

The Weak Gravity Conjecture



The Weak Gravity Conjecture

Arkani-Hamed et al. '06

- The conjecture:

“Gravity is the Weakest Force”

- For every long range gauge field there exists a particle of charge q and mass m , s.t.

$$\frac{q}{m} M_P \geq “1”$$

The Weak Gravity Conjecture

- Take a U(1) and a single family with $q < m$ (~~WGC~~)

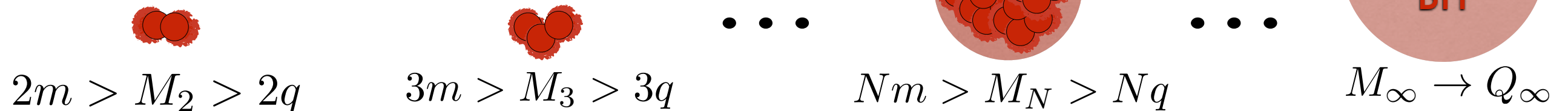


The Weak Gravity Conjecture

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- Form bound states



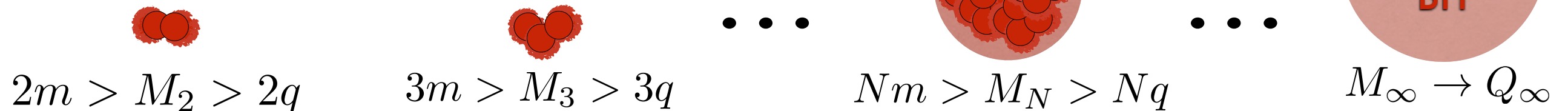
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The Weak Gravity Conjecture

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- Form bound states



- All these (BH) states are stable. Trouble w/ remnants Susskind '95
- Need a light state into which they can decay

$$\frac{q}{m} \geq \text{"1"} \equiv \frac{Q_{Ext}}{M_{Ext}}$$

The Weak Gravity Conjecture

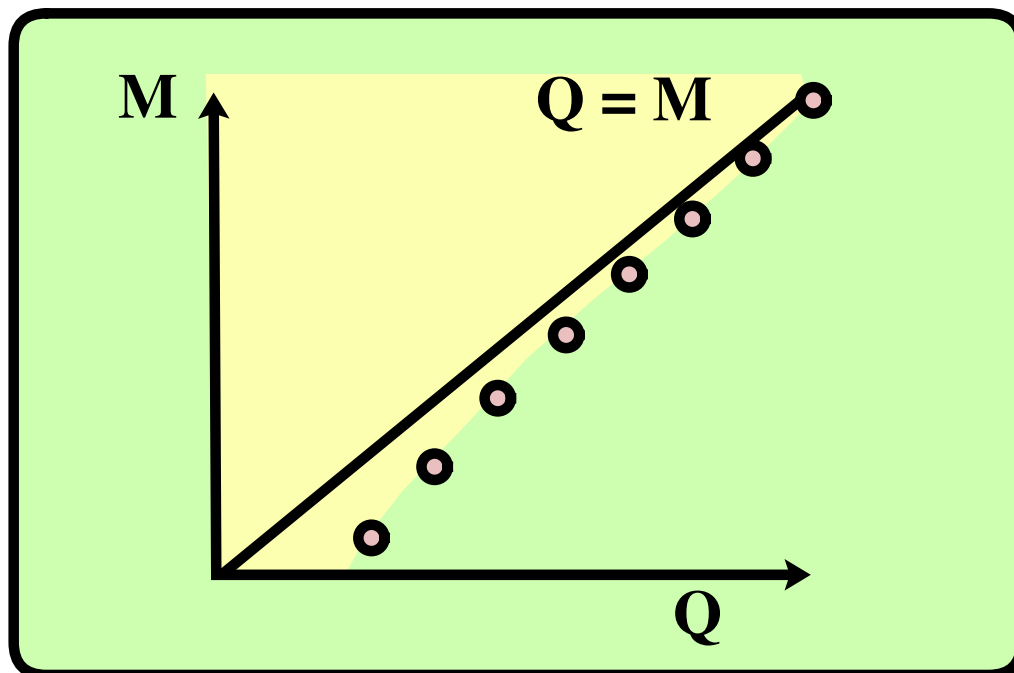
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- For bound states to decay, there must \exists a particle w/

$$\frac{q}{m} \geq "1" \equiv \frac{Q_{Ext}}{M_{Ext}}$$

Strong-WGC: satisfied by lightest charged particle

Weak-WGC: satisfied by any charged particle



E.g. Heterotic spectrum:

$$M^2 \propto Q_L^2 + 2N_L - 2$$

The Weak Gravity Conjecture

- Suggested generalization to p-dimensional objects charged under (p+1)-forms:

$$\frac{Q}{T_p} \geq \text{“1”}$$

- p=-1 applies to instantons coupled to axions:

$$e^{-S_{inst}} = e^{-m+i\phi/f} \quad \implies \quad fm \leq \text{“1”}$$

- Seems to explain difficulties in finding $f > M_P$
- Is there evidence for the p=-1 version of the WGC?

