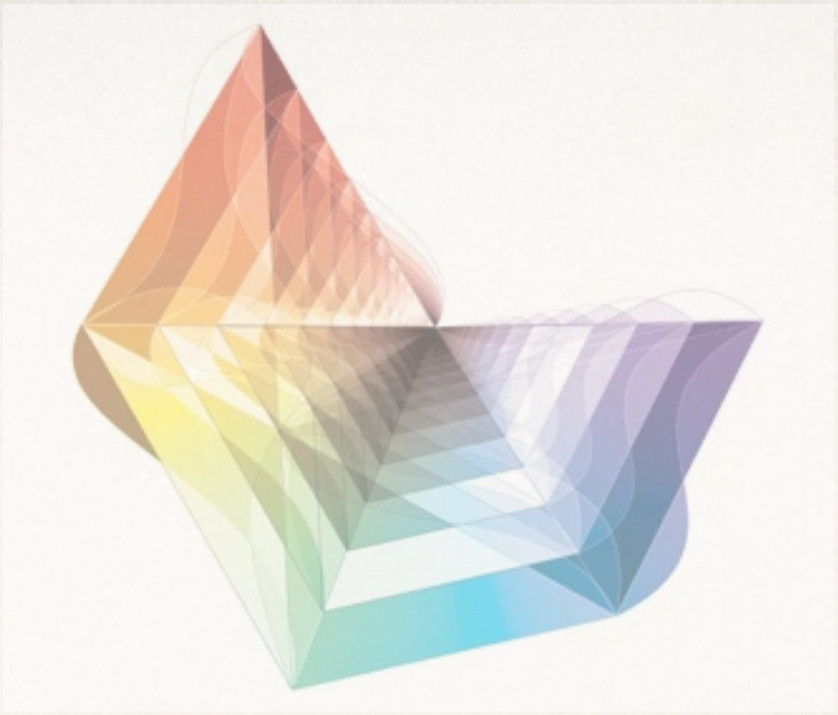

# Scattering Amplitudes
# LECTURE 1


Jaroslav Trnka

Walter Burke Institute for Theoretical Physics, Caltech

Center for Quantum Mathematics and Physics (QMAP), UC Davis


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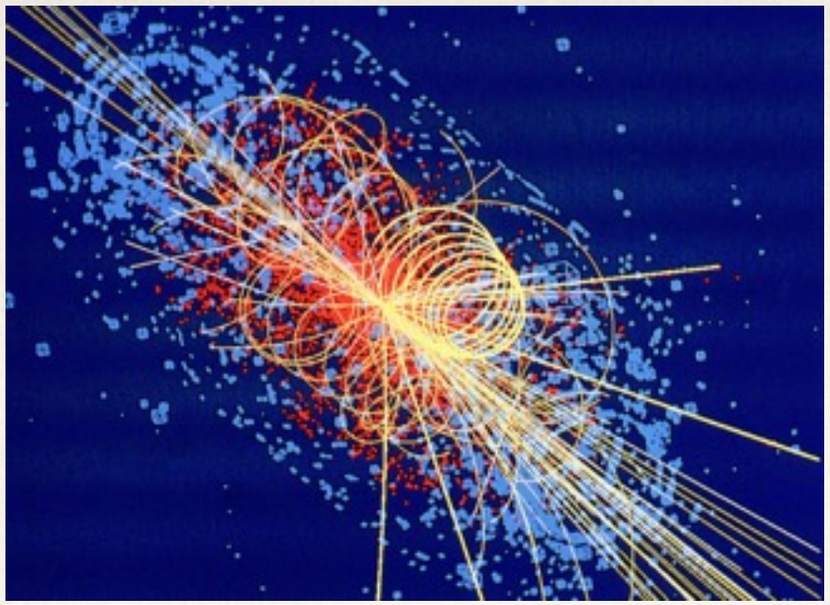
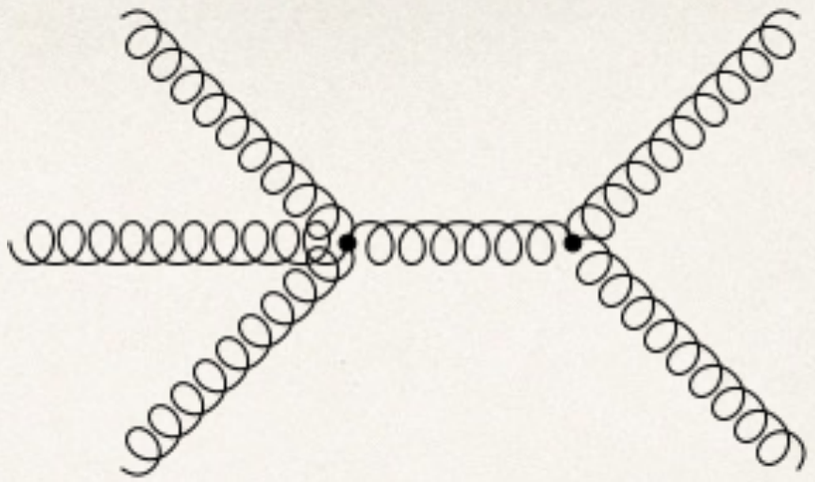
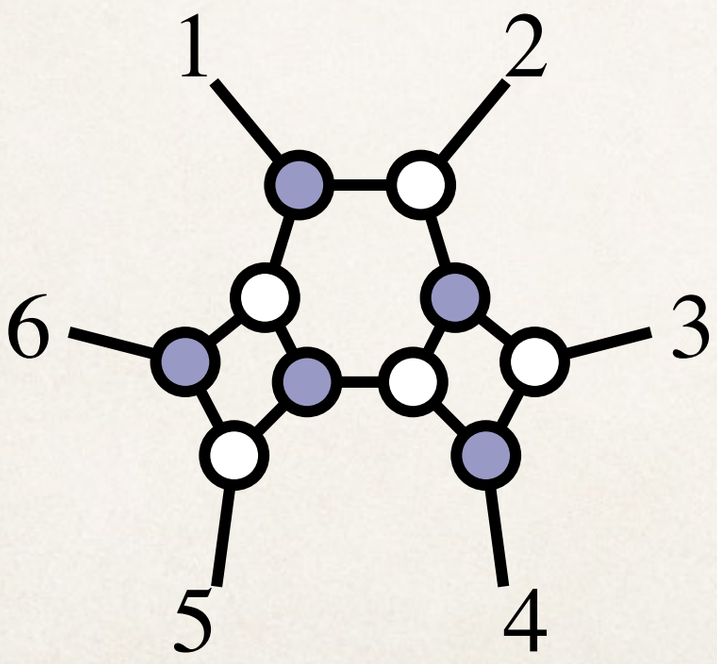
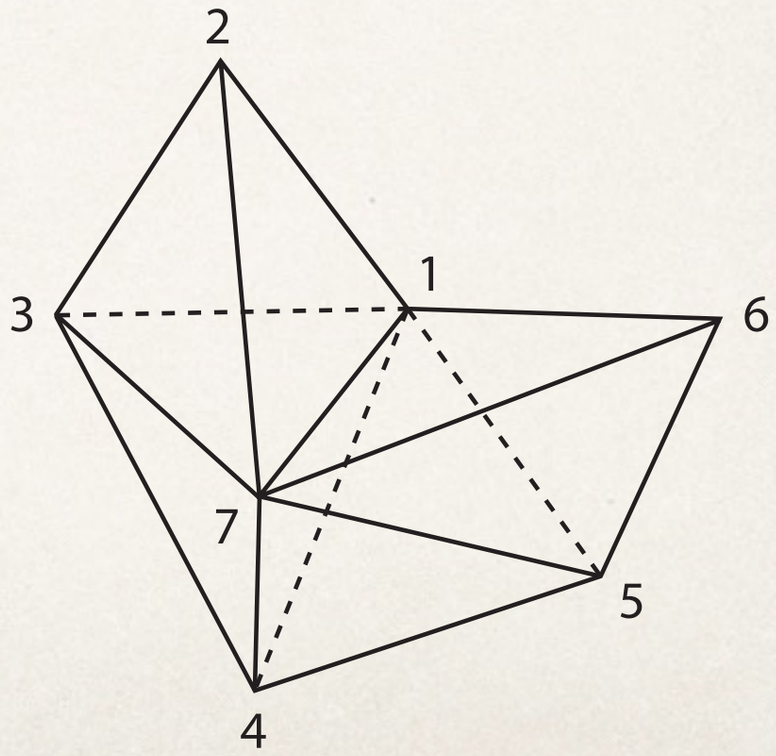
*Asian winter school, Okinawa, January 2016*





