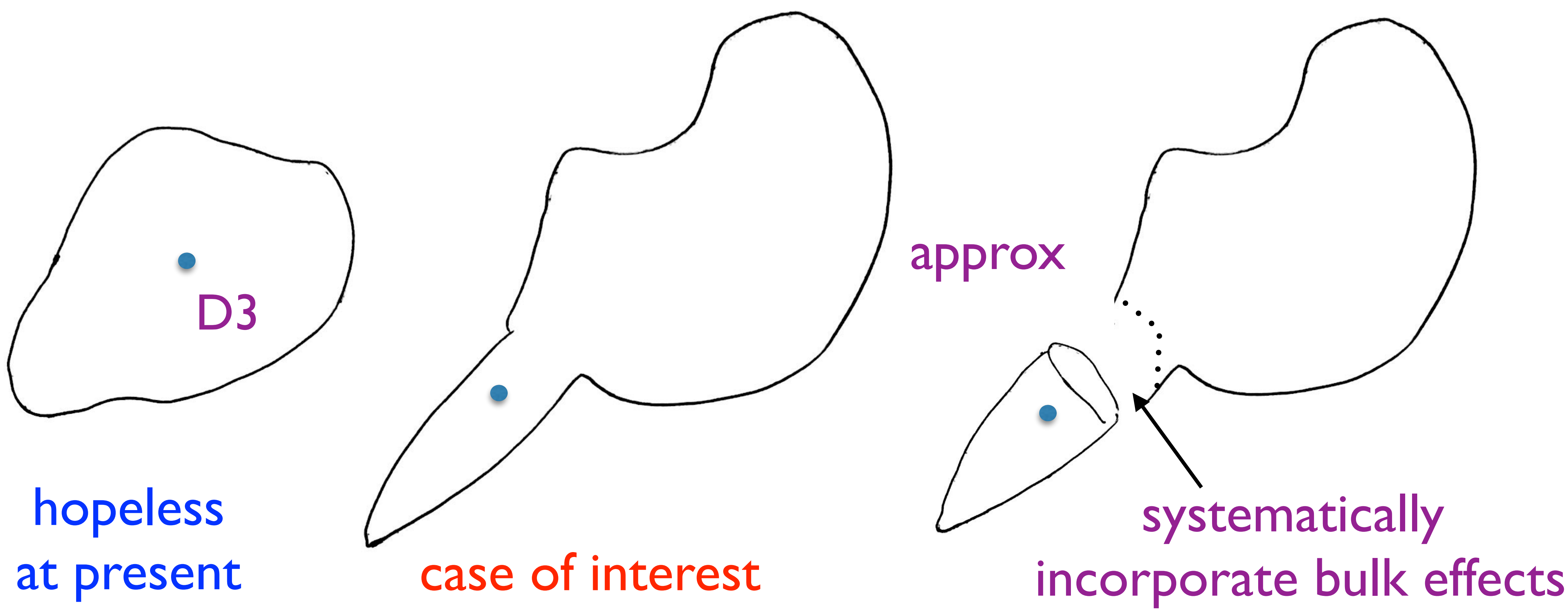
D3-brane Potential

- The Kahler potential contains:

$$K(T,\bar{T},z_\alpha,\bar{z}_\alpha) = -3\ln\left[T+\bar{T}-\gamma k(z_\alpha,\bar{z}_\alpha)\right] \qquad \nabla_\alpha\overline{\nabla}_\beta k = g_{\alpha\overline{\beta}}$$

