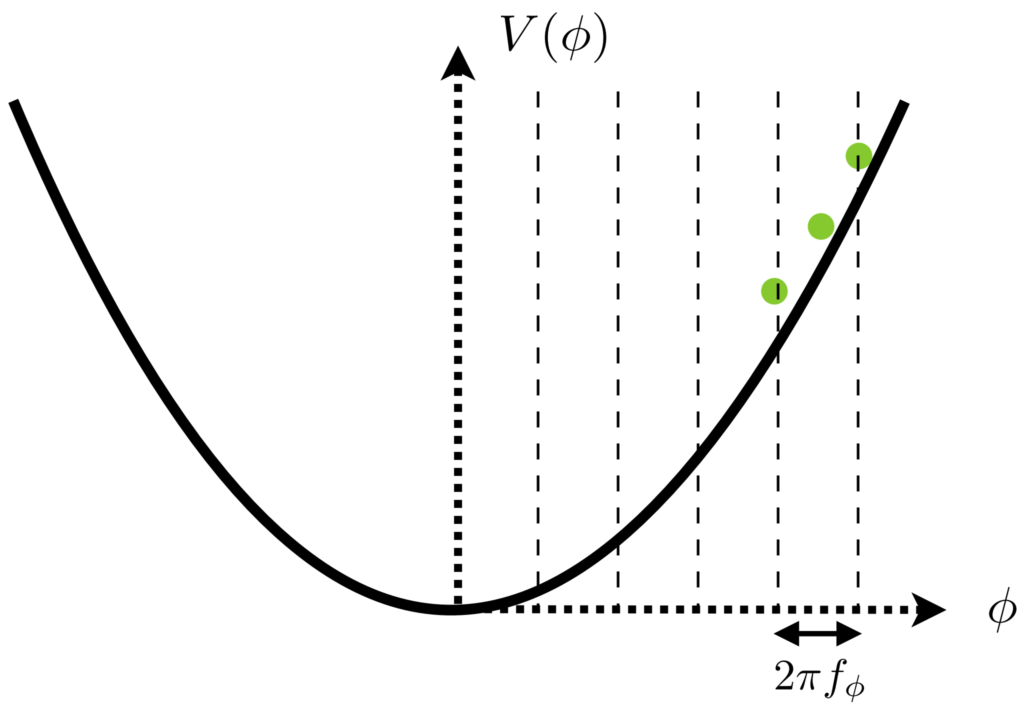
Axion Monodromy

The axion periodicity can be lifted

via brane coupling [Silverstein, Westphal '08];[McAllister, Silverstein, Westphal '08]; …,
or flux potential [Marchesano, GS, Uranga '14];[Blumenhagen, Plauschinn '14];
[Hebecker, Kraus, Witowski, '14];[McAllister, Silverstein, Westphal, Wrase '14]; …

Axion Monodromy Inflation

Idea: Combine chaotic inflation and natural inflation



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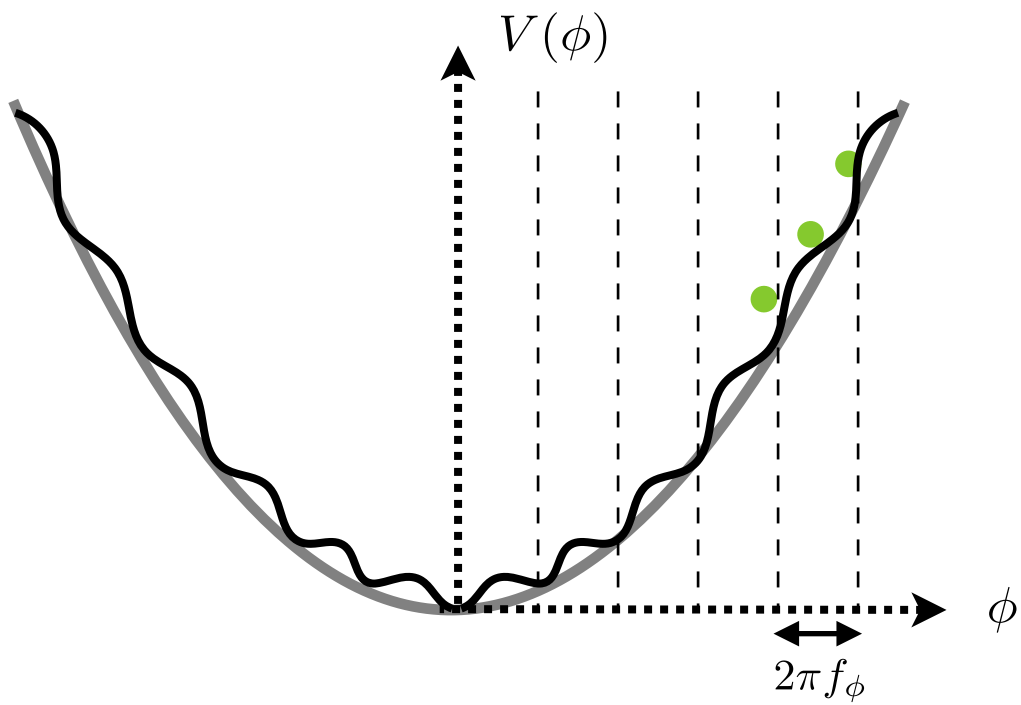