# Scattering amplitudes



# Overview of lectures

- Review of scattering amplitudes
  - Motivation
  - On-shell amplitudes, kinematics
  - Recursion relations, unitarity methods

- Geometric formulation
  - Toy model: N=4 Super Yang Mills theory
  - On-shell diagrams
  - Amplituhedron

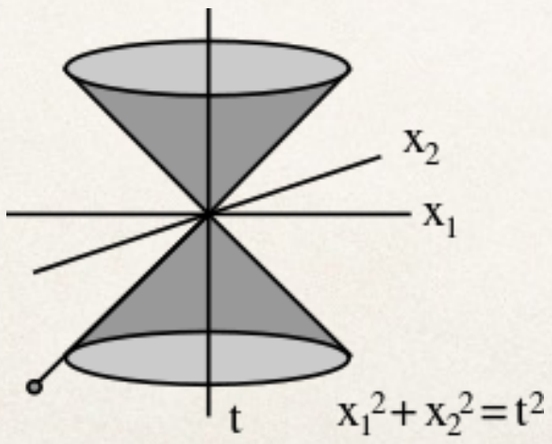
# Motivation

# Quantum Field Theory (QFT)

- Our theoretical framework to describe Nature

- Compatible with two principles

Special relativity




Quantum mechanics

$$H(t)|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

# Perturbative QFT

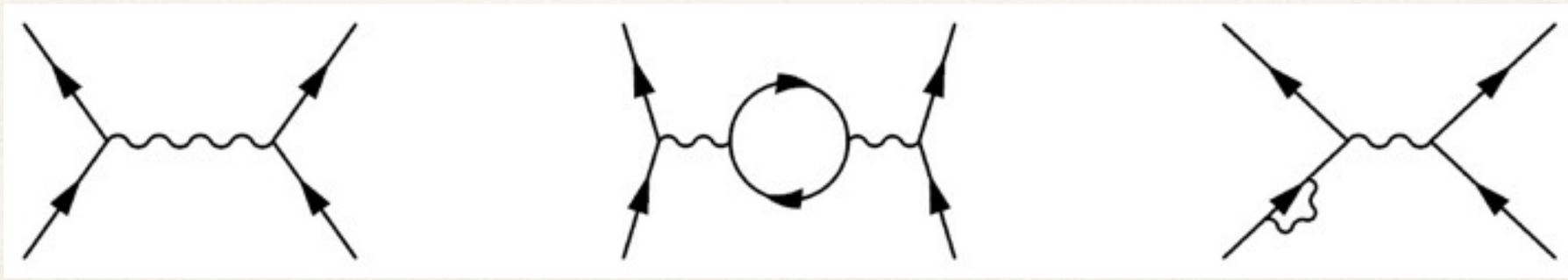
(Dirac, Heisenberg, Pauli; Feynman, Dyson, Schwinger)

- Fields, Lagrangian, Path integral

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\overline{\psi} \not{D} \psi - m\overline{\psi}\psi \qquad \int \mathcal{D}A\, \mathcal{D}\psi\, \mathcal{D}\overline{\psi}\; e^{iS(A,\psi,\overline{\psi},J)}$$

- Feynman diagrams: pictures of particle interactions

  Perturbative expansion: trees, loops



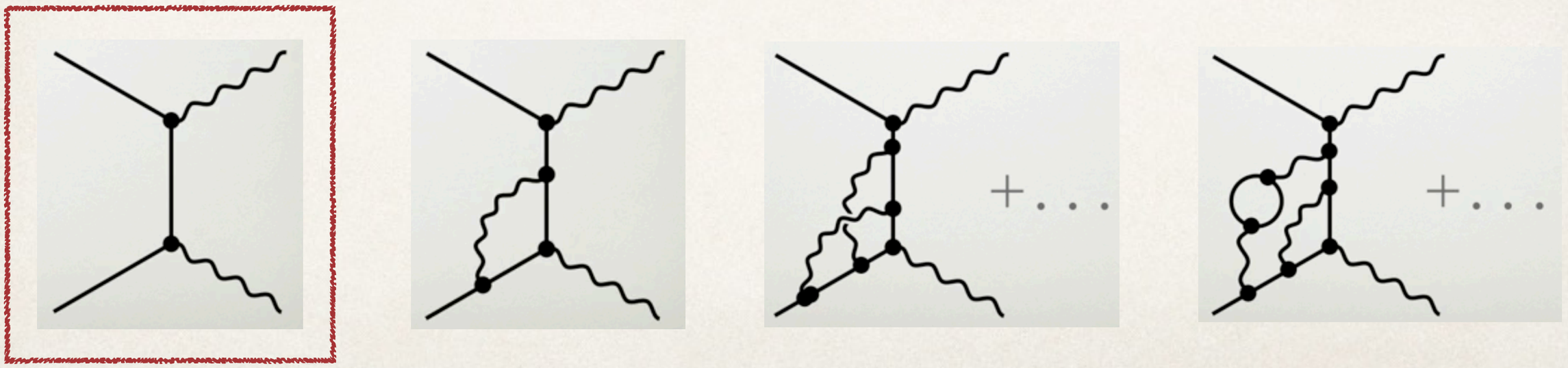
# Great success of QFT

- QFT has passed countless tests in last 70 years
- Example: Magnetic dipole moment of electron