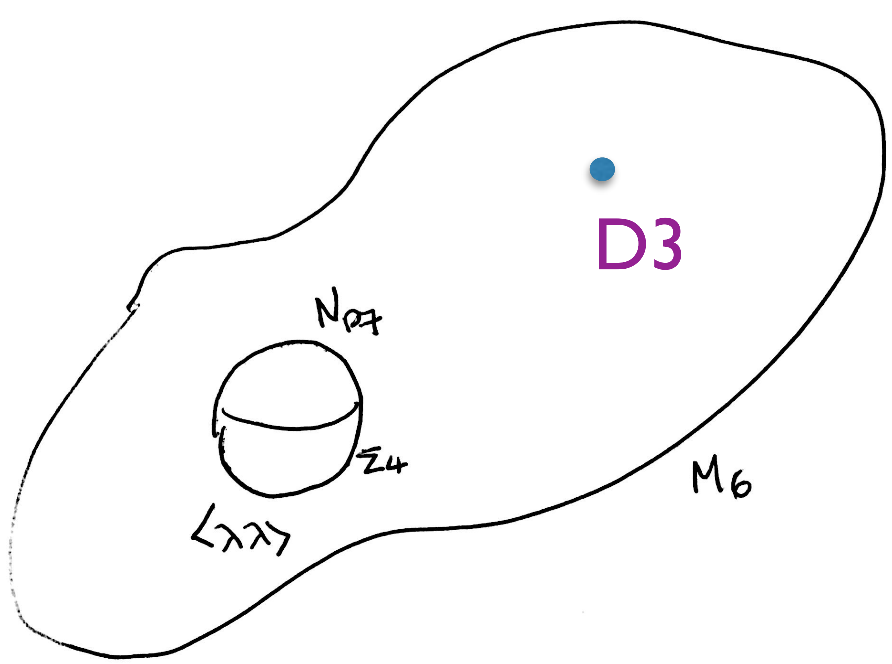
difficult to get metric for compact CY →need a simpler setting

- Idea:



D3-brane Potential

- To leading order, D3 feels no force (∵ no scale).
- What is the effect of W_{NP} on D3?



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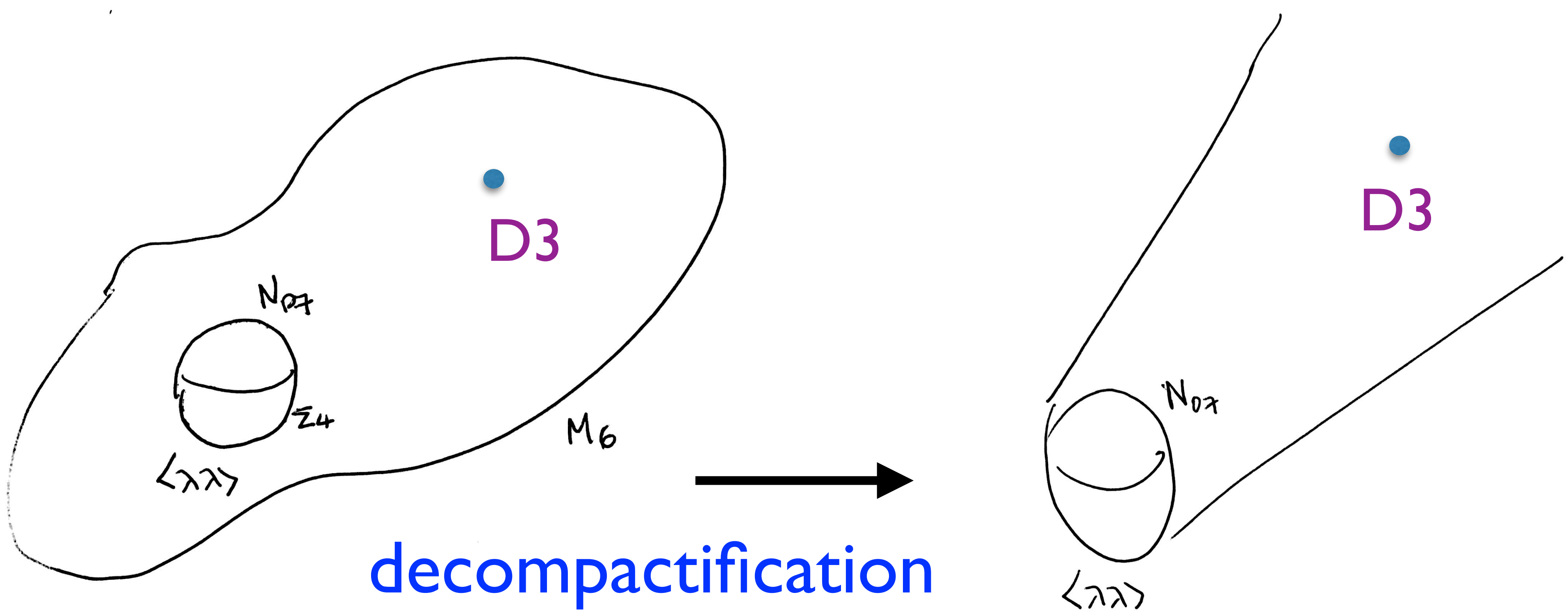
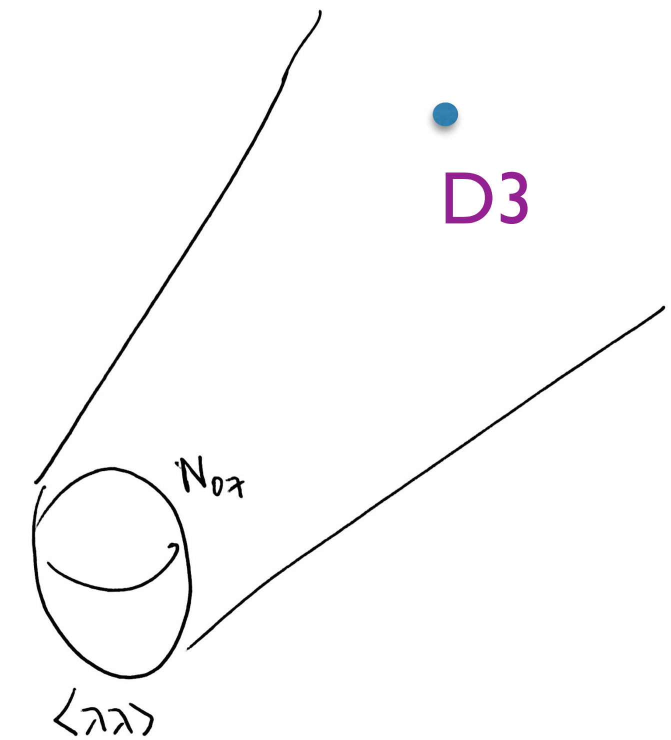


