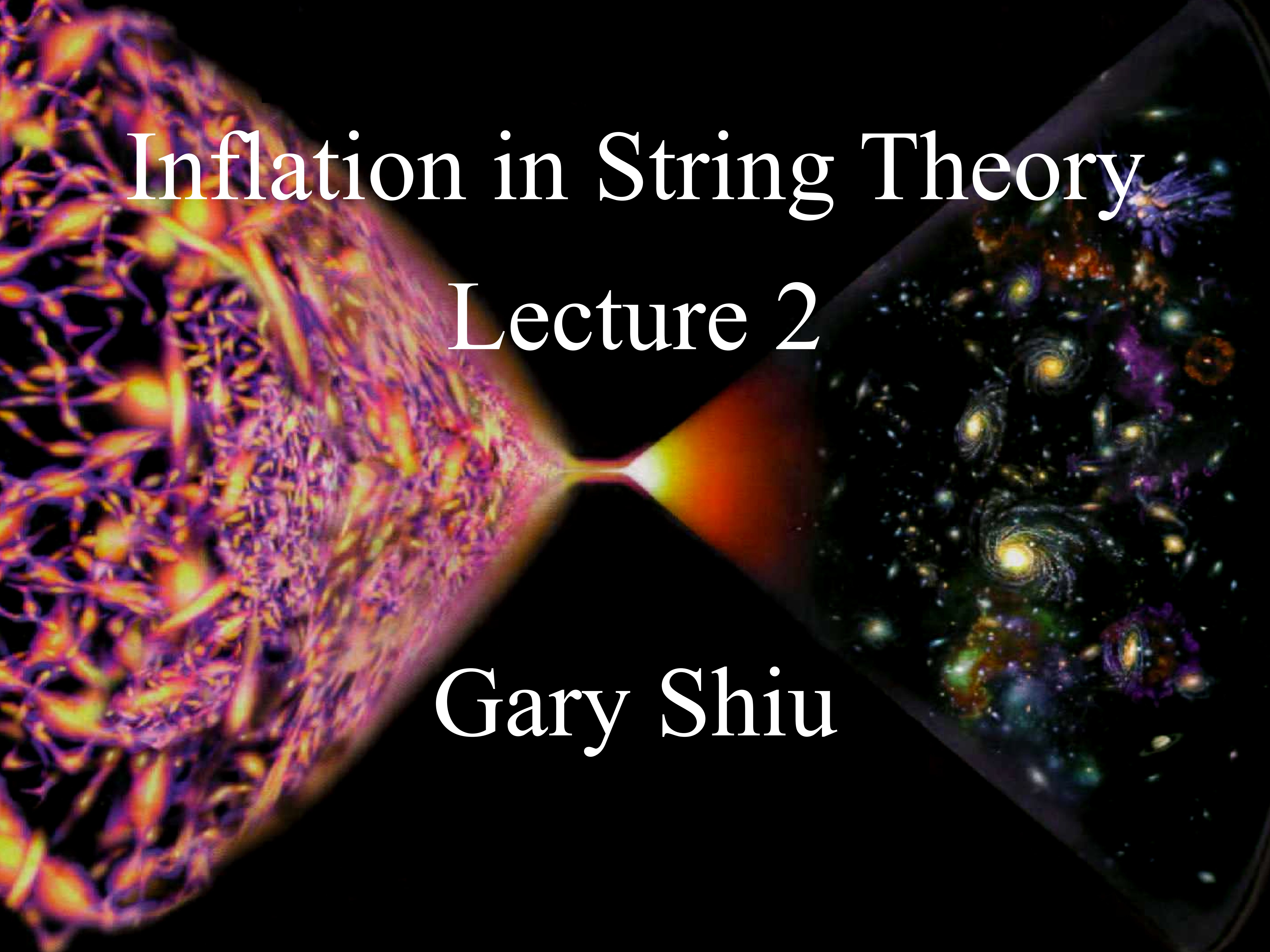
Inflation in String Theory

Lecture 2



Gary Shiu

From the first lecture …

Tensor Perturbations

- Quadratic action for $\delta g_{ij}=a^2 h_{ij}$: $S_{(2)} = \frac{M_P^2}{8}\int d\tau d^3x a^2 \left[\left(h'_{ij}\right)^2 - (\nabla h_{ij})^2\right]$
- In terms of the canonically normalized polarization modes:

$$v_{\vec{k},\gamma} \equiv \frac{a}{2} M_P h_{\vec{k},\gamma} \qquad \gamma = 1, 2$$

- The action reads: $S = \sum_\gamma \frac{1}{2}\int d\tau d^3k \left[\left(v'_{\vec{k},\gamma}\right)^2 - \left(k^2 - \frac{a''}{a}\right)\left(v_{\vec{k},\gamma}\right)^2\right]$
- For an approximate dS background:

$$P_T = 2P_h = \frac{4}{k^3}\frac{H^2}{M_P^2} \quad \Rightarrow \quad \boxed{\Delta_h^2(k) \equiv \frac{k^3}{2\pi^2} P_h(k) = \frac{2}{\pi^2}\frac{H^2}{M_{\rm pl}^2}}$$