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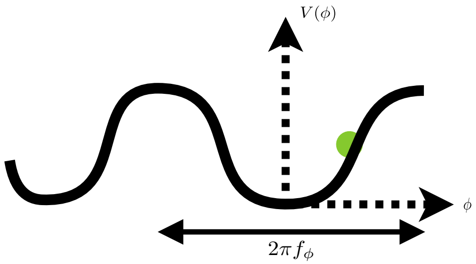
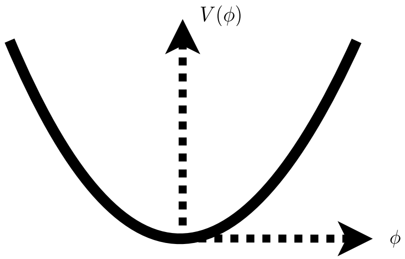
Axion Monodromy

Two sources of shift symmetry breaking:

1. NP effects → discrete symmetry (no monodromy)



2. Periodicity lifting effects (monodromy)

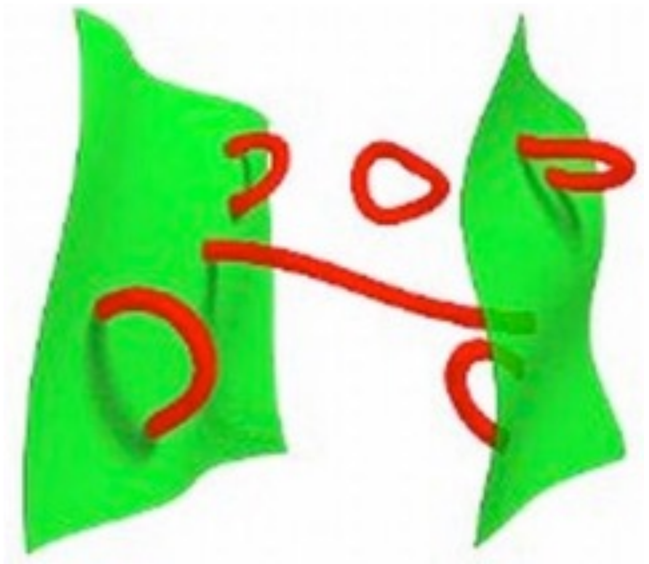


In earlier axion monodromy models (e.g. [McAllister, Silverstein and Westphal]), a wrapped brane was used to lift the axion periodicity.

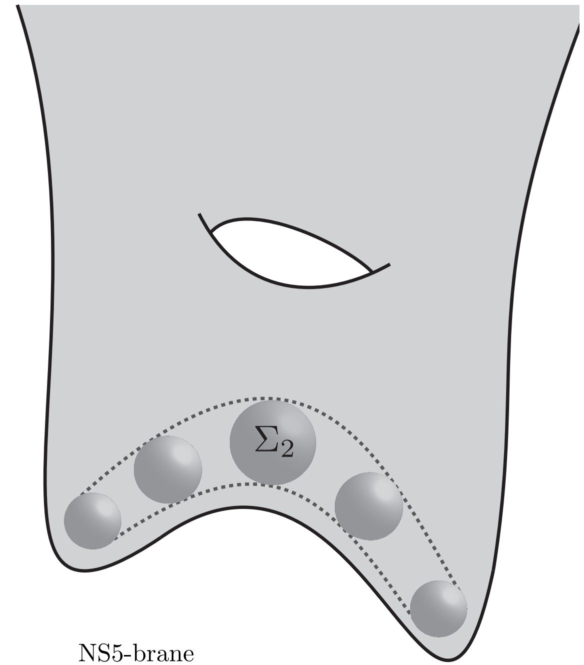
A 5-brane wrapping the 2-cycle breaks the axion shift symmetry:

$$V = T_5 \int_{\Sigma_2} \mathrm{d}^2\sigma \sqrt{-\det(G+A)}$$

$$= T_5 \sqrt{\ell^2_{\Sigma_2} + a^2}$$



Axion Monodromy



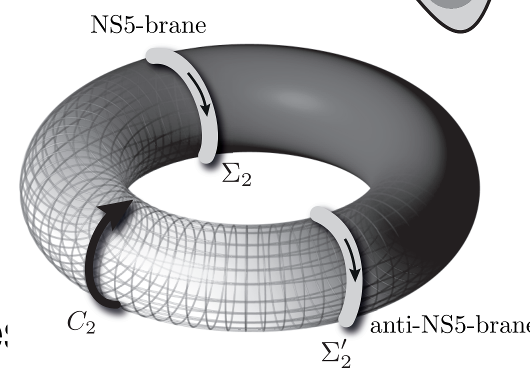
Challenges in realizing monodromy w/ **branes**:

- η problem ⇒ NS5-branes (not D5-branes)

$$W = W_0 + \mathcal{A} e^{-2\pi T} \, ,$$
$$K = -3 \ln \left(T + \bar{T} + \gamma b^2 \right)$$

$$S_{\rm D5} = \frac{1}{(2\pi)^5 g_{\rm s} (\alpha')^3} \int_{\mathcal{M}_4 \times \Sigma_2} {\rm d}^6\sigma \sqrt{-\det(G_{ab} + B_{ab})}$$

$$c_I = \frac{1}{\alpha'} \int_{\Sigma_2^I} C_2 \qquad \text{not} \qquad b_I = \frac{1}{\alpha'} \int_{\Sigma_2^I} B_2$$



- Gauss's law ⇒ $\overline{\text{NS5}}$ wrapping "homologous" cycles

- Brane annihilation ⇒ NS5 and $\overline{\text{NS5}}$ at different warped throats

- Backreaction [Conlon] ⇒ whole system embedded into another throat.

F-term Axion Monodromy Inflation

Obs: Axion Monodromy ~ Giving a mass to an axion

✦ Done in string theory within the moduli stabilization program: adding ingredients like background fluxes generate superpotentials in the effective 4d theory

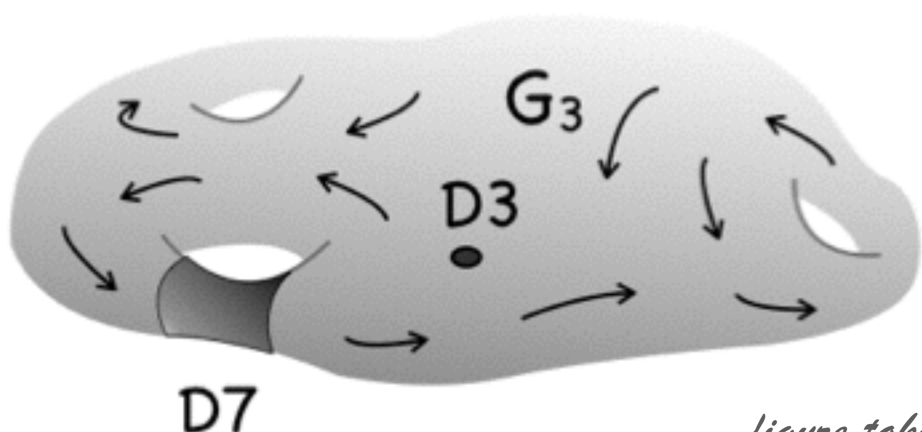


figure taken from Ibañez & Uranga '12

F-term Axion Monodromy Inflation

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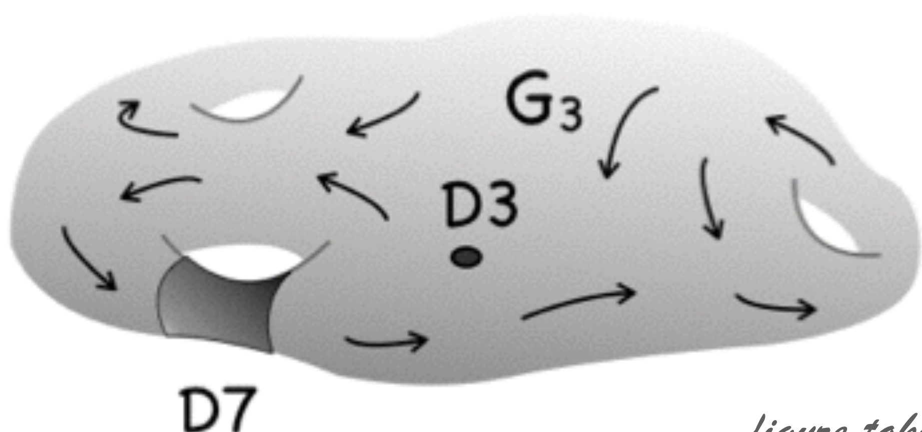
Idea: Use same techniques to generate an inflation potential



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Idea: Use same techniques to generate an inflation potential

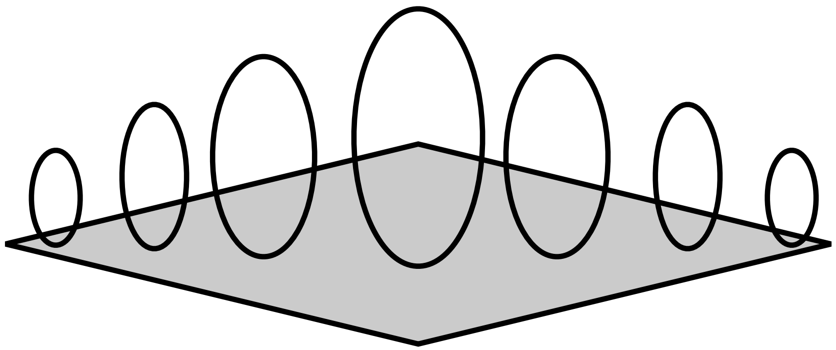
- Simpler models, all sectors understood at weak coupling
- Spontaneous SUSY breaking, no need for brane-anti-brane
- Clear endpoint of inflation, allows to address reheating