1928

$$\text{Theory:} \quad g_e = 2$$

$$\text{Experiment:} \quad g_e \sim 2$$



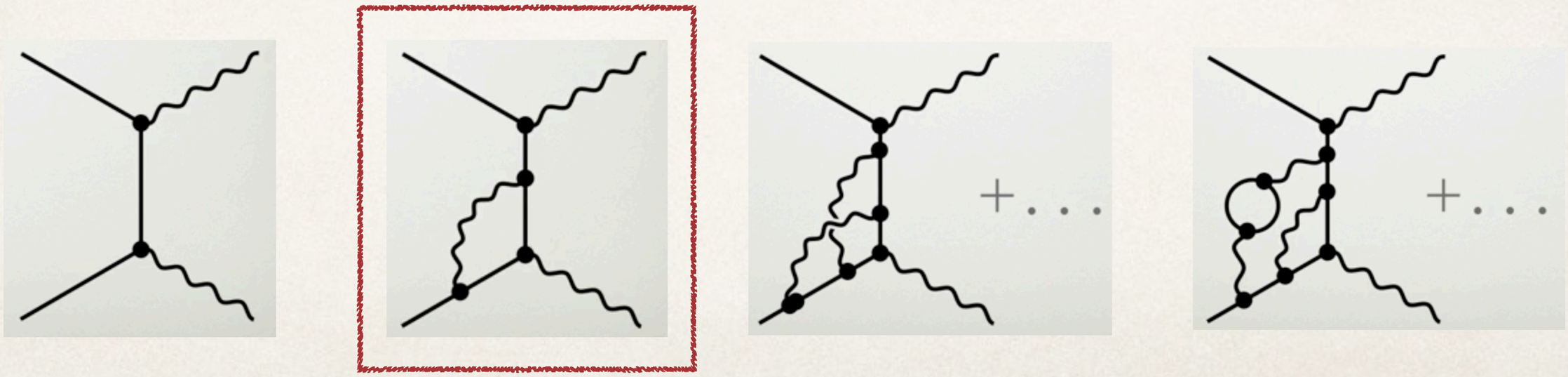
# Great success of QFT

- QFT has passed countless tests in last 70 years
- Example: Magnetic dipole moment of electron

1947

$$\text{Theory:} \quad g_e = 2.00232$$

$$\text{Experiment:} \quad g_e = 2.0023$$



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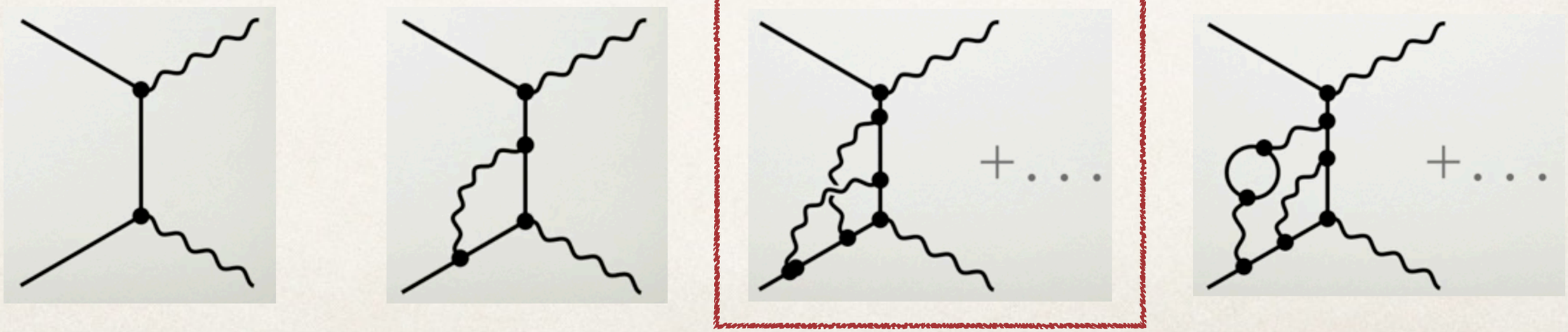
- Example: Magnetic dipole moment of electron

1957 Theory: $g_e = 2.0023193$

1972 Experiment: $g_e = 2.00231931$



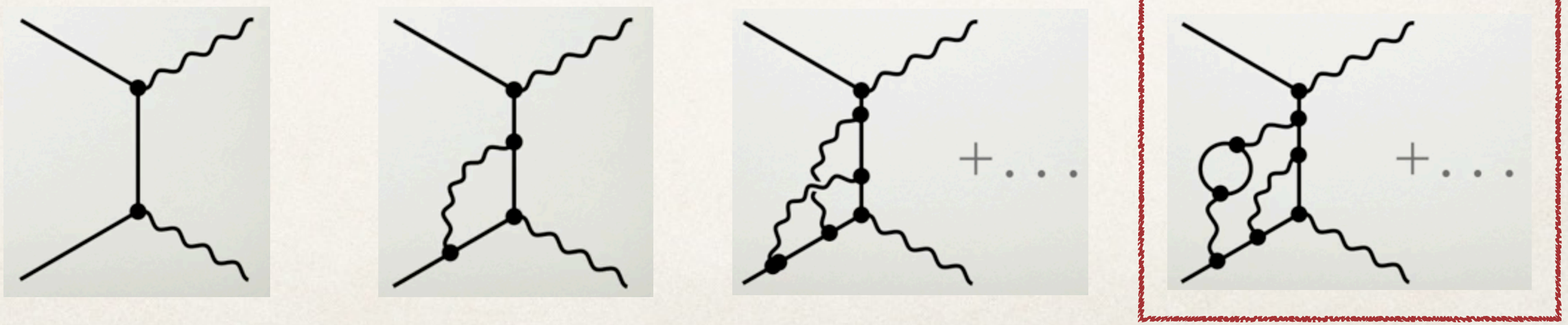
# Great success of QFT

- QFT has passed countless tests in last 70 years
- Example: Magnetic dipole moment of electron

1990

$$\text{Theory:} \quad g_e = 2.0023193044$$

$$\text{Experiment:} \quad g_e = 2.00231930438$$



# Dualities

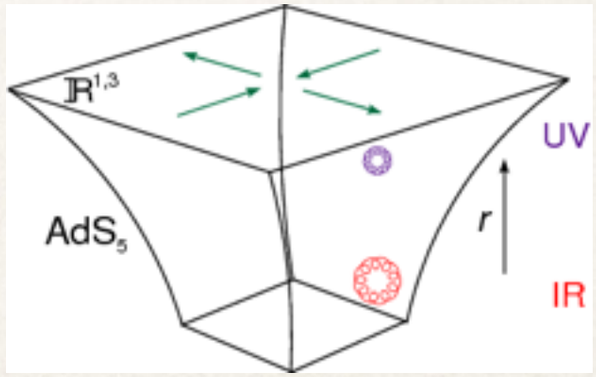
- At strong coupling: perturbative expansion breaks

- Surprises: dual to weakly coupled theory
  - Gauge-gauge dualities

    (Montonen-Olive 1977, Seiberg-Witten 1994)

  - Gauge-gravity duality

    (Maldacena 1997)



# Incomplete picture

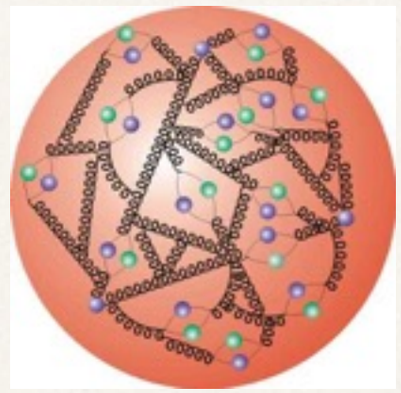
- Our picture of QFT is incomplete
- Also, tension with gravity and cosmology