WGC and Axions

Brown, Cottrell, GS, Soler

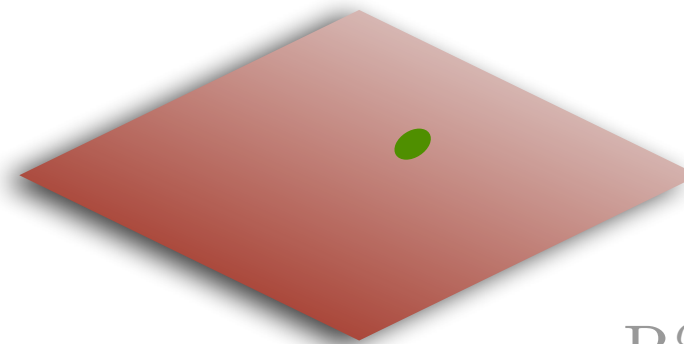
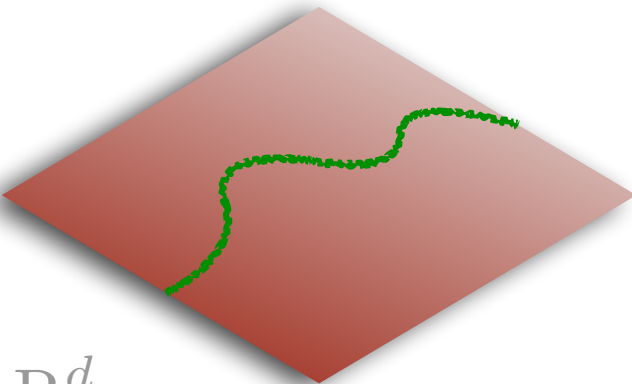
- T-duality provides a subtle connection between instantons and particles

Type IIA

Type IIB

$D(p+1)$ -Particle
(Gauge bosons)

Dp -Instanton
(Axions)

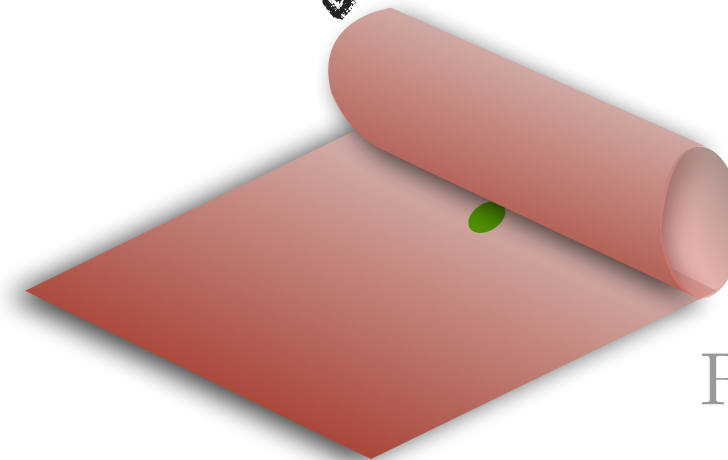
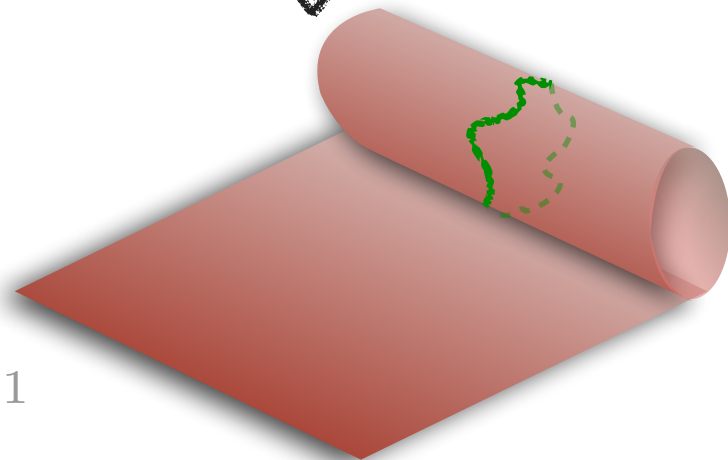


R^d

R^d

\tilde{S}^1

S^1



$R^{d-1} \times \tilde{S}^1$

$R^{d-1} \times S^1$

T-dual
 \longleftrightarrow

WGC and Axions

Brown, Cottrell, GS, Soler

Type IIA

Gauge fields: $A_i \sim \int_{\Sigma_2^{(i)}} C_3$

Particles: D2 on $\Sigma_2^{(i)}$

WGC

$$\tilde{m}_k = m_k \frac{\sqrt{g_{33}}}{2\pi l_s}$$

$$\tilde{q}_k^i = (f_k^i)^{-1} \frac{\sqrt{2}}{4\pi l_s}$$

“Couplings”:

$$\tilde{g}_s = \frac{g_s}{\sqrt{g_{33}}}$$

$$\tilde{M}_P = M_P \sqrt{g_{33}}$$

Type IIB

Axions: $\phi_i \sim \int_{\Sigma_2^{(i)}} C_2$

Instantons: D1 on $\Sigma_2^{(i)}$

$$S_{inst_k} \sim -m_k + i(f_k^i)^{-1} \phi_i$$

“Couplings”:

$$g_s$$

$$M_P$$

WGC and Axions

Brown, Cottrell, GS, Soler

**4d Type IIB
D1-instantons**

$$m_i$$

$$f_i$$

$$g_s \ll 1$$

**4d Type IIA
D2-particles**

$$\tilde{m}_i \sim m_i$$

$$\tilde{q}_i \sim f_i^{-1}$$

$$\tilde{g}_s \gg 1$$

**5d M-theory
M2-particles**

$$M_i^{(5d)} \sim m_i$$

$$Q_i^{(5d)} \sim f_i^{-1}$$

$$R_M \rightarrow \infty$$

- Apply the WGC to 5d particles:

$$\frac{Q^{(5d)}}{M_i^{(5d)}} M_P^{(5d)} = \frac{M_P^{(IIB)}}{\sqrt{2} f_i m_i} \geq \text{“1”} \equiv \left(\frac{Q}{M} M_P \right)_{\text{Ext}_{5d}} = \sqrt{\frac{2}{3}}$$

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WGC and Axions

- For each axion (gauge U(1)) there must be an instanton (particle) with

$$e^{-S_{inst}} = e^{-m + i\phi/f}$$

$$f \cdot m \leq \frac{\sqrt{3}}{2} M_P$$

Brown, Cottrell, GS, Soler

For a RR 2-form in IIB string theory. Similar bounds for axions from other p-forms in other string theories have also been obtained.