Toy Example: Massive Wilson line

- Simple example of axion: (4+d)-dimensional gauge field integrated over a circle in a compact space Π_d

$$\phi = \int_{S^1} A_1 \qquad \text{or} \qquad A_1 = \phi(x)\,\eta_1(y)$$

  - ϕ massless if $\Delta\eta_1 = 0 \Rightarrow S^1$ is a non-trivial circle in Π_d exact periodicity and (pert.) shift symmetry
  - ϕ massive if $\Delta\eta_1 = -\mu^2\,\eta_1 \Rightarrow kS^1$ homologically trivial in Π_d (non-trivial fibration)



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  (non-trivial fibration)

$$F_2 = dA_1 = \phi\, d\eta_1 \sim \mu\phi\,\omega_2$$

$\Rightarrow$ shifts in ϕ increase energy via the induced flux F_2

$\Rightarrow$ periodicity is broken and shift symmetry approximate

MWL and twisted tori

- Simple way to construct massive Wilson lines: consider compact extra dimensions Π_d with circles fibered over a base, like the twisted tori that appear in flux compactifications

- There are circles that are not contractible but do not correspond to any harmonic 1-form. Instead, they correspond to torsional elements in homology and cohomology groups

$$\text{Tor}\, H_1(\Pi_d, \mathbb{Z}) = \text{Tor}\, H^2(\Pi_d, \mathbb{Z}) = \mathbb{Z}_k$$

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- Simplest example: twisted 3-torus $\tilde{\mathbb{T}}^3$

$$H_1(\tilde{\mathbb{T}}^3, \mathbb{Z}) = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}_k$$

two normal 1-cycles

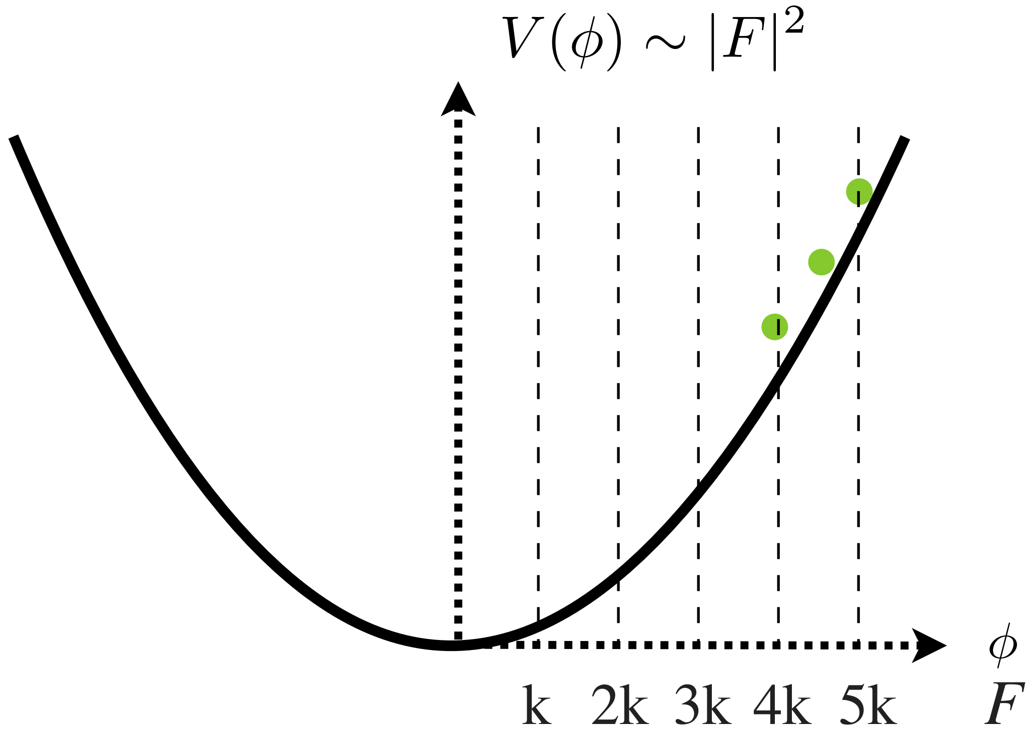
one torsional 1-cycle

$$d\eta_1 = k dx^2 \wedge dx^3 \longrightarrow F = \phi\, k\, dx^2 \wedge dx^3$$

$$\mu = \frac{kR_1}{R_2 R_3}$$

under a shift $\phi \to \phi + 1$
F_2 increases by k units

MWL and monodromy



Question:

How does monodromy and approximate shift symmetry help prevent wild UV corrections?