> If there is a new way of thinking about QFT,
> it must be seen even at weak coupling

- Explicit evidence: scattering amplitudes

# Colliders at high energies

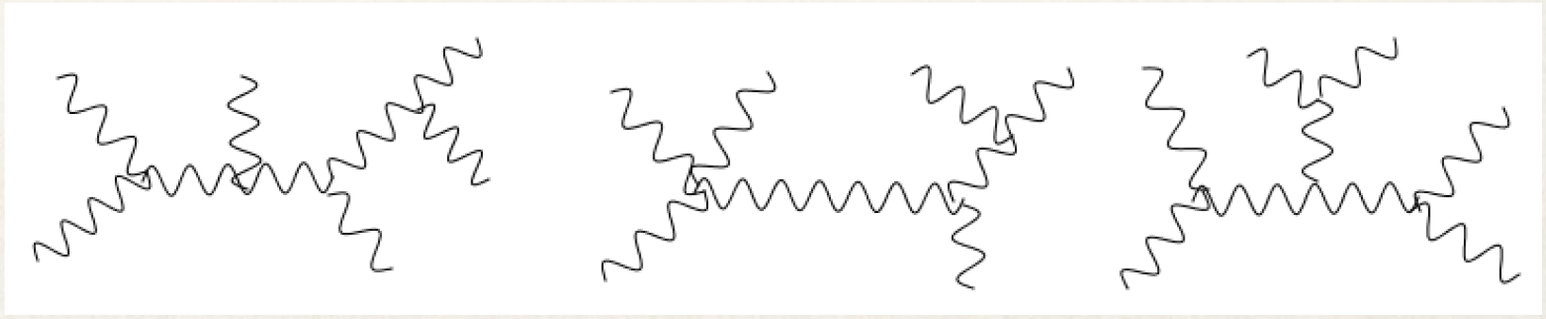
- Proton scattering at high energies



LHC - gluonic factory

- Needed: amplitudes of gluons for higher multiplicities

$$gg \to gg \ldots g$$



# Early 80s

- Status of the art: $gg \to ggg$

Brute force calculation

24 pages of result

$$(k_1 \cdot k_4)(\epsilon_2 \cdot k_1)(\epsilon_1 \cdot \epsilon_3)(\epsilon_4 \cdot \epsilon_5)$$

# New collider

- 1983: Superconducting Super Collider approved
- Energy 40 TeV: many gluons!

- Demand for calculations, next on the list: $gg \to gggg$

# Parke-Taylor formula

(Parke, Taylor 1985)

- Process $gg \to gggg$
- 220 Feynman diagrams, $\sim$100 pages of calculations
- 1985: Paper with 14 pages of result

GLUONIC TWO GOES TO FOUR

Stephen J. Parke and T.R. Taylor
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60510
U.S.A.

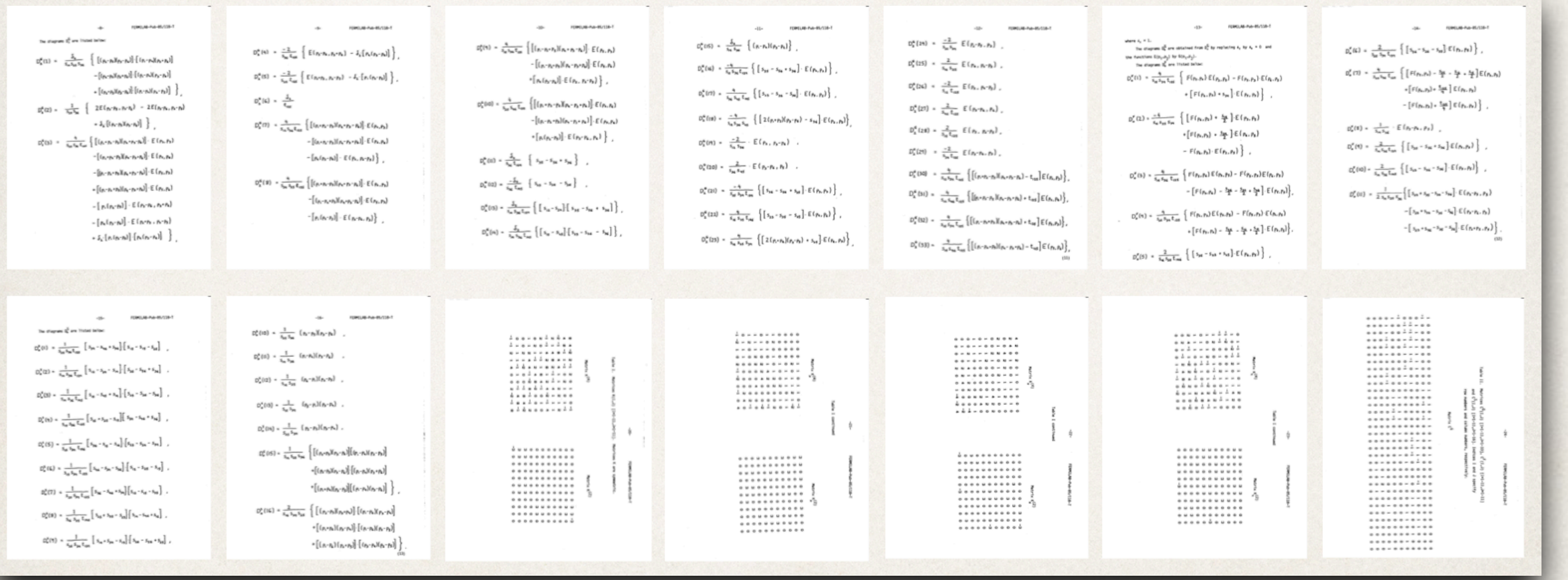
ABSTRACT

The cross section for two gluon to four gluon scattering is given in a form suitable for fast numerical calculations.

# Parke-Taylor formula

- Process $gg \to gggg$

- 220 Feynman diagrams, $\sim$100 pages of calculations



# Parke-Taylor formula

Our result has succesfully passed both these numerical checks.

Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

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- Within a year they realized

$$\mathcal{M}_6 = \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle}$$

Spinor-helicity variables

$$p^\mu = \sigma^\mu_{a\dot{a}}\lambda_a\tilde{\lambda}_{\dot{a}}$$
$$\langle 12\rangle = \epsilon_{ab}\lambda^{(1)}_a\lambda^{(2)}_b$$
$$[12] = \epsilon_{\dot{a}\dot{b}}\tilde{\lambda}^{(1)}_{\dot{a}}\tilde{\lambda}^{(2)}_{\dot{b}}$$

(Mangano, Parke, Xu 1987)

# Parke-Taylor formula

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- Within a year they realized

$$\mathcal{M}_n = \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \ldots \langle n1 \rangle}$$

AN AMPLITUDE FOR n GLUON SCATTERING

*STEPHEN J. PARKE and T. R. TAYLOR*

Fermi National Accelerator Laboratory

P.O. Box 500, Batavia, IL 60510.