Multiple Axions/
Multiple $U(1)$'s

WGC and Axions

Multiple axions/U(1)s

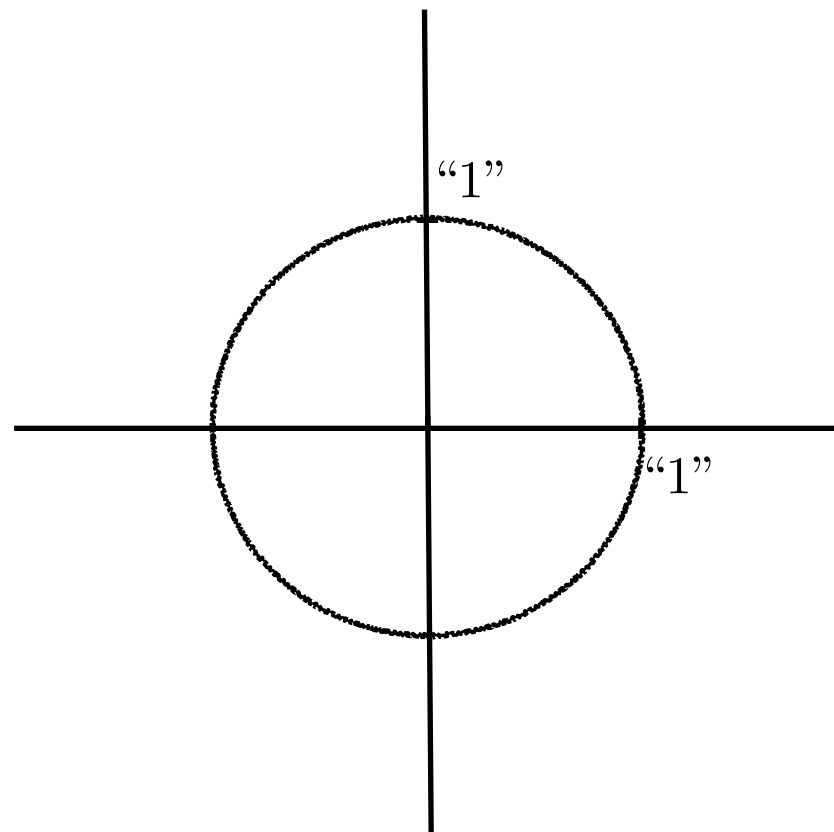
Cheung, Remmen '14
Brown, Cottrell, GS, Soler '15
Rudelius '15

- Consider two U(1) bosons (axions): there must be 2 particles (instantons) $i=1,2$, so that BH's can decay.

$$\vec{z}_i \equiv \frac{M_P}{M_i} \begin{pmatrix} Q_i^1 & Q_i^2 \end{pmatrix} \quad \left(= \frac{M_P}{\sqrt{2} m_i} \begin{pmatrix} 1/f_i^1 & 1/f_i^2 \end{pmatrix} \right)$$

