{\eta}^B \tilde{\eta}^C \tilde{\eta}^D G_-$$

- Superamplitudes: $\mathcal{A}_n = \sum_{k=2}^{n-2} \mathcal{A}_{n,k}$

Component amplitudes with power $\tilde{\eta}^{4k}$

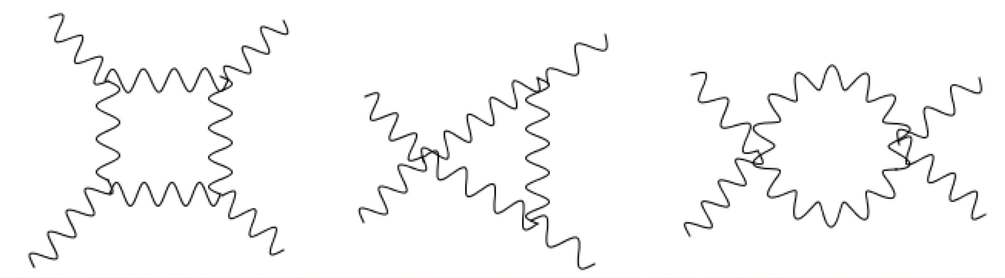
- Planarity = single trace approx, ordered particles

Simple amplitudes

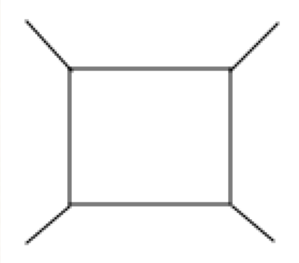
- Comparison: Feynman diagrams vs unitary methods