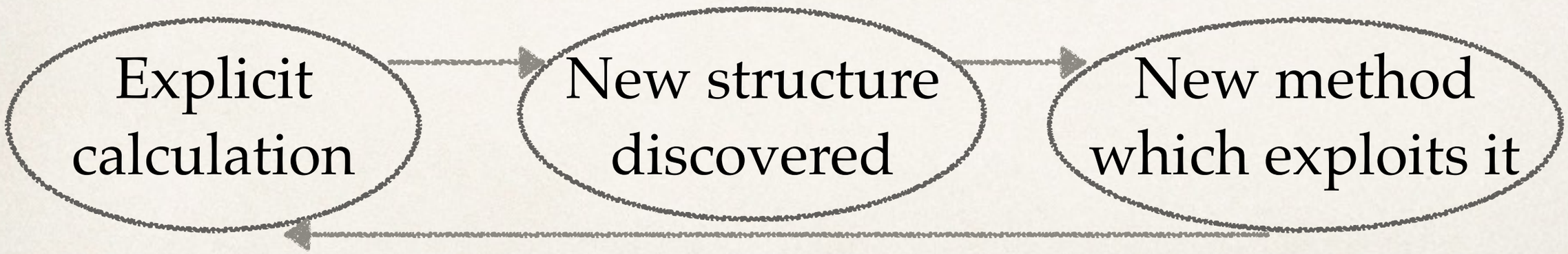
# Birth of amplitudes

- New field in theoretical particle physics

New methods and effective calculations

Uncovering new structures in QFT

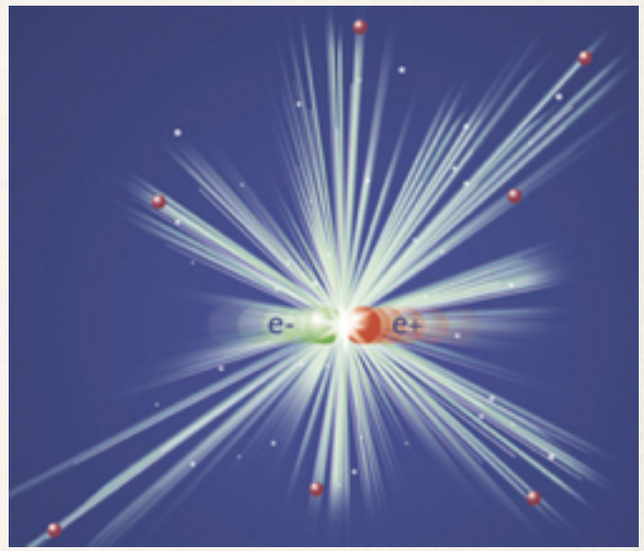
"Road map"



# What are scattering amplitudes

# Scattering process

- Interaction of elementary particles




- Initial state $|i\rangle$ and final state $|f\rangle$
- Scattering amplitude $\mathcal{M}_{if} = \langle i|f\rangle$
- Example: $e^+e^- \to e^+e^-$ or $e^+e^- \to \gamma\gamma$ etc.
- Cross section: $\sigma = \int |\mathcal{M}|^2 \, d\Omega$ probability

# Scattering amplitude in QFT

- Scattering amplitude depends on types of particles and their momenta

$$\mathcal{M}_{if} = F(p_i, s_i)$$

- Theoretical framework: calculated in some QFT

- Specified by Lagrangian: interactions and couplings

$$\mathcal{L} = \mathcal{L}(\mathcal{O}_j, g_k)$$

- Example: QED

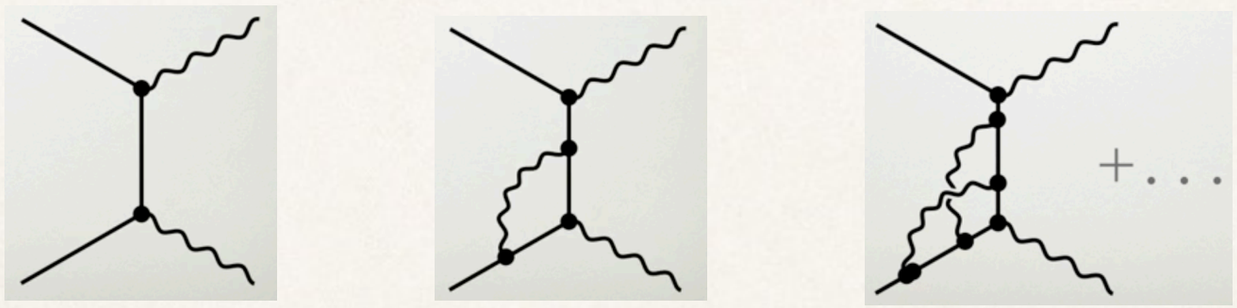
$$\mathcal{L}_{int} = e\,\overline{\psi}\gamma_\mu\psi A^\mu$$

# Perturbation theory

- Weakly coupled theory

$$\mathcal{M} = \mathcal{M}_0 + g\,\mathcal{M}_1 + g^2\,\mathcal{M}_2 + g^3\,\mathcal{M}_3 + \dots$$

- Representation in terms of Feynman diagrams

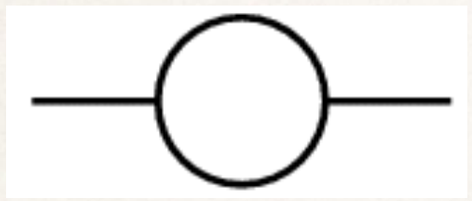



- Perturbative expansion = loop expansion

$$\mathcal{M} = \mathcal{M}^{tree} + \mathcal{M}^{1-loop} + \mathcal{M}^{2-loop} + \dots$$

# Divergencies

- Loop diagrams are generally UV divergent



$$\sim \int_{-\infty}^{\infty} \frac{d^4\ell}{(\ell^2+m^2)[(\ell+p)^2+m^2]} \sim \log\Lambda$$

- IR divergencies: physical effects, cancel in cross section

- Dimensional regularization: calculate integrals in

$4+\epsilon$ dimensions

$$\text{Divergencies} \sim \frac{1}{\epsilon^k}$$

# Renormalizable theories

- Absorb UV divergencies: counter terms
  - Finite number of them: renormalizable theory
  - Infinite number: non-renormalizable theory

- Mostly only renormalizable theories are interesting

- Exceptions: effective field theories

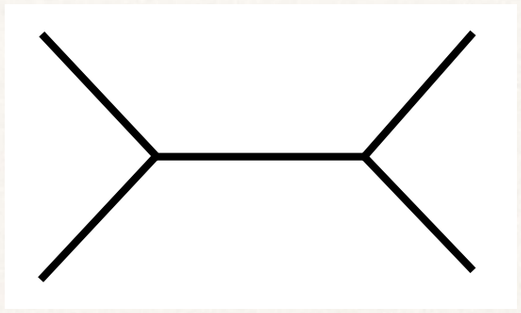
  Example: Chiral perturbation theory - derivative expansion