$$|\vec{z}_{EBH}| \equiv \text{“1”}$$

WGC



WGC and Axions

Multiple axions/U(1)s

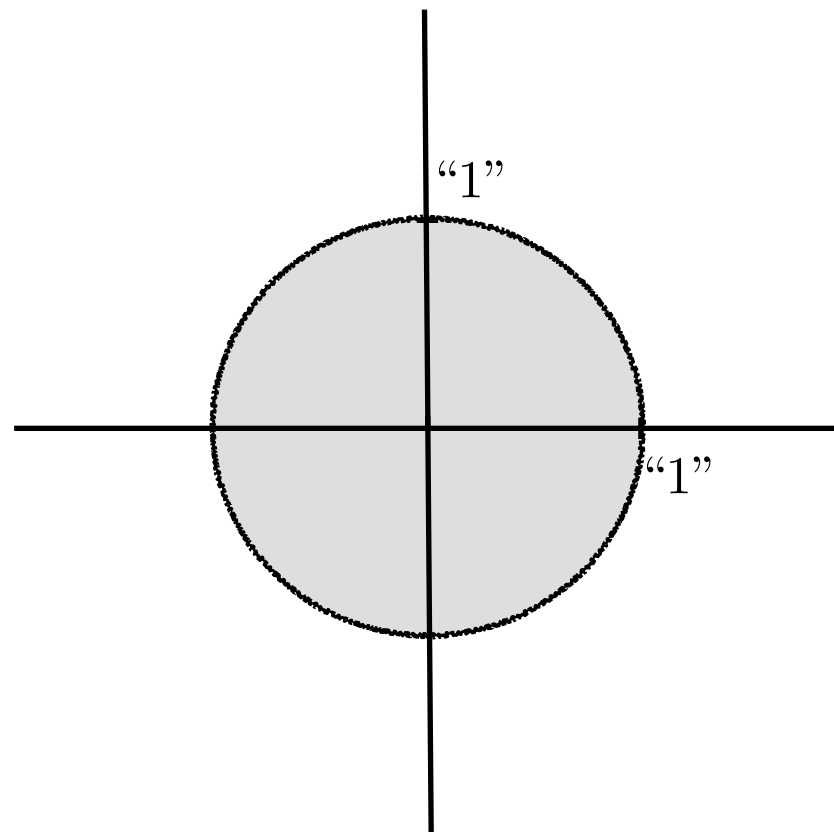
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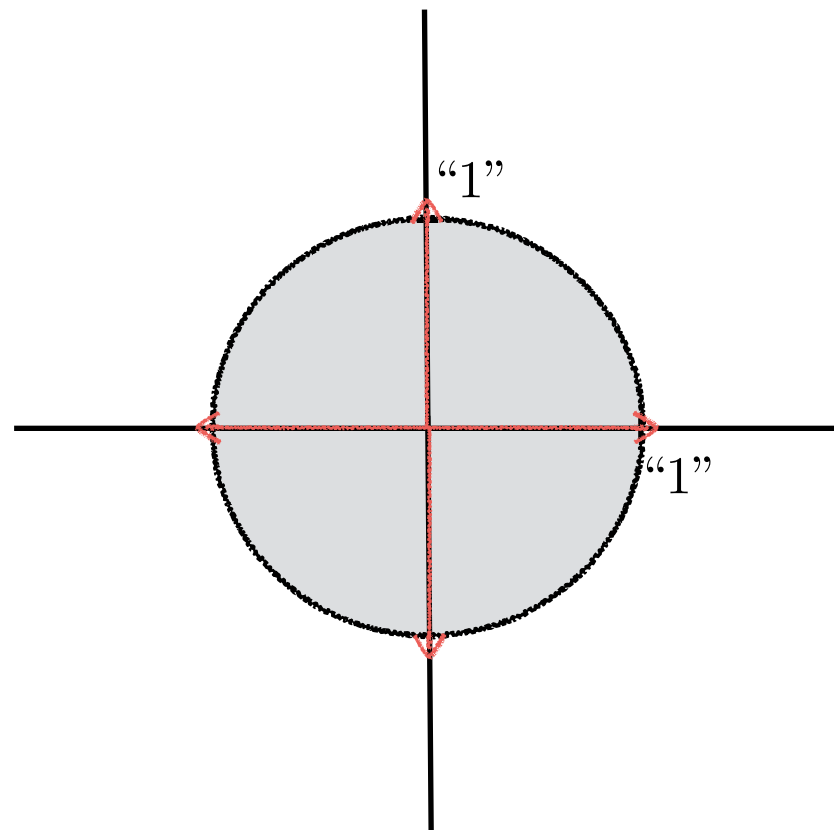
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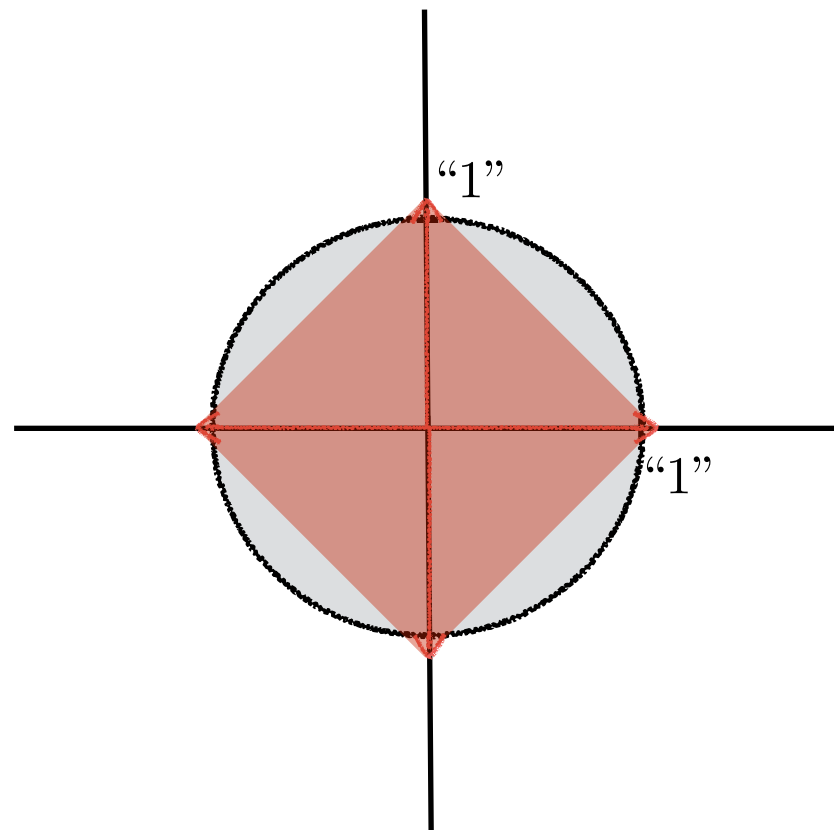
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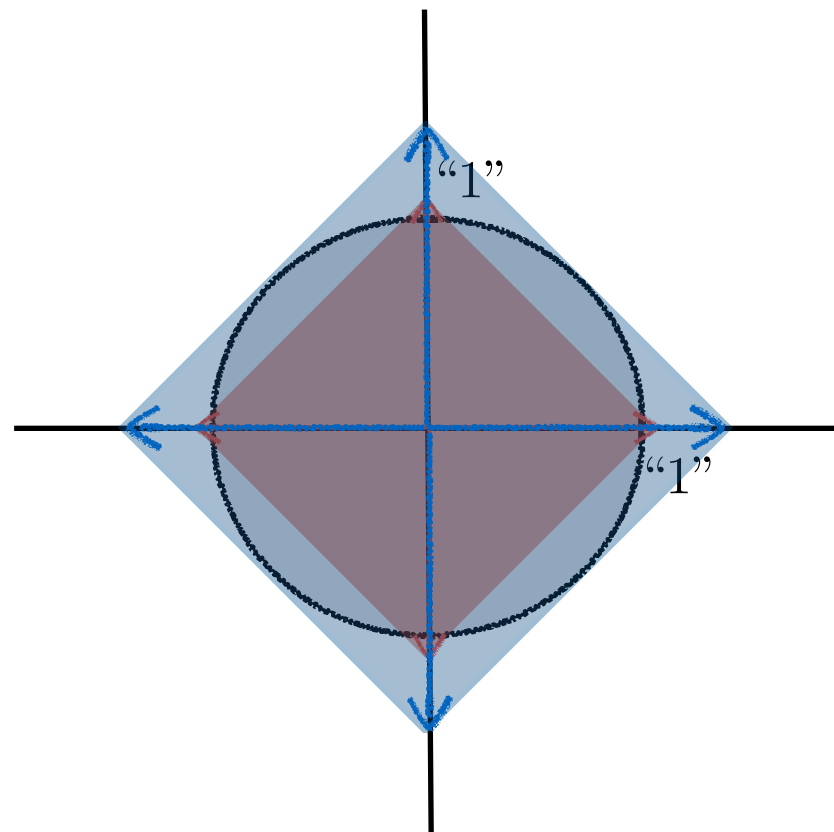
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WGC