Torsion and gauge invariance

- Twisted tori torsional invariants are not just a fancy way of detecting non-harmonic forms, but are related to a hidden gauge invariance of these axion-monodromy models
- Let us again consider a 7d gauge theory on $M^{1,3}$ x $\tilde{\mathbb{T}}^3$
  - Instead of A_1 we consider its magnetic dual V_4

$$V_4 = C_3 \wedge \eta_1 + b_2 \wedge \sigma_2 \xrightarrow{d\eta_1 = k\,\sigma_2} dV_4 = dC_3 \wedge \eta_1 + (db_2 - kC_3) \wedge \sigma_2$$

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  - From dimensional reduction of the kinetic term:

$$\int d^7x\,|dV_4|^2 \longrightarrow \int d^4x\,|dC_3|^2 + \frac{\mu^2}{k^2}|db_2 - kC_3|^2$$

  - Gauge invariance $C_3 \to C_3 + d\Lambda_2 \qquad b_2 \to b_2 + k\Lambda_2$
  - Generalization of the Stückelberg Lagrangian

Quevedo & Trugenberger '96

Effective 4d theory

- The effective 4d Lagrangian

$$\int d^4x\, |dC_3|^2 + \frac{\mu^2}{k^2}|db_2 - kC_3|^2$$

describes a massive axion, has been applied to QCD axion ⇒ generalized to arbitrary V(ϕ)

Kallosh et al. '95
Dvali, Jackiw, Pi '05
Dvali, Folkerts, Franca '13

- Reproduces the axion-four-form Lagrangian proposed by Kaloper and Sorbo as 4d model of axion-monodromy inflation *with mild UV corrections*

$$\int d^4x\, |F_4|^2 + |d\phi|^2 + \phi F_4 \qquad \begin{aligned} F_4 &= dC_3 \\ d\phi &= *_4 db_2 \end{aligned}$$

Kaloper & Sorbo '08

- It is related to an F-term generated mass term

Groh, Louis, Sommerfeld '12

Effective 4d theory

- Effective 4d Lagrangian

$$\int d^4x\, |dC_3|^2 + \frac{\mu^2}{k^2}|db_2 - kC_3|^2$$

$$F_4 = dC_3$$
$$d\phi = *_4 db_2$$

- Gauge symmetry ⇒ UV corrections only depend on F_4

$$\mathcal{L}_{\text{eff}}[\phi] = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\mu^2\phi^2 + \Lambda^4 \sum_{i=1}^{\infty} c_i \frac{\phi^{2i}}{\Lambda^{2i}}$$

$$\sum_n c_n \frac{F^{2n}}{\Lambda^{4n}} \longrightarrow \mu^2\phi^2 \sum_n c_n \left(\frac{\mu^2\phi^2}{\Lambda^4}\right)^n$$

⇒ suppressed corrections up to the scale where $V(\phi) \sim \Lambda^4$

⇒ effective scale for corrections $\Lambda \to \Lambda_{\text{eff}} = \Lambda^2/\mu$

Effective 4d theory

- Effective 4d Lagrangian

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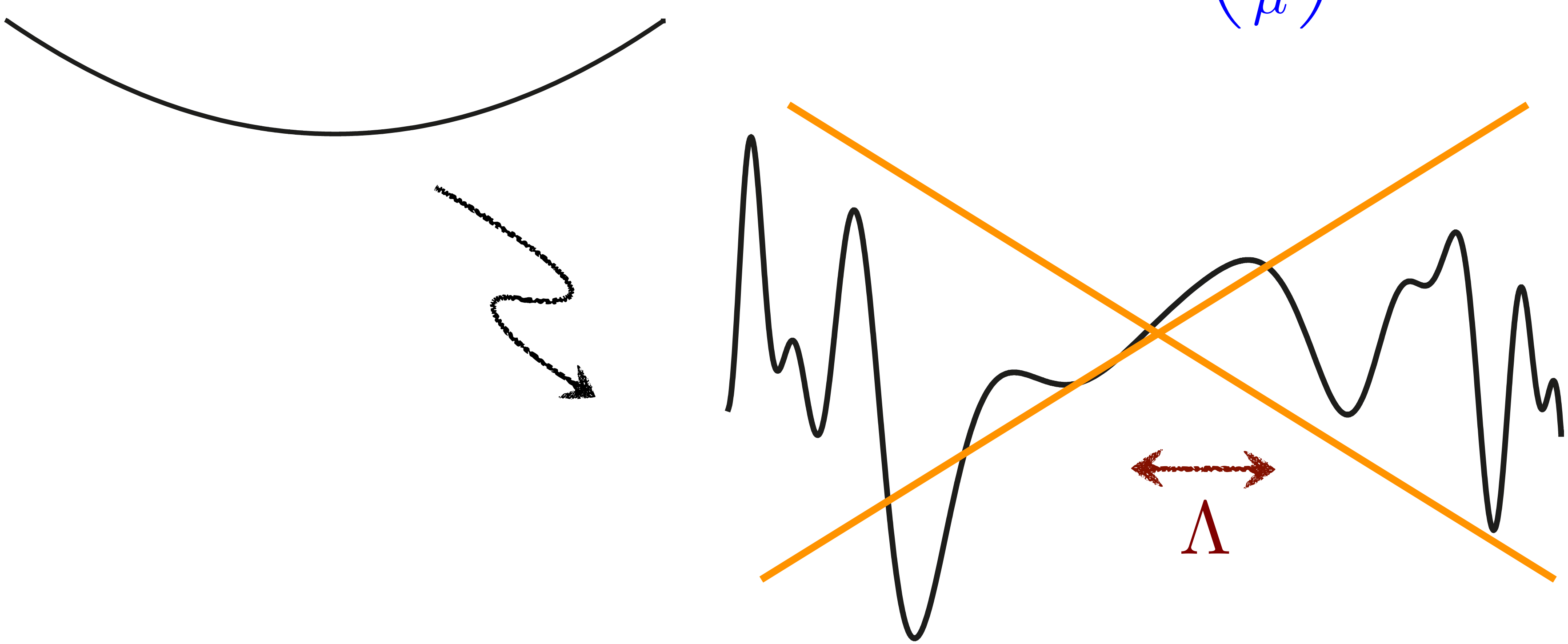