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \mathcal{L}_8 + \dots$$

  Different loop orders are mixed

# Analytic structure of amplitudes

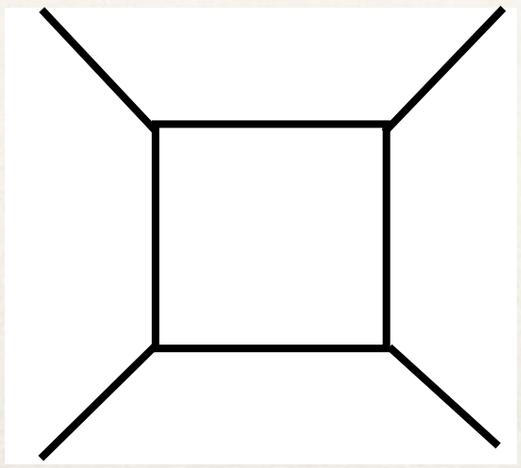
- Tree-level: rational functions



$$\sim \frac{g^2}{(p_1+p_2)^2}$$

Only poles

- Loops: polylogarithms and more complicated



$$\sim \log^2(s/t)$$

Branch cuts

# Kinematics of massless particles

# Massless particles

- Parameters of elementary particles of spin S
  - Spin $s = (-S, S)$
  - Mass $m$
  - Momentum $p^\mu$

On-shell (physical) particle

$$p^2 = m^2$$

- Massless particle: $m = 0 \quad p^2 = 0$

spin = helicity: only two extreme values $h = \{-S, S\}$

Example: photon $h = (+, -)$

$s = 0$ missing

# Spin functions

- At high energies particles are massless

  Fundamental laws reveal there

- Spin degrees of freedom: spin function
  - s=0: Scalar - no degrees of freedom
  - s=1/2: Fermion - spinor $u$
  - s=1: Vector - polarization vector $\epsilon^\mu$
  - s=2: Tensor - polarization tensor $h^{\mu\nu}$

# Spin functions

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  - s=2: Tensor - polarization tensor $h^{\mu\nu}$

# Polarization vectors

- Spin 1 particle is described by vector $\epsilon^\mu$

2 degrees of freedom

4 degrees of freedom

- Null condition: $\epsilon \cdot \epsilon^* = 0$ 3 degrees of freedom left

- We further impose: $\epsilon \cdot p = 0$

Identification

$\epsilon_\mu \sim \epsilon_\mu + \alpha p_\mu$

Feynman diagrams depend on $\alpha$

gauge dependence

# Spinor helicity variables

- Standard SO(3,1) notation for momentum

$$p^\mu = (p_0, p_1, p_2, p_3) \qquad p_j \in \mathbb{R}$$

- We use SL(2,C) representation

$$p_{ab} = \sigma^\mu_{ab} p_\mu = \begin{pmatrix} p_0 + ip_1 & p_2 + p_3 \\ p_2 - p_3 & p_0 - ip_1 \end{pmatrix}$$

On-shell: $p^2 = \det(p_{ab}) = 0$

$$\boxed{\text{Rank}\,(p_{ab}) = 1}$$

# Spinor helicity variables

- We can then write $p_{ab} = \lambda_a \kappa_b$
- SL(2,C): dotted notation $p_{a\dot{b}} = \lambda_a \widetilde{\lambda}_{\dot{b}}$

$\widetilde{\lambda}$ is complex conjugate of $\lambda$

- Little group transformation

$$\lambda \to t\lambda \qquad \widetilde{\lambda} \to \frac{1}{t}\widetilde{\lambda}$$

leaves momentum unchanged $\quad p \to p$

3 degrees of freedom

# Spinor helicity variables

- Momentum invariant $\quad (p_1 + p_2)^2 = (p_1 \cdot p_2)$

$$p_1^\mu = \sigma^\mu_{a\dot{a}}\, \lambda_{1a} \widetilde{\lambda}_{1\dot{a}} \qquad p_2^\mu = \sigma^\mu_{b\dot{b}}\, \lambda_{2b} \widetilde{\lambda}_{2\dot{b}}$$

- Plugging for momenta

$$(p_1 \cdot p_2) = (\sigma^\mu_{a\dot{a}} \sigma_{\mu\, b\dot{b}})\, (\lambda_{1a}\lambda_{2b})(\widetilde{\lambda}_{1\dot{a}}\widetilde{\lambda}_{2\dot{b}})$$

$$\epsilon_{ab}\, \epsilon_{\dot{a}\dot{b}} \qquad = (\epsilon_{ab}\lambda_{1a}\lambda_{2b})(\epsilon_{\dot{a}\dot{b}}\widetilde{\lambda}_{1\dot{a}}\widetilde{\lambda}_{2\dot{b}})$$

Define: $\quad \langle 12 \rangle \equiv \epsilon_{ab}\lambda_{1a}\lambda_{2b}$ $\qquad$ $[12] \equiv \epsilon_{\dot{a}\dot{b}}\widetilde{\lambda}_{1\dot{a}}\widetilde{\lambda}_{2\dot{b}}$

# Invariant products

- Momentum invariant

$$s_{ij} = (p_i + p_j)^2 = \langle ij \rangle [ij]$$

*Angle brackets* $\langle ij \rangle = \epsilon_{ab} \lambda_{ia} \lambda_{jb}$

*Square brackets* $[ij] = \epsilon_{\dot{a}\dot{b}} \lambda_{i\dot{a}} \lambda_{j\dot{b}}$

- Antisymmetry

$$\langle 21 \rangle = -\langle 12 \rangle \qquad [21] = -[12]$$

- More momenta

$$(p_1 + p_2 + p_3)^2 = \langle 12 \rangle [12] + \langle 23 \rangle [23] + \langle 13 \rangle [13]$$

# Invariant products

- Shouten identity

$$\langle 13\rangle\langle 24\rangle = \langle 12\rangle\langle 34\rangle + \langle 14\rangle\langle 23\rangle$$

- Mixed brackets

$$\langle 1|2+3|4] \equiv \langle 12\rangle[24] + \langle 13\rangle[34]$$

- Momentum conservation

$$\sum_{i=1}^{n} \lambda_{ia}\widetilde{\lambda}_{i\dot{a}} = 0$$

Non-trivial conditions:

Quadratic relation between components

# Polarization vectors

- Two polarization vectors

$$\epsilon_+^\mu = \sigma^\mu_{a\dot{a}} \frac{\eta_a \widetilde{\lambda}_{\dot{a}}}{\langle \eta \lambda \rangle} \qquad \epsilon_-^\mu = \sigma^\mu_{a\dot{a}} \frac{\lambda_a \widetilde{\eta}_{\dot{a}}}{[\widetilde{\eta}\, \widetilde{\lambda}]}$$

where $\eta, \widetilde{\eta}$ are auxiliary spinors

Note that $(\epsilon_+ \cdot \epsilon_-) = 1$

- Freedom in choice of $\eta, \widetilde{\eta}$ corresponds to

$$\epsilon^\mu \sim \epsilon^\mu + \alpha\, p^\mu$$

- Gauge redundancy of Feynman diagrams

# Scaling of amplitudes

- Consider some amplitude $A(-+--+\cdots-)$

$$A = (\epsilon_1\epsilon_2\ldots\epsilon_n)\cdot Q$$

depends only on momenta

- Little group scaling

$$\lambda \to t\lambda \qquad \tilde{\lambda} \to \frac{1}{t}\tilde{\lambda}$$

$$\longrightarrow \quad p \to p \qquad \epsilon_+ \to \frac{1}{t^2}\cdot\epsilon_+ \qquad \epsilon_- \to t^2\cdot\epsilon_- \quad \longrightarrow$$

$$A(i^-) \to t^2\cdot A(i^-) \qquad A(i^+) \to \frac{1}{t^2}\cdot A(i^+)$$

# Back to Parke-Taylor formula

- Let us consider $A(1^-2^-3^+4^+5^+6^+)$
- Scaling $A\left(t\lambda_i, \frac{1}{t}\widetilde{\lambda}_i\right) = t^2 \cdot A(\lambda_i, \widetilde{\lambda}_i)$ for particles 1,2

$$A\left(t\lambda_i, \frac{1}{t}\widetilde{\lambda}_i\right) = \frac{1}{t^2} \cdot A(\lambda_i, \widetilde{\lambda}_i) \quad \text{for particles } 3,4,5,6$$