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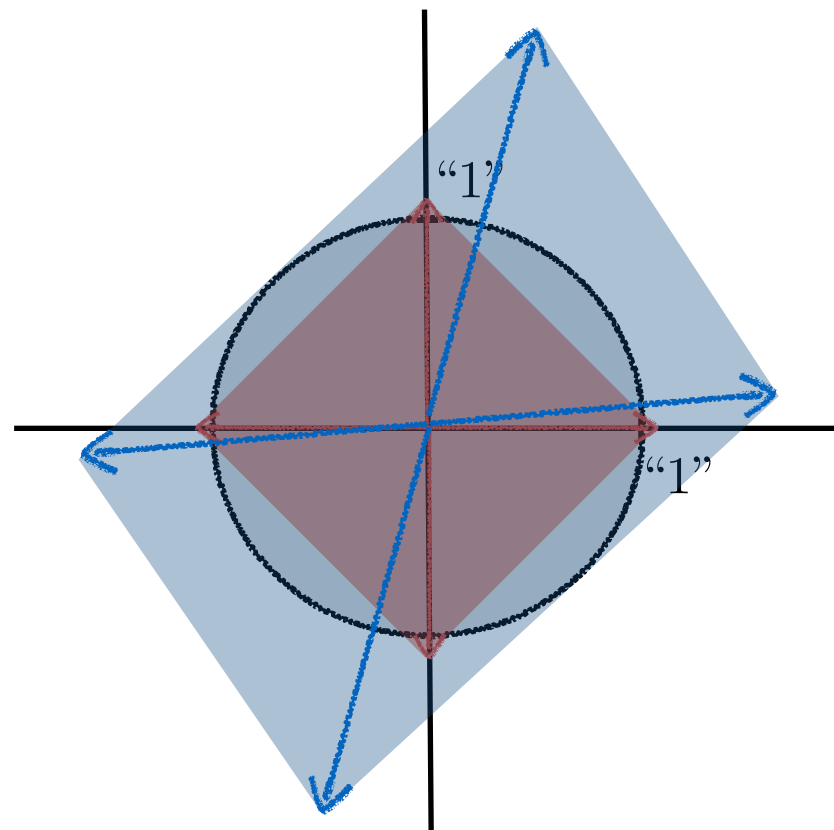
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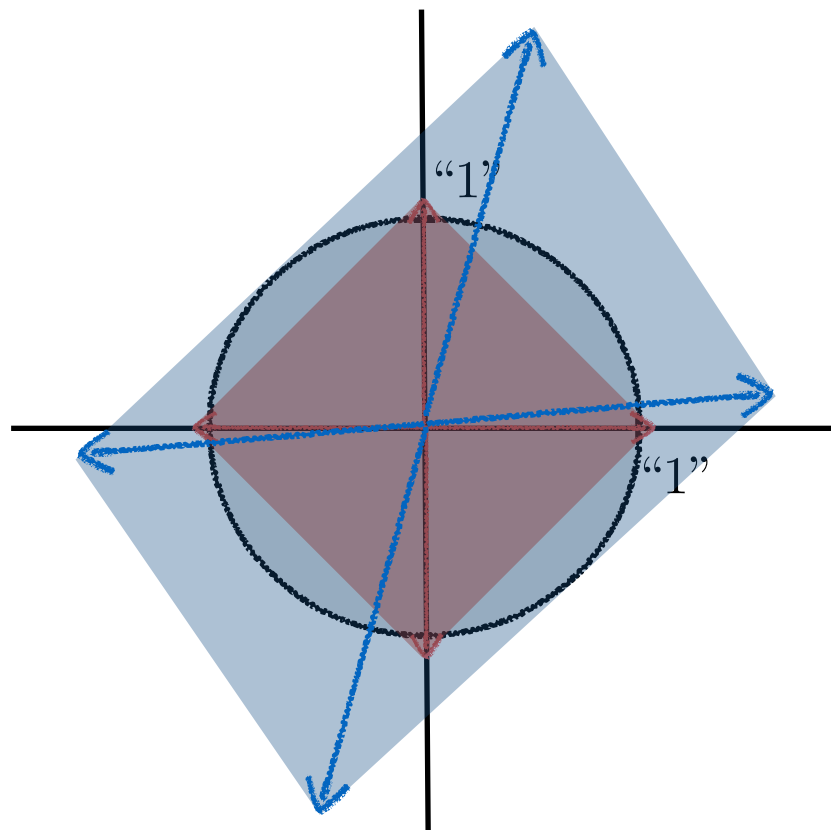
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WGC

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Convex Hull $\{\vec{z}_{p1}, \vec{z}_{p2}\}$