Discrete symmetries and domain walls

- The integer k in the Lagrangian

$$\int d^4x\,|F_4|^2 + \frac{\mu^2}{k^2}|db_2 - kC_3|^2$$

corresponds to a discrete symmetry of the theory broken spontaneously once a choice of four-form flux is made. This amounts to choose a branch of the scalar potential

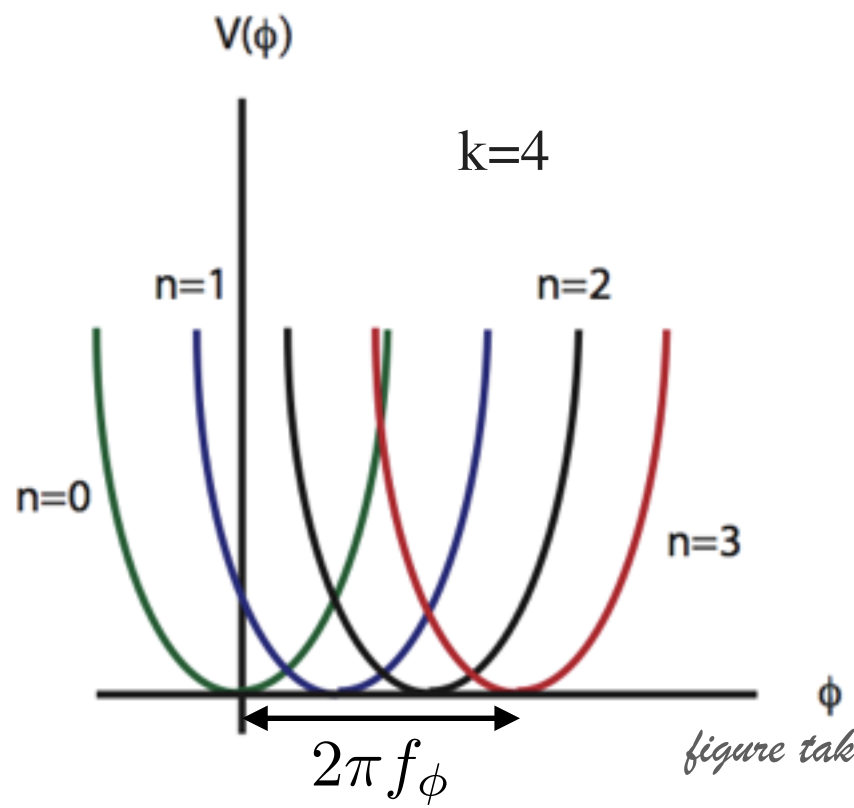

figure taken from Kaloper & Lawrence '14

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- Branch jumps are made via nucleation of domain walls that couple to C_3, and this puts a maximum to the inflaton range

- Domain walls analysed in string constructions:

Berasaluce-Gonzalez, Camara, Marchesano, Uranga '12

  - They correspond to discrete symmetries of the superpotential/ landscape of vacua, and appear whenever axions are stabilized
  - k domain walls decay in a cosmic string implementing $\phi \to \phi+1$

Massive Wilson lines in string theory

- Simple example of MWL in string theory: D6-brane on $M^{1,3}$ x $\tilde{\mathbb{T}}^3$
- An inflaton vev induces a non-trivial flux F_2 proportional to ϕ but now this flux enters the DBI action

$$\sqrt{\det\left(G + 2\pi\alpha' F_2\right)} \;=\; d\mathrm{vol}_{M^{1,3}}\;\left(|F_2|^2 + \mathrm{corrections}\right)$$

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- For small values of ϕ we recover chaotic inflation, but for large values the corrections are important and we have a potential of the form

$$V = \sqrt{L^4 + \langle\phi\rangle^2} - L^2$$

Similar to the D4-brane model of Silverstein and Westphal except for the inflation endpoint

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Massive Wilson lines and flattening