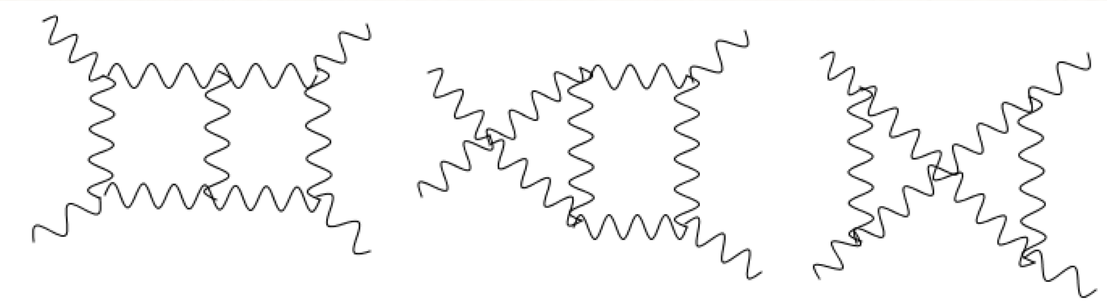
$gg \to gg$

Number of graphs

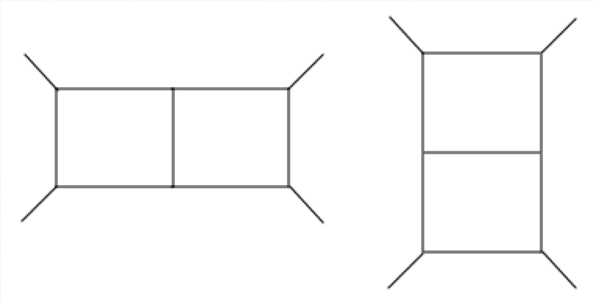


87 vs 1



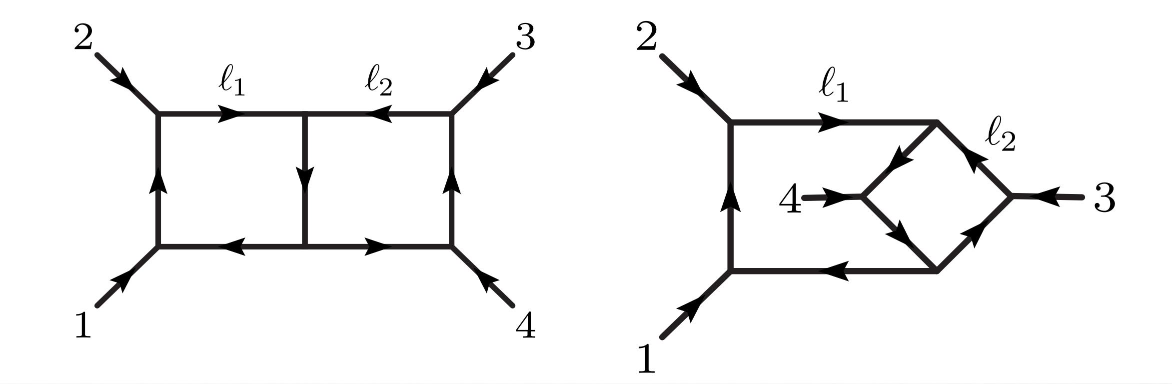


$\sim$1000 vs 2

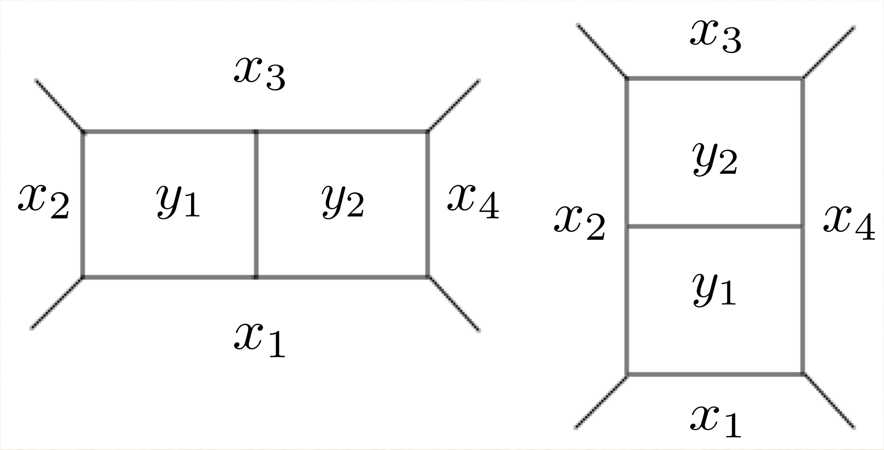


Dual variables

- Generally, each diagram has its own variables
  - No global loop momenta
  - Each diagram: its own labels



- Planar limit: dual variables



$$k_1 = (x_1 - x_2) \quad k_2 = (x_2 - x_3)$$

$$\ell_1 = (x_3 - y_1) \quad \ell_2 = (y_2 - x_3)$$

etc

Global variables

Integrand

- Using these variables: define a single function

$$\mathcal{M} = \int d^4y_1 \dots d^4y_L \, \mathcal{I}(x_i, y_j)$$

Integrand

- Ideal object to study: rational function, no divergencies
- Standard wisdom: $\mathcal{I} \sim \mathcal{I} + \text{Total derivative}$
- Planar N=4 SYM: unique function

Dual conformal invariance (DCI)

- Tree-level amplitudes + integrand in planar N=4 SYM:

(Drummond, Henn, Smirnov, Sokatchev 2006)

Dual conformal symmetry

- Dual variables $p_i = x_{i+1} - x_i$ $\sum_i x_i = 0$
- Conformal symmetry in the dual space
- Superconformal symmetry + Dual -> Yangian

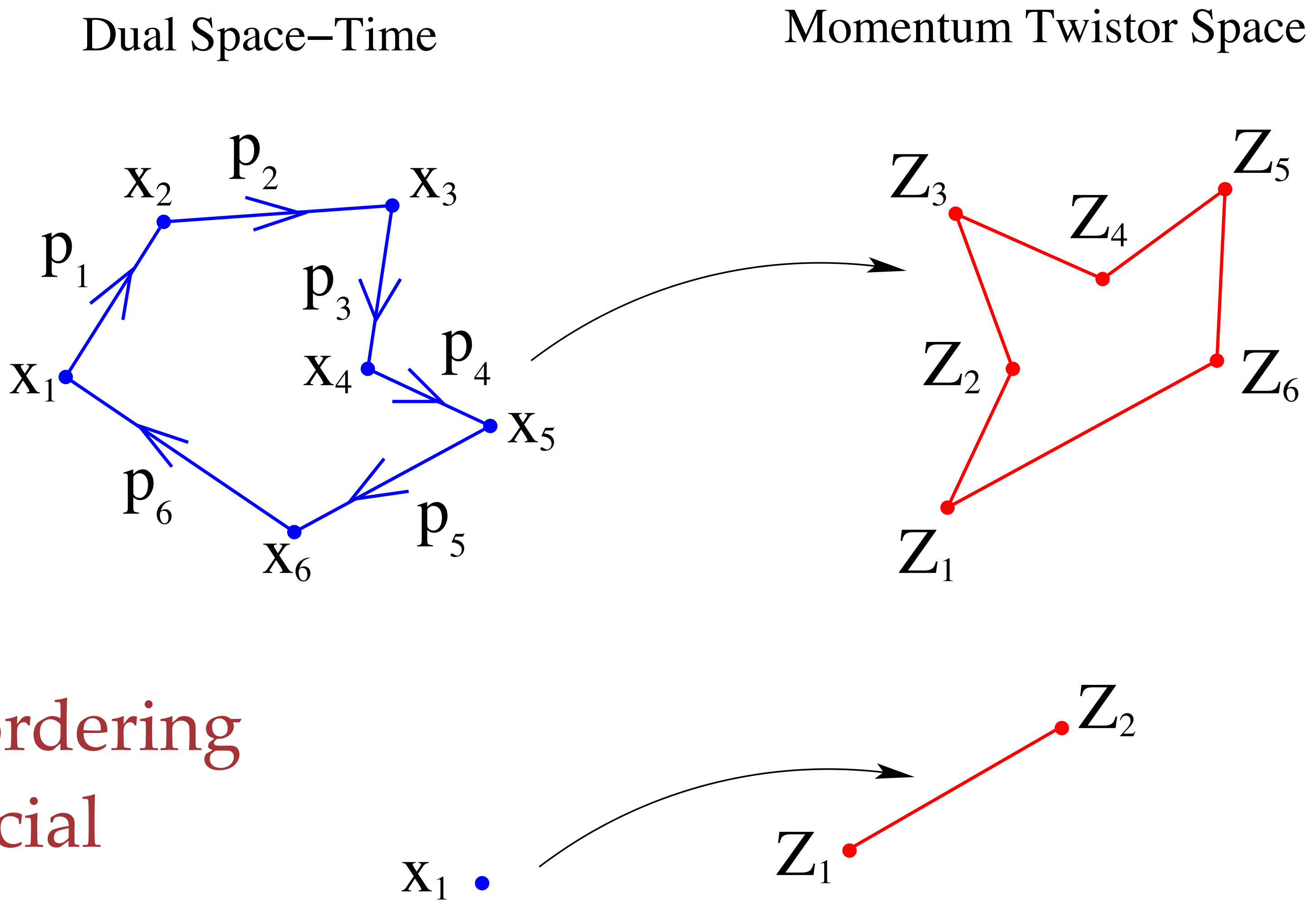
(Drummond, Henn, Korchemsky, Sokatchev 2008) (Drummond-Henn-Plefka 2009)