D3-brane Potential

- N=1 SU(N_c) SYM with 4D G_N =0
- Not pure glue but N_f = 1
- Mass of 37 matter = ϕ - ϕ_7 $\propto$ l_{37}
- Using "Seiberiology" for SQCD w/ N_f=1



$$W(m) = \left(N_c \Lambda^{3N_c - N_f} \det m\right)^{\frac{1}{N_c}} \to m^{\frac{1}{N_c}} \times (\text{independent of } m)$$

see e.g., Intriligator and Seiberg, hep-th/9509066

$$m \propto \phi \Rightarrow W \propto \phi^{\frac{1}{N_c}} \Rightarrow V(\phi) \neq 0$$

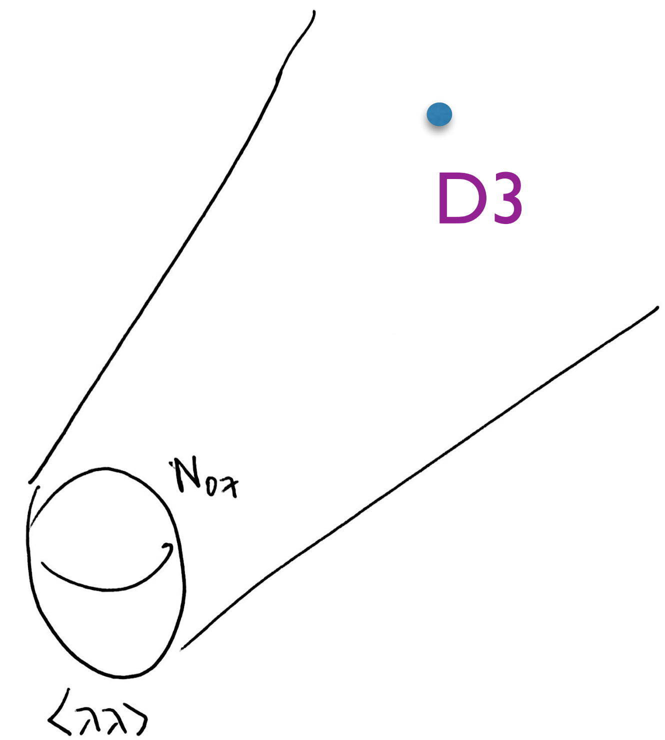
D3-brane Potential

- Non-perturbative superpotential:

$$W_{NP} = A_0 e^{-aT} \to A_0 f(z)^{\frac{1}{N_c}} e^{-aT}$$

$\uparrow$

f(z)=0 ↔ locus of Σ_4



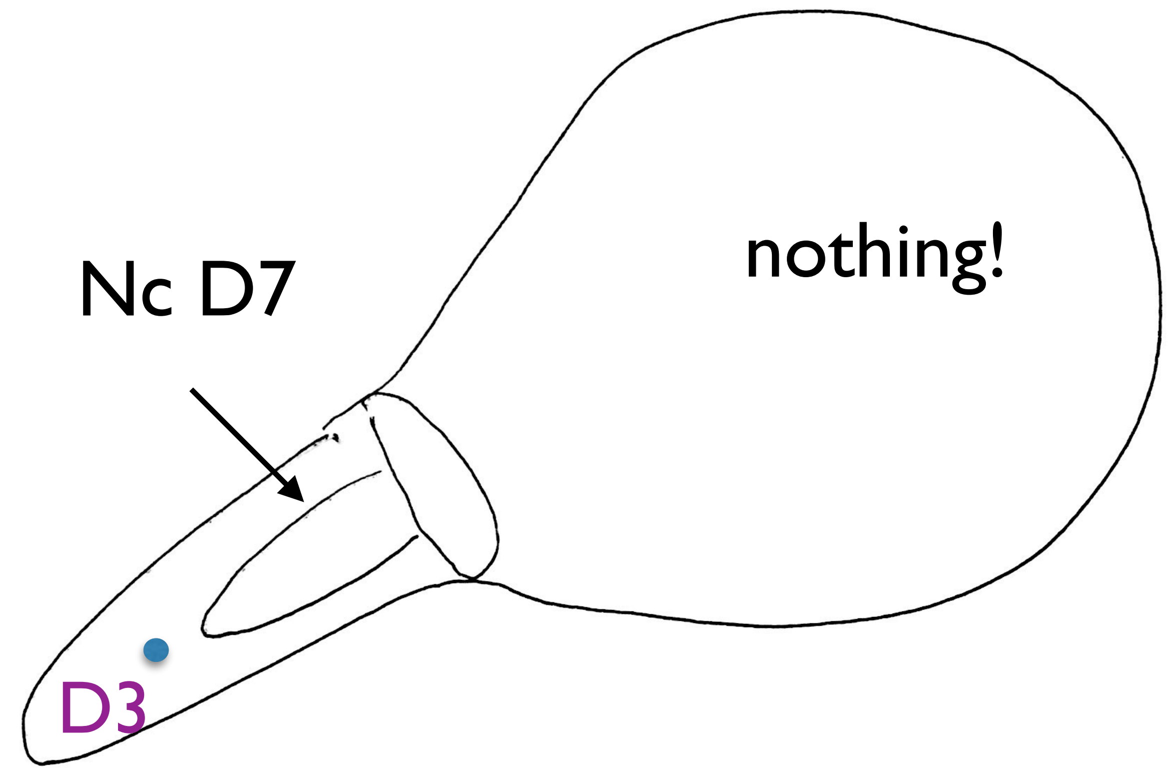
- Many ways to derive this:

  1. open string 1-loop corr. to gauge kin, function (BHK '04)
  2. general topological considerations (Ganor '96)
  3. closed string (i.e., SUGRA) (BDKMMM '06)