- The DBI modification

$$\langle\phi\rangle^2 \to \sqrt{L^4 + \langle\phi\rangle^2} - L^2$$

can be interpreted as corrections due to UV completion

- E.g., integrating out moduli such that H < m_{mod} < M_{GUT} will correct the potential, although not destabilise it

Kaloper, Lawrence, Sorbo '11

- In the DBI case the potential is flattened: argued general effect due to couplings to heavy fields

Dong, Horn, Silverstein, Westphal '10

- Large vev flattening also observed in examples of confining gauge theories whose gravity dual is known [Witten'98]

Dubovsky, Lawrence, Roberts '11

Other string examples

- We can integrate a bulk p-form potential C_p over a p-cycle to get an axion

$$F_{p+1} = dC_p, \quad C_p \to C_p + d\Lambda_{p-1} \qquad\qquad c = \int_{\pi_p} C_p$$

- If the p-cycle is torsional we will get the same effective action

$$\int d^{10}x |F_{9-p}|^2 \longrightarrow \int d^4x \, |dC_3|^2 + \frac{\mu^2}{k^2} |db_2 - kC_3|^2$$

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- The topological groups that detect this possibility are

$$\text{Tor}\, H_p(\mathbf{X}_6, \mathbb{Z}) = \text{Tor}\, H^{p+1}(\mathbf{X}_6, \mathbb{Z}) = \text{Tor}\, H^{6-p}(\mathbf{X}_6, \mathbb{Z}) = \text{Tor}\, H_{5-p}(\mathbf{X}_6, \mathbb{Z})$$

one should make sure that the corresponding axion mass is well below the compactification scale (e.g., using warping)

Franco, Galloni, Retolaza, Uranga '14

Other string examples

- Axions also obtain a mass with background fluxes
- Simplest example: $\phi = C_0$ in the presence of NSNS flux H_3

$$W = \int_{\mathbf{X}_6} (F_3 - \tau H_3) \wedge \Omega \qquad \tau = C_0 + i/g_s$$

- We also recover the axion-four-form potential

$$\int_{M^{1,3} \times \mathbf{X}_6} C_0 H_3 \wedge F_7 = \int_{M^{1,3}} C_0 F_4 \qquad F_4 = \int_{\mathrm{PD}[H_3]} F_7$$

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- M-theory version: *Beasley, Witten '02*
- A rich set of superpotentials obtained with type IIA fluxes

$$\int_{\mathbf{X}_6} e^{J_c} \wedge (F_0 + F_2 + F_4) \qquad J_c = J + iB$$

→ **potentials higher than quadratic**

- Massive axions detected by torsion groups in K-theory

Features of Axion Monodromy

- Axion monodromy inflation takes this form:

$$V(\phi) = V_{\text{non-periodic}}(\phi) + \Lambda^4 \cos\left(\frac{\phi}{f}\right)$$

- Simplest example: linear potential (including flattening)

$$V(\phi) = \mu^3 \phi + \Lambda^4 \cos\left(\frac{\phi}{f}\right) = \mu^3 \left[\phi + bf \cos\left(\frac{\phi}{f}\right)\right]$$

- However, V can be more general:

$$V_{\text{non-periodic}}(\phi) = \mu^{4-p}\phi^p \quad \text{or even} \quad V_{\text{non-periodic}}(\phi) = \sum_n c_n \phi^n$$

- This leads to modulations in power-spectrum & bispectrum:

$$\Delta^2_{\mathcal{R}}(k) = \Delta^2_{\mathcal{R}}(k_*) \left(\frac{k}{k_*}\right)^{n_s-1} \left[1 + \delta n_s \cos\left(\frac{\phi_k}{f}\right)\right] \approx \Delta^2_{\mathcal{R}} \left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{\delta n_s}{\ln(k/k_*)} \cos\left(\frac{\phi_k}{f}\right)}$$

$$\frac{G(k_1,k_2,k_3)}{k_1 k_2 k_3} = f_{res} \sin\left(\frac{2}{\phi f} \ln K + \text{phase}\right)$$