Momentum twistors

(Hodges 2009)

- New variables: points in $\mathbb{P}^3$

$$Z = \begin{pmatrix} \lambda_a \\ x_{a\dot{a}}\lambda_a \end{pmatrix}$$



Cyclic ordering crucial

Momentum twistors

- Dual conformal: SL(4) on momentum twistors
- Dual conformal invariants: $\langle 1234\rangle = \epsilon_{abcd} Z_1^a Z_2^b Z_3^c Z_4^d$

$$\langle 1234\rangle = \langle 12\rangle\langle 23\rangle\langle 34\rangle[23]$$

- Yangian invariants:

$$[12345] = \frac{(\eta_1\langle 2345\rangle + \cdots + \eta_5\langle 1234\rangle)^4}{\langle 1234\rangle\langle 2345\rangle\langle 3451\rangle\langle 4512\rangle\langle 5123\rangle}$$

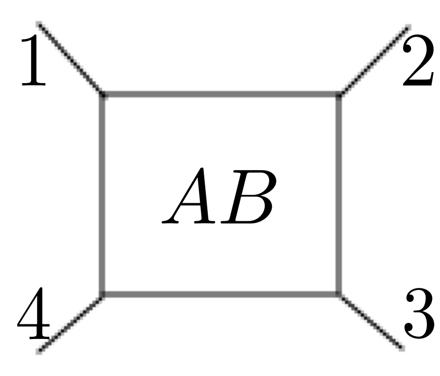
where $\widetilde{\eta}_a = \dfrac{\langle a-1\, a+1\rangle\eta_a + \langle a\, a-1\rangle\eta_{a+1} + \langle a+1\, a\rangle\eta_{a-1}}{\langle a\, a+1\rangle\langle a\, a-1\rangle}$

Momentum twistors

- Loop variable: pair of momentum twistors

$$\ell \leftrightarrow Z_A Z_B$$

- Example:



$$\frac{d^4\ell\, st}{\ell^2(\ell+k_1)^2(\ell+k_1+k_2)^2(\ell-k_4)^2}$$

$$\frac{\langle ABd^2A\rangle\langle ABd^2B\rangle\langle 1234\rangle^2}{\langle AB12\rangle\langle AB23\rangle\langle AB34\rangle\langle AB41\rangle}$$

where $\ell^2 = \frac{\langle AB41\rangle}{\langle AB\rangle\langle 41\rangle}$ $\quad \langle ij\rangle = \epsilon_{abcd}Z_i^a Z_j^b I^{cd}$

Infinity twistor: breaking of DCI

DCI of trees and integrand

- In planar N=4 SYM

$$\mathcal{R}^{\ell-loop}_{n,k} = \frac{\mathcal{I}^{\ell-loop}_{n,k}}{\mathcal{M}^{tree}_{n,k=2}}$$

Yangian invariant (covariant)

Parke-Taylor amplitude

- Example of 6pt tree-level amplitude

$$\mathcal{M}_6 = [12345] + [34561] + [56123]$$

All terms include spurious poles in momentum space

Tension between hidden symmetry and locality

Thank you for attention!