WGC and Axions

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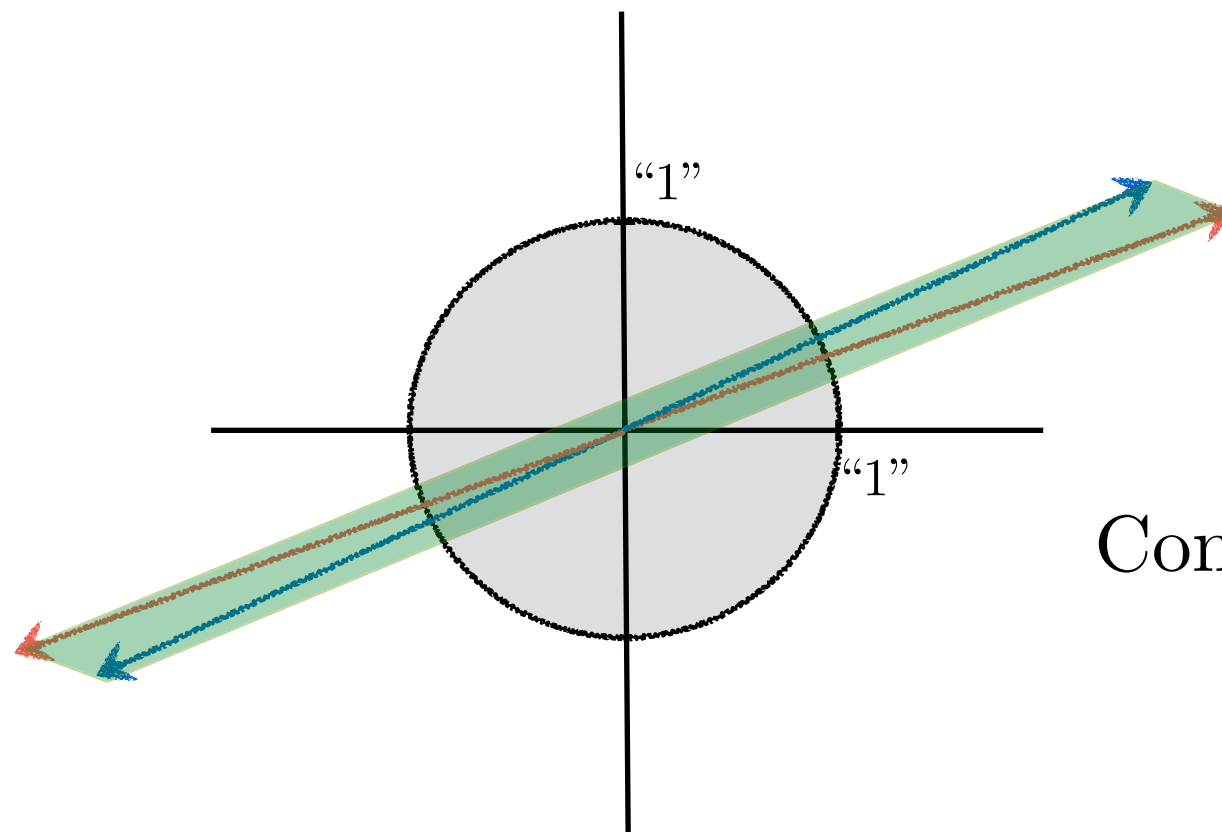
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KNP

WGC



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\cap

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WGC and Axions

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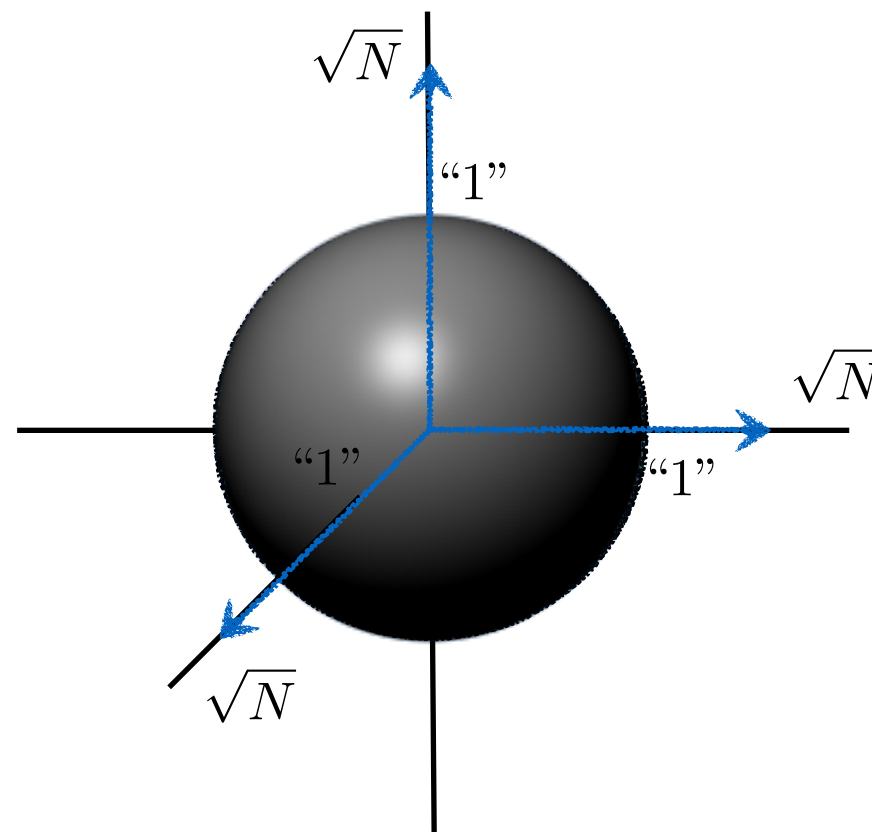
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N-flation

$$z_i^k \geq \sqrt{N} \delta_i^k$$



WGC

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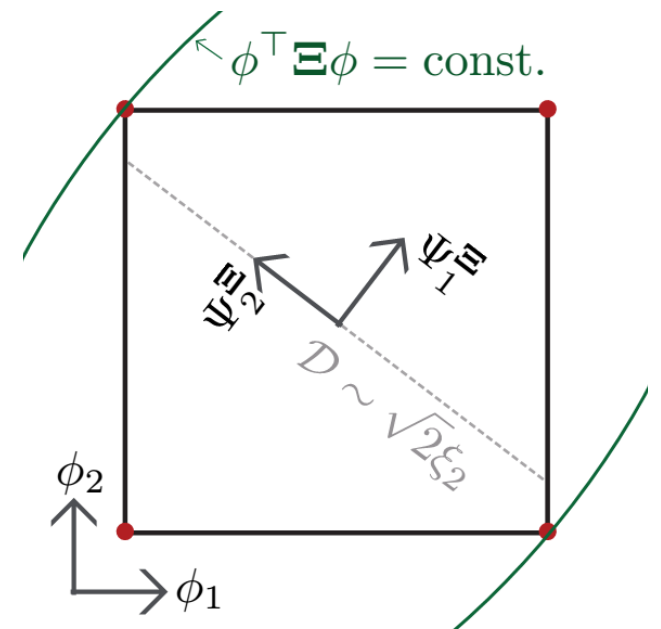
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Convex Hull $\{\vec{z}_{p1}, \vec{z}_{p2}\}$

WGC and Multi-axion Inflation

- Generally, given a set of instantons that gives a super-Planckian “diameter” in axion field space

$$\mathcal{L} = \frac{1}{2} \partial \phi^\top \Xi \partial \phi - \sum_{i=1}^N \Lambda_i^4 [1 - \cos(\phi_i)]$$



Bachlechner et al '15

we showed that the convex hull generated by these instantons **does not** contain the extremal ball, to have parametric control.