- Check for explicit expression

$$A_6 = \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle}$$

If only $\langle ij\rangle$ allowed
the form is unique

# Helicity amplitudes

- In Yang-Mills theory we have + or - "gluons"

$$A_6(1^-2^-3^+4^+5^+6^+)$$

- We denote k: number of - helicity gluons

- Some amplitudes are zero

$$A_n(+++\cdots+) = 0$$
$$A_n(-++\cdots+) = 0$$
$$A_n(---\cdots-) = 0$$
$$A_n(+--\cdots-) = 0$$

First non-trivial: k=2

$$A_n(--+\cdots+)$$

Parke-Taylor formula for tree level

# Three point amplitudes

# Three point kinematics

- Gauge invariant building blocks: on-shell amplitudes

$$p_1^2 = p_2^2 = p_3^2 = 0 \qquad\qquad p_1 + p_2 + p_3 = 0$$

- Plugging second equation into the first

$$(p_1 + p_2)^2 = (p_1 \cdot p_2) = 0$$

- Similarly we get for other pairs

$$(p_1 \cdot p_2) = (p_1 \cdot p_3) = (p_2 \cdot p_3) = 0$$

These momenta are very constrained!

# Three point kinematics

- Use spinor helicity variables trivializes on-shell condition

$$p_1 = \lambda_1\widetilde{\lambda}_1, \quad p_2 = \lambda_2\widetilde{\lambda}_2, \quad p_3 = \lambda_3\widetilde{\lambda}_3$$

- The mutual conditions then translate to

$$(p_1 \cdot p_2) = \langle 12\rangle[12] = 0$$

- And similarly for other two pairs

$$(p_1 \cdot p_3) = \langle 13\rangle[13] = 0 \qquad (p_2 \cdot p_3) = \langle 23\rangle[23] = 0$$

# Two solutions

- We want to solve conditions

$$\langle 12\rangle[12] = \langle 13\rangle[13] = \langle 23\rangle[23] = 0$$

- Solution 1: $\langle 12\rangle = 0$ which implies $\lambda_2 = \alpha\lambda_1$

  Then we also have $\langle 23\rangle = \alpha\langle 13\rangle$

  And we set $\langle 13\rangle = 0$ by demanding $\lambda_3 = \beta\lambda_1$

$$\boxed{\lambda_1 \sim \lambda_2 \sim \lambda_3}$$

# Two solutions

- Solution 2: $[12] = [23] = [13] = 0$

$$\widetilde{\lambda}_1 \sim \widetilde{\lambda}_2 \sim \widetilde{\lambda}_3$$

- Let us take this solution

$$p_1 = \lambda_1\widetilde{\lambda}_1, \quad p_2 = \alpha\lambda_2\widetilde{\lambda}_1, \quad p_3 = (-\lambda_1 - \alpha\lambda_2)\widetilde{\lambda}_1$$

complex momenta

No solution for real momenta

# Three point amplitudes

- Gauge theory: scattering of three gluons (not real)
- Building blocks: $\langle 12\rangle, \langle 23\rangle, \langle 13\rangle, [12], [23], [13]$
- Mass dimension: each term $\sim m$
- Three point amplitude $A_3 \sim \epsilon^3 p \sim p \sim m$

# Three point amplitudes

- Two options

$$A_3^{(1)} = \langle 12\rangle^{a_1}\langle 13\rangle^{a_2}\langle 23\rangle^{a_3} \qquad A_3^{(2)} = [12]^{b_1}[13]^{b_2}[23]^{b_3}$$

- Apply to $A_3(1^-, 2^-, 3^+)$

$$A_3(t\lambda_1, t^{-1}\widetilde{\lambda}_1) = t^{a_1+a_2}\cdot A_3 \qquad a_1 + a_2 = 2 \qquad a_1 = 3$$
$$A_3(t\lambda_2, t^{-1}\widetilde{\lambda}_2) = t^{a_1+a_3}\cdot A_3 \longrightarrow a_1 + a_3 = 2 \longrightarrow a_2 = -1$$
$$A_3(t\lambda_3, t^{-1}\widetilde{\lambda}_3) = t^{a_2+a_3}\cdot A_3 \qquad a_2 + a_3 = -2 \qquad a_3 = -1$$

# Three point amplitudes

- Similarly for $A_3(1^+, 2^+, 3^-)$

- Two fundamental amplitudes

$$A_3(1^-, 2^-, 3^+) = \frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle}$$

$$A_3(1^+, 2^+, 3^-) = \frac{[12]^3}{[13][23]}$$

This is true to all orders: just kinematics

# Three point amplitudes

- Collect all $(--+)$ amplitudes

$$A_3(1^-,2^-,3^+) = \frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle}$$

$$A_3(1^-,2^+,3^-) = \frac{\langle 13\rangle^3}{\langle 12\rangle\langle 23\rangle}$$

$$A_3(1^+,2^-,3^-) = \frac{\langle 23\rangle^3}{\langle 12\rangle\langle 13\rangle}$$

$$\frac{\langle a\,b\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}$$

where a,b are - helicity gluons

- Similarly for $(++-)$ amplitudes

$$\frac{[ab]^4}{[12][23][31]}$$

where a,b are + helicity gluons

# Three point amplitudes

- Using similar analysis we find for gravity

$$\frac{\langle ab\rangle^8}{\langle 12\rangle^2\langle 23\rangle^2\langle 31\rangle^2}$$

where a,b are - helicity gravitons

$$\frac{[ab]^8}{[12]^2[23]^2[31]^2}$$

where a,b are + helicity graviton

- Note that there is no three-point scattering

  <u>They exist only for complex momenta</u>

- Important input into on-shell methods

# Tree-level amplitudes

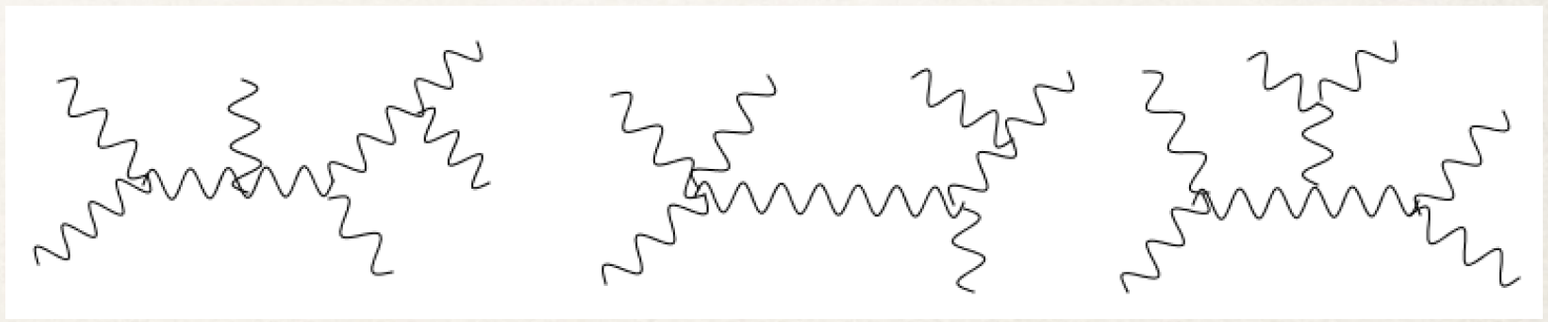
# Feynman diagrams

- Yang-Mills Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \sim (\partial A)^2 + A^2 \partial A + A^4$$