↑ **2 polarizations**

Tensor Perturbations

- Tensor tilt: $$n_t \equiv \frac{d \ln \Delta_h^2}{d \ln k} = -2\varepsilon$$ (consistency condition)

- Tensor-to-scalar ratio: $$r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2} \quad \Rightarrow \quad \frac{H}{M_{\rm pl}} = \pi \, \Delta_{\mathcal{R}}(k_\star) \sqrt{\frac{r}{2}}$$

  direct probe of inflation scale

- On substituting $\Delta_{\mathcal{R}}(k_\star) = 4.7 \times 10^{-5}$ (measured)

$$E_{\rm inf} \equiv (3H^2 M_{\rm pl}^2)^{1/4} = 8 \times 10^{-3} \left(\frac{r}{0.1}\right)^{1/4} M_{\rm pl}$$

GUT scale physics

Lyth Bound

- Tensor-to-scalar ratio: $r \equiv \frac{P_T}{P_S} = \frac{8}{M_P^2}\left(\frac{\dot{\varphi}}{H}\right)^2 = \frac{8}{M_P^2}\left(\frac{d\varphi}{dN}\right)^2$
- The inflaton field range: $\frac{\Delta\varphi}{M_P} = \int \sqrt{\frac{r(N)}{8}}\, dN$
- r cannot change greatly (at least for single-field slow-roll):

$$\frac{d}{dN}\log r = -\left[(n_s - 1) + \frac{r}{8}\right] \quad \Rightarrow \quad \frac{\Delta\varphi}{M_P} = N_{\mathrm{eff}}\sqrt{\frac{r_*}{8}} \qquad \text{(HW)}$$

- Taking a conservative estimate $N_{\mathrm{eff}} \gtrsim 30$ (while typically $N_{\mathrm{eff}} \gtrsim 50$)

$$\Delta\varphi \gtrsim \left(\frac{r}{0.01}\right)^{1/2} M_P$$

- Example: Chaotic $m^2\varphi^2$ inflation requires $\Delta\varphi = 15 M_P$ for $\Delta N = 60$.

Super-Planckian Field Range?

- EFT is sensitive to massive dofs. when $\Delta\Phi > M_P$, consider e.g.,

$$V(\phi,\psi_i) = V(\phi) + \sum_i \left[\Lambda^2\psi_i^2 + c_4 g_i^2\phi^2\psi_i^2 + c_6 g_i^4\phi^4\frac{\psi_i^2}{\Lambda^2} + \ldots\right]$$

- Mass shifted by $\Delta M \sim \Lambda$ for $\Delta\Phi = \Lambda/g$

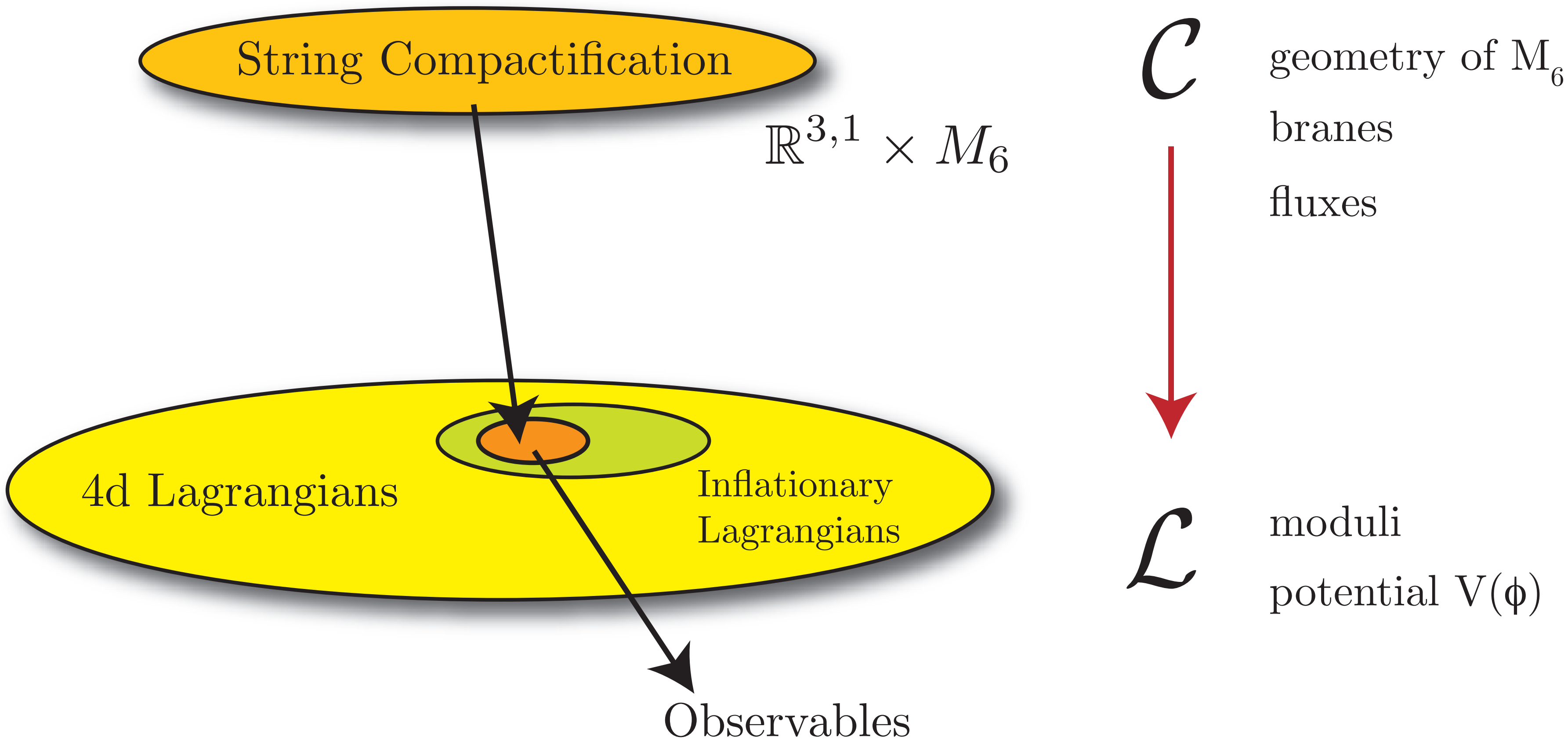
- Super-Planckian field range requires $g<<\Lambda/M_P$ (weaker than G_N strength)!
- Why does Φ couple so weakly? → a question for **Planck scale theory**!