- Our conclusions agree with the gravitational instanton diagnostics of [Montero, Uranga, Valenzuela '15] in some instances but go beyond theirs in other cases.

Is there a way around this?



Loophole suggested in Brown, Cottrell, GS, Soler,
“Fencing in the Swampland”, arXiv:1503.04783 [hep-th]

A possible loophole

- The WGC requires $f \cdot m < 1$ for ONE instanton, but not ALL

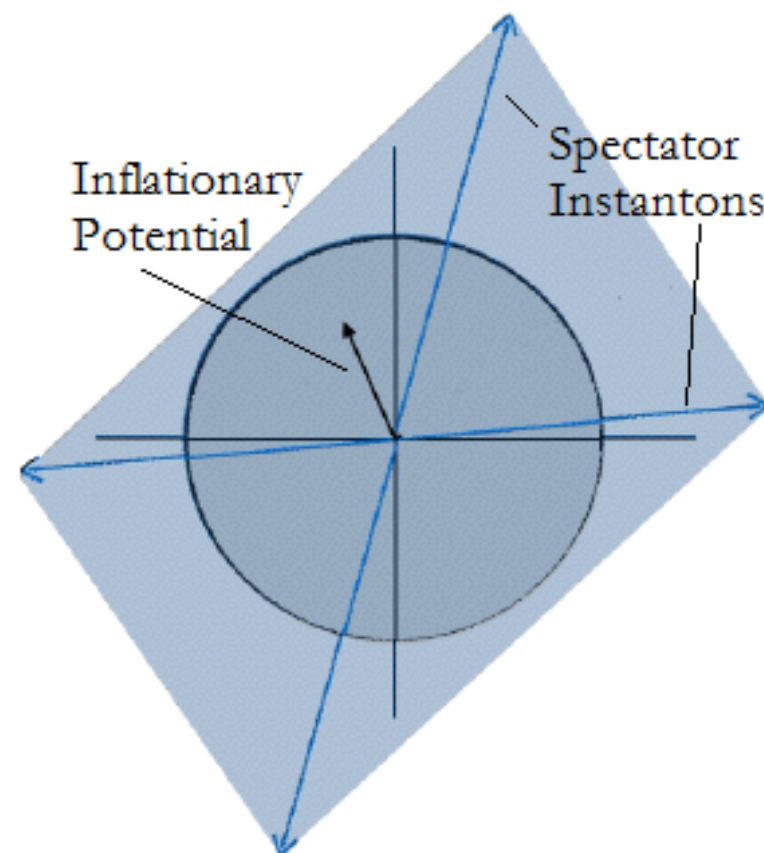
$$V = e^{-m} \left[1 - \cos \left(\frac{\Phi}{F} \right) \right] + e^{-M} \left[1 - \cos \left(\frac{\Phi}{f} \right) \right]$$

With $1 < m \ll M$, $F \gg M_P > f$, $M \times f \ll 1$

- The second instanton fulfills the WGC, but is negligible, an “spectator”. Inflation is governed by the first term.

A possible loophole

- In the presence of “spectator” (negligible) instantons that fulfill the WGC, dominant instantons can generate an inflationary potential