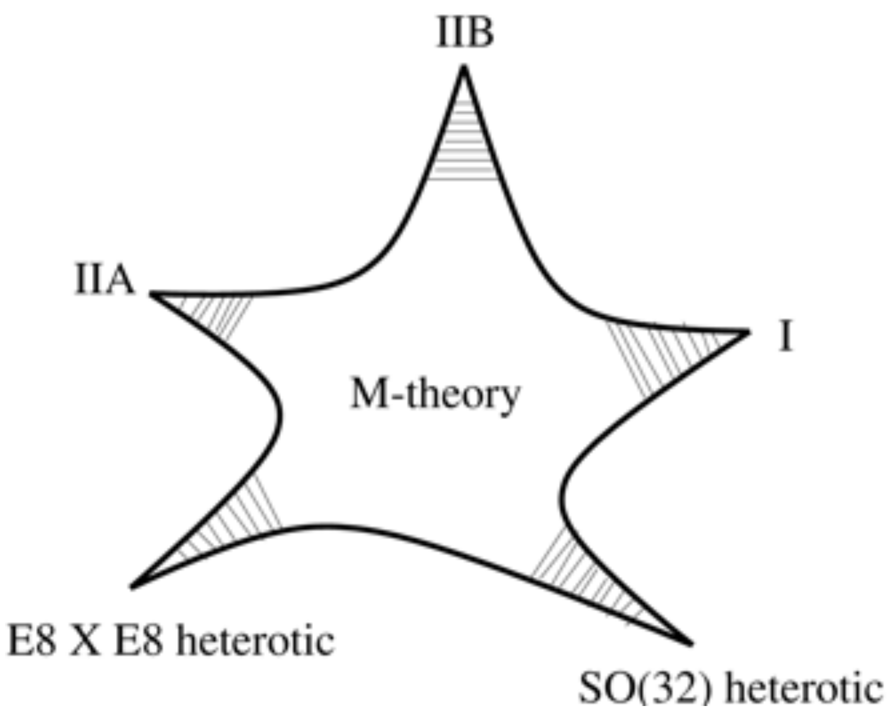
motivate template for PLANCK

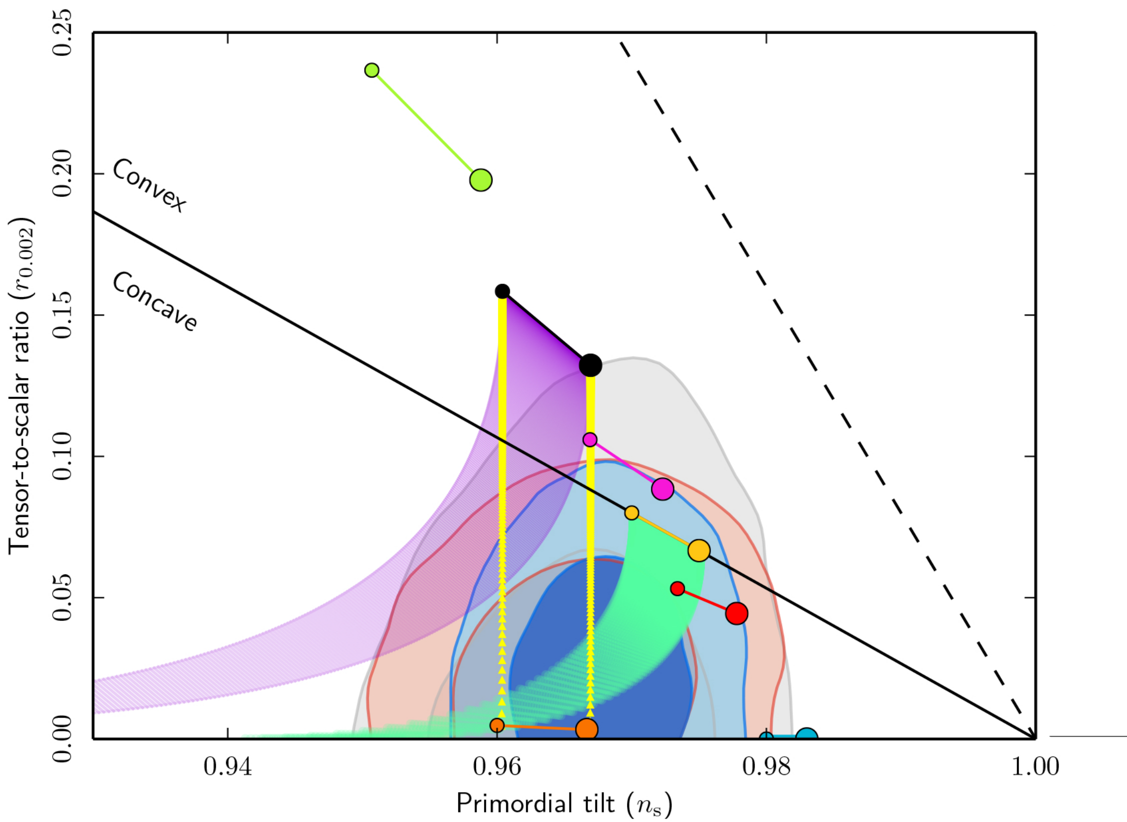
Short Summary

- Axion monodromy is an elegant idea that combines chaotic and natural inflation, aiming to prevent disastrous UV corrections to the inflaton potential.
- We have discussed its ***concrete*** implementation in a ***new*** framework, dubbed F-term axion monodromy inflation compatible with spontaneous supersymmetry breaking.
- In a simple set of models the inflaton is a massive Wilson line, but there are many more flux monodromy models.
- Key to taming UV corrections is the axion-four-form coupling.
- Discrete symmetries classified by K-theory torsion groups.
- α' corrections important for inflation ("flattening") and moduli stabilization.

Short Summary

- A broad class of large field inflationary scenarios that can be implemented in any limit of string theory w/ rich pheno:





- Moduli stabilization needs to be addressed in detailed models. (see e.g., [Blumenhagen, Herschmann, Plauschinn],…).

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