Small vs Large Field Inflation

- Small Field Inflation $\Delta\Phi < M_P$
  - Unobservable tensors ($r << 0.01$)
  - dim-6 Planck suppressed operator $\mathcal{O}_6/M_P^2$ can give $\Delta\eta \sim 1$
  - higher-dim. operators are harmless as $\Delta\eta \lesssim (\Delta\Phi/M_P)^{\Delta-6} << 1$
  - can systematically enumerate all $\mathcal{O}_\Delta$ with $\Delta \geq 6$ and balance them against each other (tuning)
- Large Field Inflation $\Delta\Phi > M_P$
  - Detectable tensors ($r >> 0.01$)
  - $\mathcal{O}_\Delta$ becomes more and more important for larger Δ
  - enumeration is hopeless, need a powerful *symmetry!*

Lecture 2

Inflation in String Theory



- Inflation is UV sensitive in any case!
- Though string inflation $\subset$ inflation, some models/signatures would not have been uncovered if not $\because$ string theory, e.g., unusual $V(\Phi) \sim \Phi^{\sqrt{28}-2}$, DBI inflation, NG, cosmic strings, oscillations in power/bi-spectrum, …
- Is there a swampland beyond the landscape? (Lecture 4)

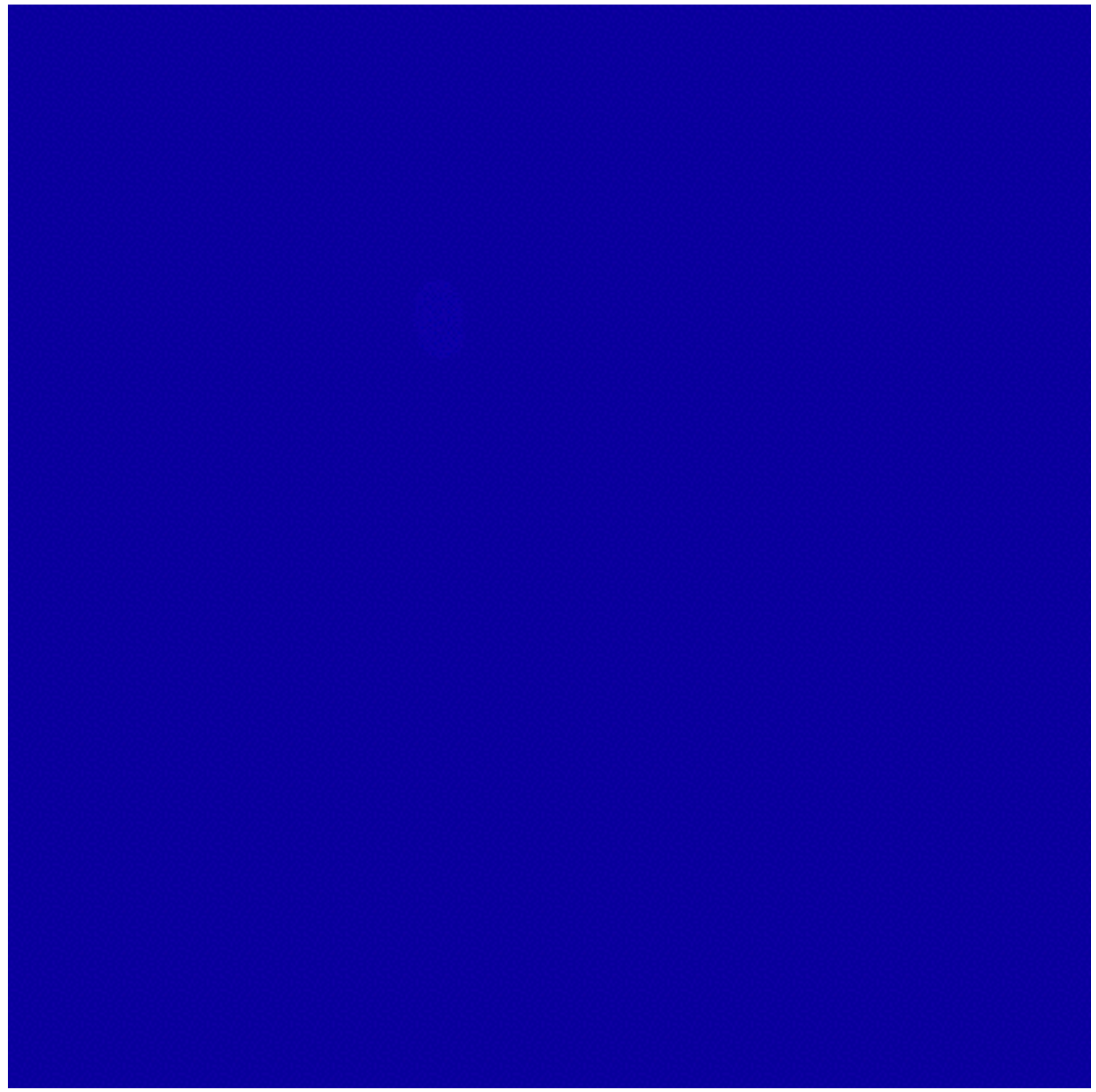
Inflation in String Theory

- Embedding inflation in string theory requires understanding fully moduli stabilization & its effects on 4D EFT: difficult task!
- Two approaches to inflation in string theory:

Compute the inflaton action by brute force, in a tractable example

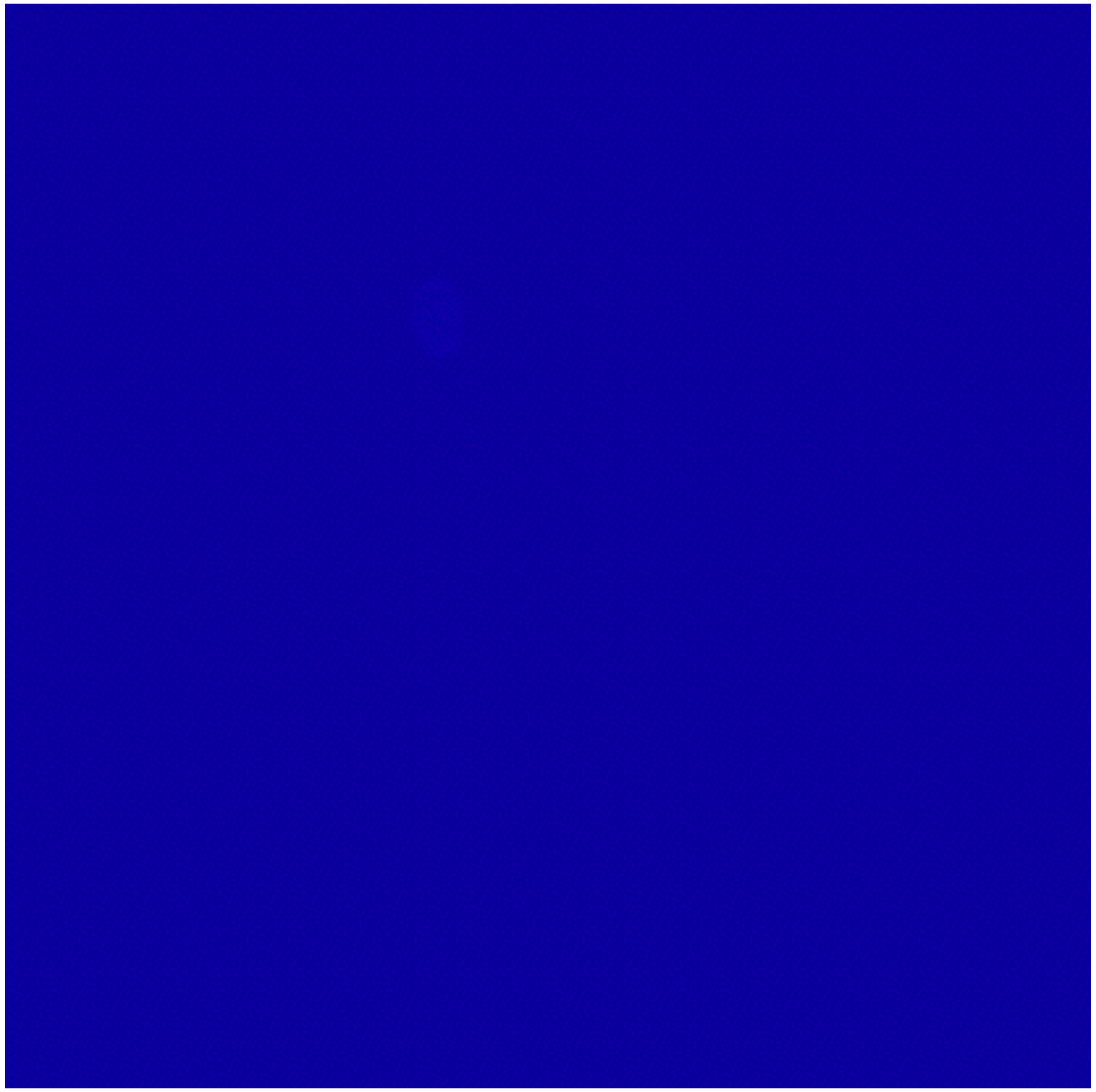
Find a symmetry that simplifies the task