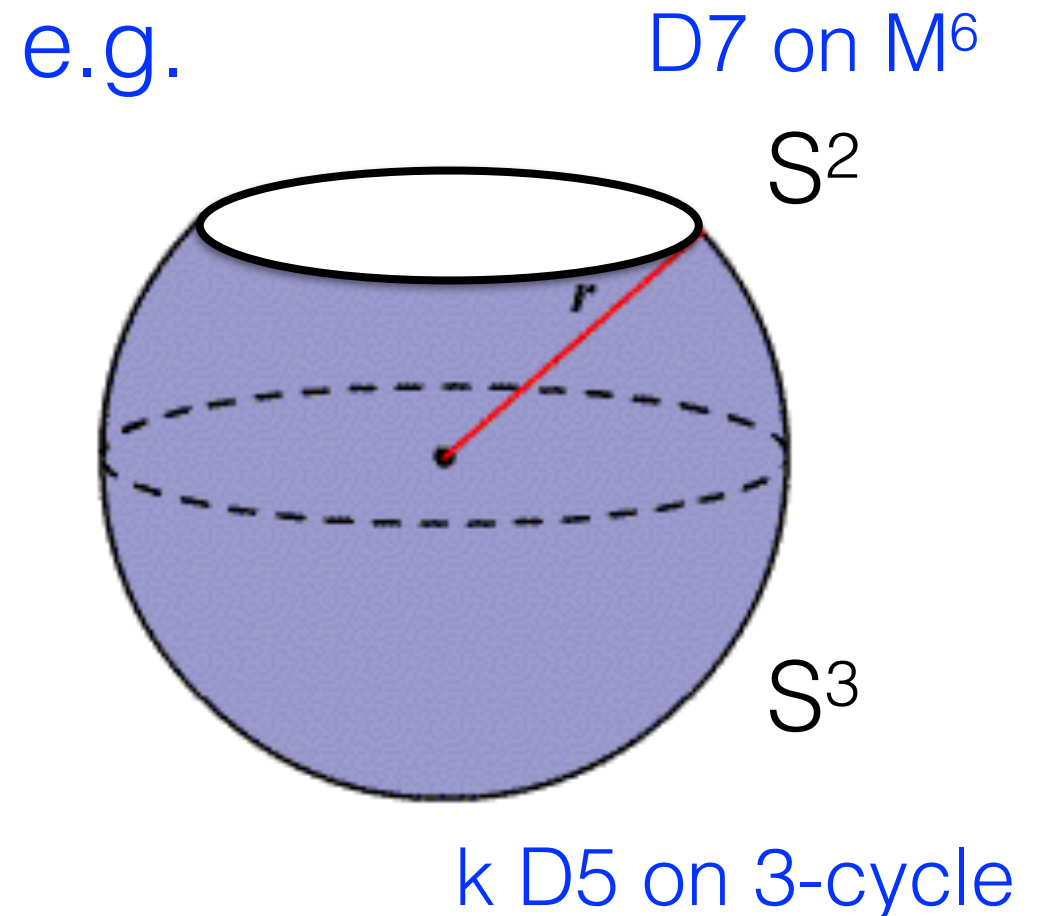
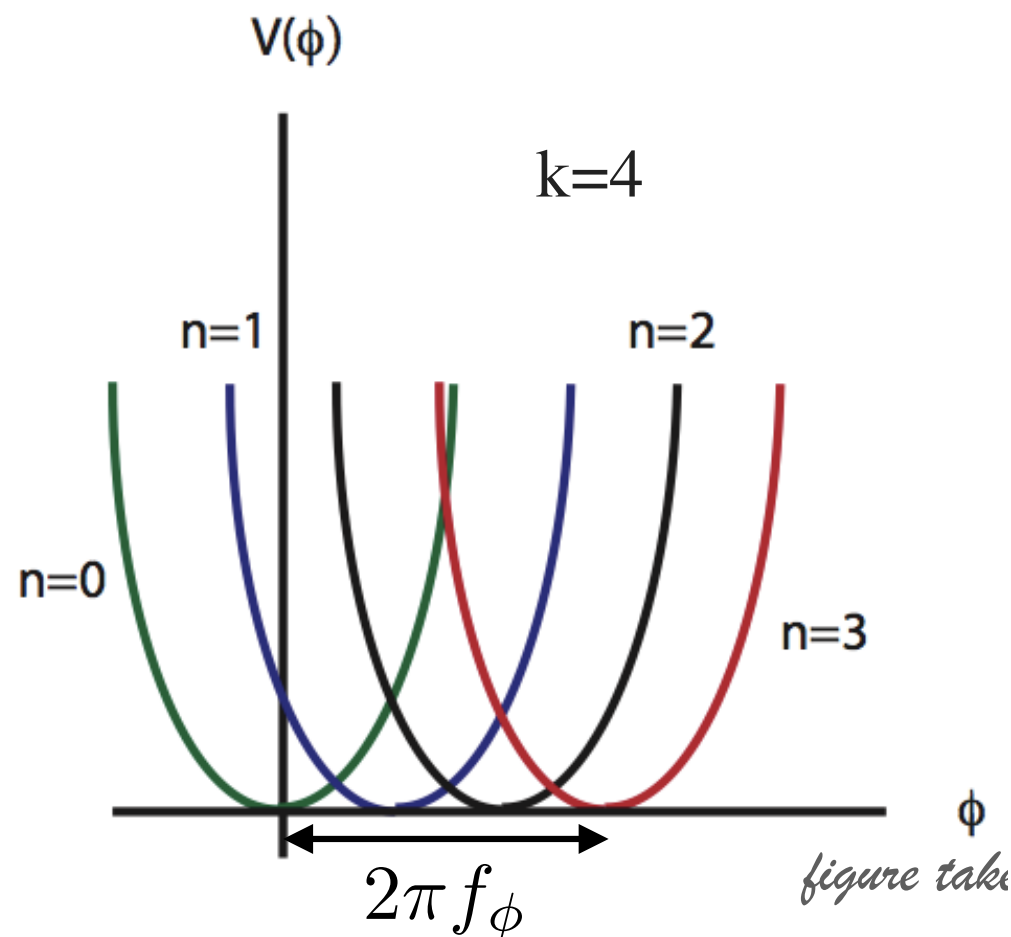
- These scenarios generically violate the Strong-WGC:
“The LIGHTEST charged states satisfy $Q/M > 1$ ”
- Oscillations in power spectrum [Choi, Kim];[Kappl, Nilles,Winkler]

Further Evidences for the WGC

- ∞ many stable remnants in a finite mass range lead to pathologies [Susskind]; WGC is a stronger requirement but
 - It is suggestive based on analyticity, unitarity of scattering amplitudes [Cheung, Remmen]
 - It seems compatible with holography [Nakayama, Nomura]
- For WGC to be compatible w/ dim. reduction, an *even stronger form* (lattice WGC) [Heidenreich, Reece, Rudelius], than our original strong form, was proposed.
- Attempts to evade the WGC (including our loophole) amount to hiding our ignorance about the UV.
- Realizing large field inflation in string theory to tame UV sensitivity; burden of proof on claim of counterexamples.

Axion Monodromy

- Axion is mapped to a **massive** gauge field.
- Possible tunneling to different branches of the potential:



- Suppressing this tunneling can lead to a bound on field range (hence r)

Brown, Garcia-Etxebarria, Marchesano, GS, in progress

Conclusions

Conclusions

- Inflation is sensitive to UV physics. Large field inflation requires *even more input* from quantum gravity.
- F-term axion monodromy inflation
- We have made the WGC precise for (a large class of) axions which can be dualized to $U(1)$ gauge fields.
- Constraints on multiple axions in terms of convex hull (bound on the “diameter” of axion space):
 - KNP, N-flation, kinetic mixing,...

Conclusions

- Exciting interface between BH in quantum gravity and inflation.
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THANKS