$$\sim f^{abc}\, g_{\mu\nu} p_\alpha \qquad \sim f^{abe} f^{cde}\, g_{\mu\nu} g_{\alpha\beta}$$

- Draw diagrams
  Feynman rules
  Sum everything



# Locality and unitarity

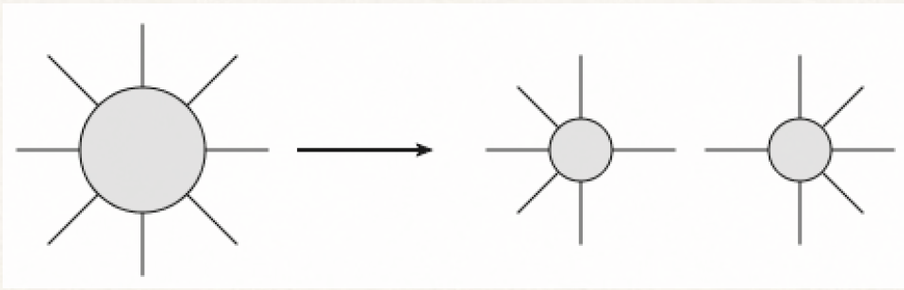
- Only poles: Feynman propagators

*Locality*

$$\frac{1}{P^2} \quad \text{where} \quad P = \sum_{k \in \mathcal{P}} p_k$$

- On the pole

*Unitarity*



$$\mathcal{M} \xrightarrow[P^2=0]{} \mathcal{M}_L \frac{1}{P^2} \mathcal{M}_R$$

Feynman diagrams recombine on both sides into amplitudes

# Factorization on the pole

On $P^2 = 0$

$$\text{Res}\,\mathcal{M} = \mathcal{M}_L \frac{1}{P^2} \mathcal{M}_R$$

- For $P^2 = 0$ the internal leg: on-shell physical particle
- Both sub-amplitudes are on-shell, gauge invariant
- On-shell data: statement about on-shell quantities

# On-shell constructibility

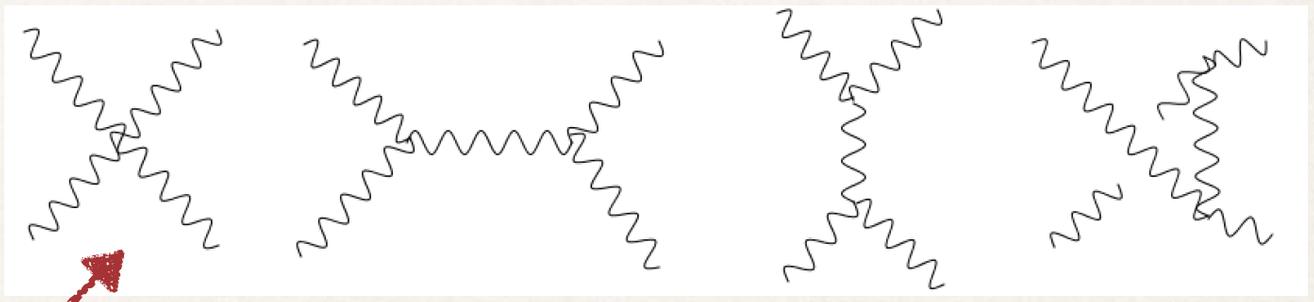
- Question:

Is knowing all residues on all poles enough to specify the amplitude?

- Naively, this is false for Yang-Mill theory

$$gg \to gg$$

Four point amplitude



Contact term

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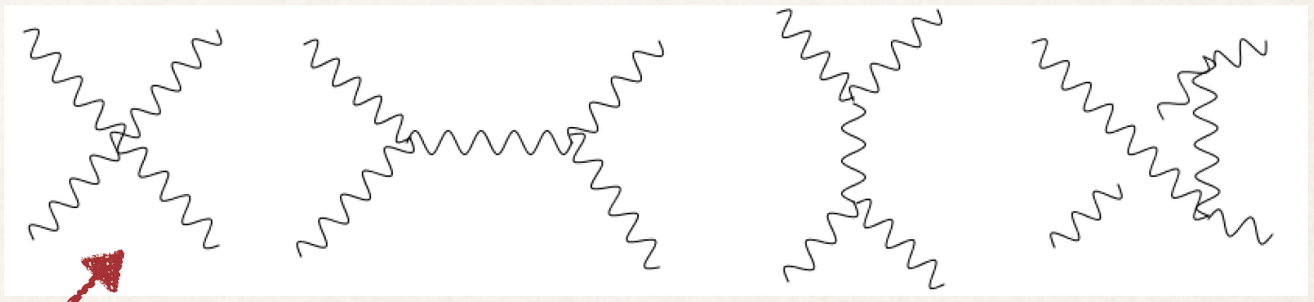

Contact term

Imposing gauge invariance fixes it

# On-shell constructibility

- In gravity we have infinity tower of terms

$$\mathcal{L} \sim \sqrt{g}\, R \sim h^2 + h^3 + h^4 + \dots$$

- Only $h^3$ terms important, others fixed by diffeomorphism symmetry

- On-shell constructibility of Yang-Mills, GR

**Only function which factorizes properly on all poles is the amplitude.**

# Color decomposition

- Sum of Feynman diagrams in Yang-Mills

$$\mathcal{M} = \sum_{FD} (\text{Color}) \times (\text{Kinematics})$$

Polarization vectors

Gauge dependent

- Color factors

$$\text{Tr}(T^{a_1} T^{a_2} T^{a_3} \dots T^{a_n})$$

- Decomposition

$$\mathcal{M} = \sum_{\sigma} \text{Tr}(T^{\sigma_1} T^{\sigma_2} T^{\sigma_3} \dots T^{\sigma_n})\, A(123\dots n)$$

# Color decomposition

- Sum of Feynman diagrams in Yang-Mills

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Gauge dependent

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- Decomposition

$$\mathcal{M} = \sum_{\sigma} \text{Tr}(T^{\sigma_1} T^{\sigma_2} T^{\sigma_3} \dots T^{\sigma_n}) A(123\dots n)$$

# Color ordered amplitude

$$A(123\ldots n)$$

Particles are ordered, other orderings: permutations

## Gauge invariant

- This is a key object of our interest

- Consider:
  - All particles massless and on-shell
  - All momenta incoming
  - Helicities fixed

Thank you for attention!