D3-brane Inflation



[Dvali and Tye]; [Burgess, Majumdar, Nolte, Quevedo, Rajesh, Zhang];[Dvali, Shafi, Solganik]; [Shiu, Tye];[Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi] and many others.

D3-brane Inflation



[Dvali and Tye]; [Burgess, Majumdar, Nolte, Quevedo, Rajesh, Zhang];[Dvali, Shafi, Solganik]; [Shiu, Tye];[Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi] and many others.

D-brane Inflation

- A specific (and tractable) example of small-field inflation
- **Inflaton:** can be an open string or closed string scalar
- Type IIB strings on a CY 3-fold with O3/O7-planes (easy lift to F-theory), promising choice: ϕ = D3-brane position [Dvali, Tye]
- 10D SUGRA action contains the bulk terms:

$$S_{\rm IIB} = \frac{1}{2\kappa^2}\int d^{10}X\sqrt{-G_E}\left[R_E - \frac{|\partial\tau|^2}{2({\rm Im}(\tau))^2} - \frac{|G_3|^2}{2\,{\rm Im}(\tau)} - \frac{|\tilde{F}_5|^2}{4}\right] - \frac{i}{8\kappa^2}\int\frac{C_4\wedge G_3\wedge\bar{G}_3}{{\rm Im}(\tau)}\,.$$

where

$$G_3 \equiv F_3 - \tau H_3$$

$$\tau \equiv C_0 + i e^{-\Phi}$$

$$\tilde{F}_5 = F_5 - \tfrac{1}{2}C_2\wedge H_3 + \tfrac{1}{2}B_2\wedge F_3$$

Warped Compactification

- This bulk action is supplemented by $S_{\text{localized}}$ (more later).
- This is leading SUGRA, of course, ∃ g_s, α' corrections.
- Compacitifcation on a **warped CY,** e.g., [Giddings, Kachru, Polchinski]:

$$ds^2 = e^{-6u(x)} e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{2u(x)} e^{-6A(y)} \hat{g}_{mn} dy^m dy^n$$

the *breathing mode* *u(x)* is defined s.t. kinetic terms don't mix, and

$$\hat{g} = \text{a CY metric}, \;\; A = \text{warp factor}$$

- For moduli *dynamics*, need a better ansatz to solve the Einstein constraint equations [Shiu, Torroba, Underwood, Douglas (STUD)]:

$$ds^2 \to ds^2 + 2e^{2A(y)} \left(\partial_\mu \partial_\nu u^I(x) K_I(y) dx^\mu dx^\nu + \partial_\mu u^I(x) B_{mI} dx^\mu dy^m \right)$$

Background Solutions

- For our purpose, the simpler ansatz suffices. We also take:

$$\tilde{F}_5 = (1 + \star_{10})\, \mathrm{d}\alpha(y) \wedge \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3$$

- In general, compact warped solutions are not easily found.
- For IIB flux compactification, Bianchi id. for α + trace of EE:

$$\nabla^2 \left(e^{4A} - \alpha\right) = \frac{e^{8A}}{24\,\mathrm{Im}(\tau)} |iG_3 - \star_6 G_3|^2 + e^{-4A} |\partial(e^{4A} - \alpha)|^2$$
$$+\ 2\kappa^2 e^{2A} \left(\mathcal{J}_{\rm loc} - \mathcal{Q}_{\rm loc}\right) \ ,$$

where $\mathcal{J}_{\rm loc} \equiv \frac{1}{4} \left(\sum_{M=4}^{9} T^M{}_M - \sum_{M=0}^{3} T^M{}_M \right)_{\rm loc}$ stress-energy tensor from localized sources

- One finds solutions if the BPS-like condition is satisfied:

$$\mathcal{J}_{\rm loc} \geq \mathcal{Q}_{\rm loc}$$

saturated by D3, O3, & D7 on calibrated Σ_4

ISD Compactification

- If all sources satisfy the inequality, the 3-form flux is ISD:

$$\star_6 G_3 = i G_3 \ , \qquad e^{4A} = \alpha$$

and all localized sources saturate $\mathcal{J}_{\rm loc} \geq \mathcal{Q}_{\rm loc}$

- We refer to these solutions as **ISD compactifications.**

- u(x) is unconstrained, and the moduli space $\mathcal{M}$ of D3 =CY.

- The action for the localized sources: $S_{\rm localized} = S_{\rm DBI} + S_{\rm CS}$

$$S_{\rm DBI} = -g_{\rm s} T_p \int {\rm d}^{p+1}\sigma \, e^{-\Phi} \sqrt{-\det(G_{ab} + \mathcal{F}_{ab})} \ ,$$

$$S_{\rm CS} = i\, \mu_p \int_{\Sigma_{p+1}} \sum_n C_n \wedge e^{\mathcal{F}}$$

ISD Compactification

- The induced metric on the D3-brane:

$$G_{ab} = \frac{\partial x^M}{\partial \xi^a}\frac{\partial x^N}{\partial \xi_b} g_{MN} \quad \text{where } M, N = 0, \ldots, 9, \text{and } a, b = 0, \ldots, 3$$

- For homogeneous config., the localized source terms give:

$$\mathcal{L}_{DBI+CS} = -T_3 \; e^{4A-12u}\sqrt{1 - e^{8u-4A}\hat{g}_{mn}\dot{y}_m\dot{y}_n} + T_3 \; \alpha e^{-12u}$$

- For low velocities: $\mathcal{L}_{DBI+CS} = -T_3\left(e^{4A} - \alpha\right)e^{-12u} + \frac{1}{2}T_3 \; \hat{g}_{mn}\dot{y}^m\dot{y}^n e^{-4u}$

- Define: $\Phi_\pm \equiv e^{4A} \pm \alpha$ and $\phi^m = \sqrt{T_3}\; y^m$ (mass dim. 1)

- For Φ_- =0 (ISD soln), we have $V(\phi) \equiv 0$, D3 moves freely in CY.

4D SUGRA

- Package these results in terms of 4D N=1 SUGRA data:

$$W_0 = \frac{c}{\alpha'} \int G_3 \wedge \Omega$$

[Gukov-Vafa-Witten]

$$K_0 = -2\ln(\mathcal{V}) - \ln\left(-i(\tau - \bar{\tau})\right) - \ln\left(-i\int \Omega \wedge \bar{\Omega}\right)$$

4D SUGRA

- Package these results in terms of 4D N=1 SUGRA data:

$$W_0 = \frac{c}{\alpha'} \int G_3 \wedge \Omega$$

[Gukov-Vafa-Witten]

$$K_0 = -2\ln(\mathcal{V}) - \ln\left(-i(\tau - \bar{\tau})\right) - \ln\left(-i\int \Omega \wedge \bar{\Omega}\right)$$

$$\uparrow$$

$$\mathcal{V} = \frac{1}{6}\int_{X_6} J \wedge J \wedge J = \frac{1}{6} c_{ijk} t^i t^j t^k$$

4D SUGRA

- Package these results in terms of 4D N=1 SUGRA data:

$$W_0 = \frac{c}{\alpha'} \int G_3 \wedge \Omega$$

[Gukov-Vafa-Witten]

$$K_0 = -2\ln(\mathcal{V}) - \ln\left(-i(\tau - \bar{\tau})\right) - \ln\left(-i \int \Omega \wedge \bar{\Omega}\right)$$

$$\uparrow$$

$$\mathcal{V} = \frac{1}{6}\int_{X_6} J \wedge J \wedge J = \frac{1}{6} c_{ijk} t^i t^j t^k$$

- For $h^{1,1}=1$, $\mathcal{V} = \left(T + \overline{T}\right)^{3/2} \Rightarrow -2\ln\mathcal{V} \rightarrow -3\ln(T + \overline{T})$

4D SUGRA

- Package these results in terms of 4D N=1 SUGRA data:

$$W_0 = \frac{c}{\alpha'} \int G_3 \wedge \Omega$$

[Gukov-Vafa-Witten]

$$K_0 = -2\ln(\mathcal{V}) - \ln\left(-i(\tau - \bar{\tau})\right) - \ln\left(-i\int \Omega \wedge \bar{\Omega}\right)$$

$$\uparrow$$

$$\mathcal{V} = \frac{1}{6}\int_{X_6} J \wedge J \wedge J = \frac{1}{6} c_{ijk} t^i t^j t^k$$

- For $h^{1,1}=1$, $\mathcal{V} = \left(T + \overline{T}\right)^{3/2} \Rightarrow -2\ln\mathcal{V} \to -3\ln(T + \overline{T})$

- The scalar potential enjoys a **no-scale property**:

$$V_F = e^{K_0}\left[K_0^{I\bar{J}} D_I W_0 \overline{D_J W_0} - 3|W_0|^2\right] \quad \text{with} \quad W_0 = W_0\left(\zeta_\alpha, \tau\right)$$

$$\sum_{I,J=T_i} K_0^{I\bar{J}} \partial_I K_0 \partial_{\bar{J}} K_0 = 3 \qquad \Rightarrow \qquad V_F = e^{K_0} \sum_{I,J\neq T_i} K_0^{I\bar{J}} D_I W_0 \overline{D_J W_0}$$

No-scale Structure

- D3-branes enter the Kahler potential as:

$$K(T,\bar{T},z_\alpha,\bar{z}_\alpha) = -3\ln\left[T+\bar{T}-\gamma k(z_\alpha,\bar{z}_\alpha)\right] \qquad \nabla_\alpha\overline{\nabla}_\beta k = g_{\alpha\overline{\beta}}$$

- The scalar potential remains no-scale **(homework)**:

$$\sum_{I,J=T_i,z_\alpha} K_0^{I\bar{J}}\partial_I K_0 \partial_{\bar{J}} K_0 = 3$$

- What about I=axio-dilaton τ & complex-structure moduli ζ_α?

$$D_I W_0 \equiv \partial_I W_0 + (\partial_I K) W_0 = 0 \ , \ \Rightarrow \qquad G_3|_{(3,0)} = 0, \quad G_3|_{(1,2)} = 0.$$

By eq. counting, τ and ζ_α (α=1,.. $h^{1,2}$) are all generically stabilized.

- At tree-level SUGRA, V_{min}=0 though W≠0 (generically), SUSY is generically broken for all T< ∞.

Flat Direction & Runaway

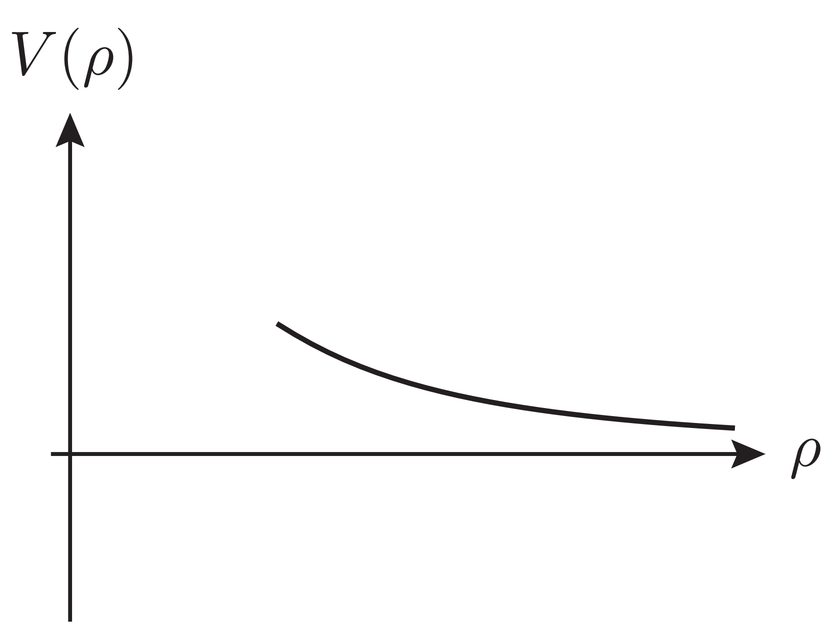
While ∃ (generically) vacua where τ and ζ_α receives a potential, T does not …. but for a good reason:

$$T = \int_{\Sigma_4} \sqrt{g} + i \int_{\Sigma_4} C_4$$

$C_4 \to C_4$ + const. is a sym.

This is a flat direction in **no-scale compactification**

Adding any SUSY breaking sources (IASD flux, or anti D3-brane),
→ decompactification instability, e.g.,



$$V_{\overline{D3}} = T_3 e^{-12u} \Phi_+ \neq 0$$
$$V_{D3} = T_3 e^{-12u} \Phi_- \neq 0 \text{ if non-ISD flux}$$

V=0 only at u=∞ (runaway)!

Stabilizing the Volume

$W \supset T^{\alpha}$ is forbidden because of the shift symmetry but:

1. $W_{NP} = e^{-T}$ is allowed
2. K_P can receive corrections, no longer no-scale $\Rightarrow V(T)$, e.g.,
   - leading $\alpha'^3 \zeta(3) R^4$ term gives upon dim reduction:

$$K = -2\ln\left(\mathcal{V} + \zeta(3)\chi\left(\mathcal{M}\right)\right)$$

[Becker, Becker, Hack, Louis]

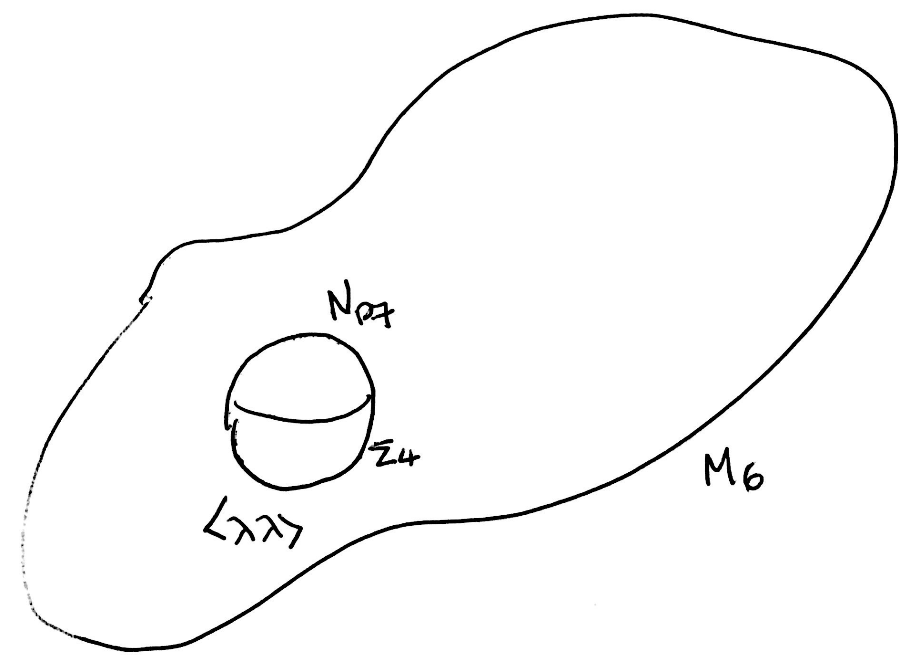
   - other α' corrections to K [Garcia-Etxebarria, Hayashi, Savelli, GS]; [Junghans, GS];[Minasian, Pugh, Savelli]; …

Depending on W_0:

- $W_0 \ll 1$: W_{NP} dominates [Kachru, Kallosh, Linde, Trivedi] (KKLT)
- $W_0 \simeq 1$: K_P dominates [Balasubramanian, Berglund, Conlon, Quevedo] (LVS)

Non-Perturbative W

Consider N D7 on Σ_4 (holomorphic)



Deformations on Σ_4 ⇔

adj. matter in 4D N=1 SYM on D7

If Σ_4 is rigid, D7-theory is pure glue N=1 SYM;
at low energies, confines and generates a $\langle\lambda\lambda\rangle$ W:

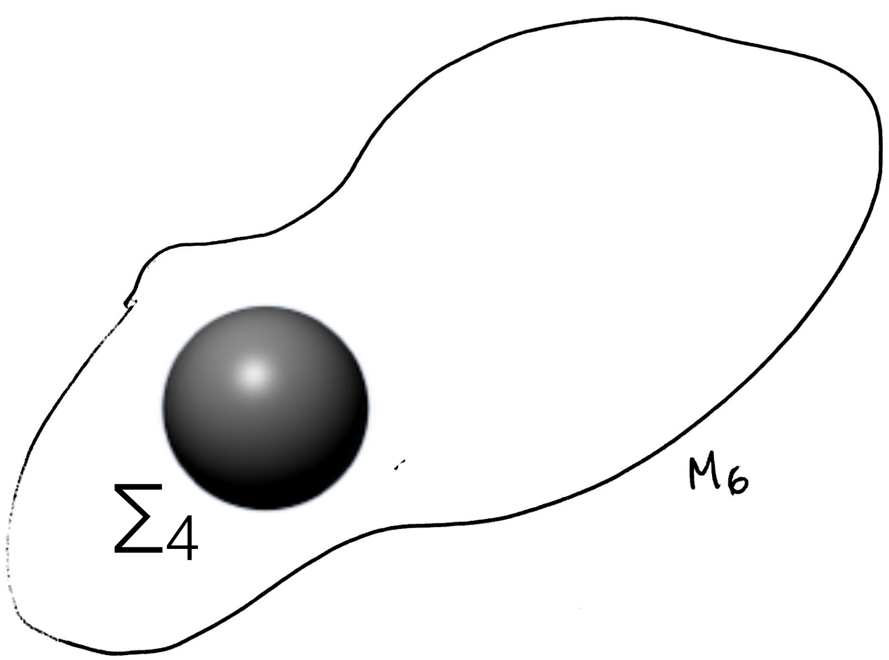
$$W = M_{UV}^3 \exp\left(-\frac{8\pi}{g_{YM}^2}\frac{1}{C_2(G)}\right) \text{ where } C_2(G) = N \text{ for SU(N)}$$

HW: $$\frac{8\pi}{g_{YM}^2} = T_3 \int_{\Sigma_4} d^4\xi \sqrt{g_4^{\text{induced}}} e^{-4A} \equiv T_3 V_4^W \quad \Rightarrow \quad W = e^{-T_3 V_4^W \frac{1}{N_{D7}}}$$

Non-Perturbative W

Similar contribution from ED3

$$W = e^{-T_3 V_4^W}$$



Identifying $\quad T_3 V_4^w = 2\pi T$

(long story to show this is a good Kahler coordinate, see e.g., [Baumann et al];[Chen, Nakayama, GS])

Either case, we can write: $\quad W = W_0 + e^{-aT}$

$\partial_T W \neq 0 \quad$ no-scale is spoiled

Generically solution: $\quad D_T W = 0 \text{ at } T = T_* \quad$ (SUSY minima)

Single vs Multi-Field

Natural question: can ϕ_{D3} be the inflaton?

After all, V(ϕ)=0 at **leading order.**

Can this picture be realized upon stabilization of T?

Mass of ϕ is much lower than that of τ & ζ_α but can be comparable to that of T

- Can tune $m_T \gg m_\phi$, or
- study 2-field inflation