- **Implications:** W_{NP} gives non-negligible contribution to $V(\phi)$

Compactification Effects

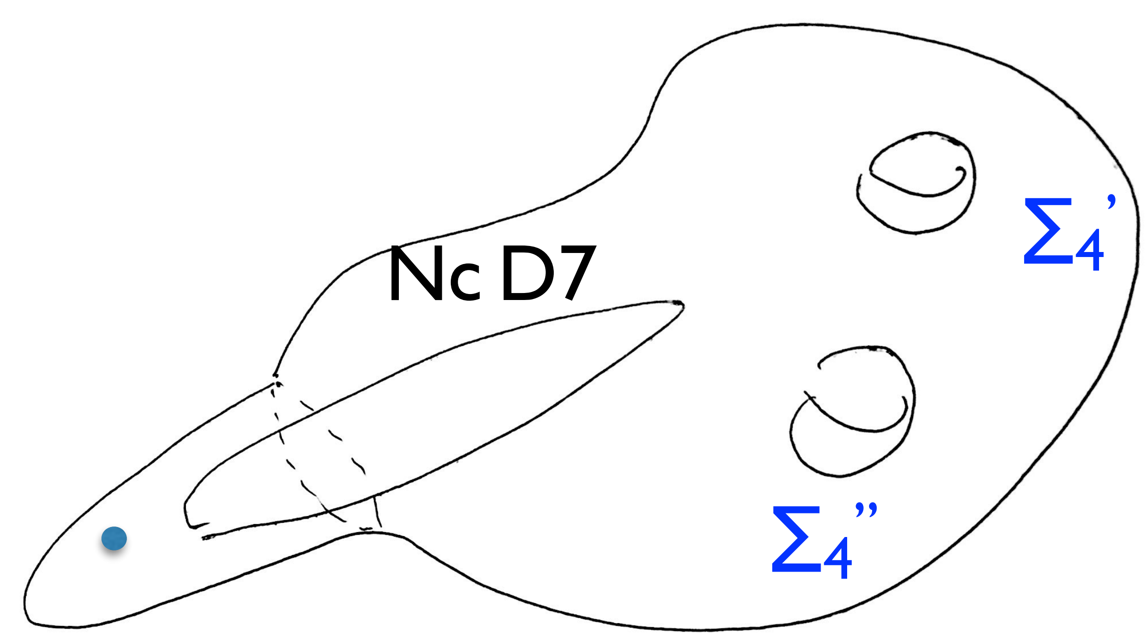
- For a toy model:



we can compute $V\phi)$ in full!

- Distant Σ_4 can give non-negligible contribution to $V(\phi)$

we'll *parametrize* these effects
for Calabi-Yau cones



Calabi-Yau Cones

- Let Y_5 be a Sasaki-Einstein manifold

if $Y_5 \times \mathbb{R}^+$ is Kahler $\quad R_{ab} \propto g_{ab}$

- The cone $ds_{10}^2 = dr^2 + r^2 ds_{Y_5}^2$ has a CY metric
- Taking N D3 at the tip of the cone, and in near-horizon limit,

$$ds_{10}^2 = \left(\frac{r}{R}\right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \left(\frac{R}{r}\right)^2 \left(dr^2 + r^2 ds_{Y_5}^2\right)$$

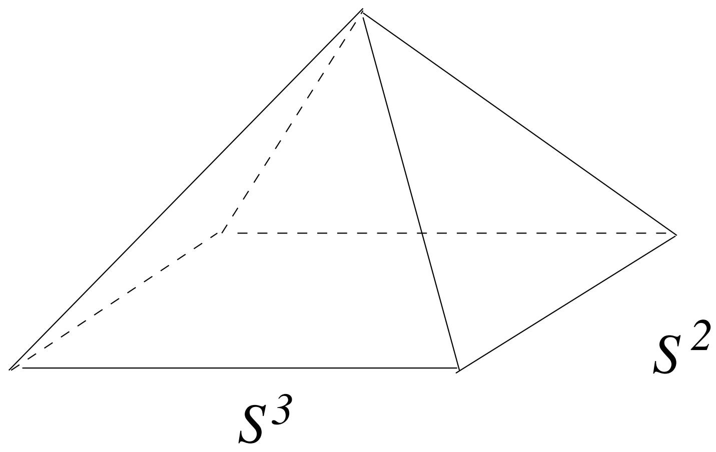
$$\Leftrightarrow AdS_5 \times Y_5 \text{ with } R^4 = \frac{4\pi^4 g_s N \alpha'^2}{\mathrm{Vol}(Y_5)}$$

- This is dual to an N=1 SCFT (useful later on).

Conifold

- Simplest example of CY cones is the conifold:

$$\sum_{A=1}^{4} z_A^2 = 0$$



- Topologically, the conifold is a cone over $S^2 \times S^3$:

$$z^A = x^A + iy^A \qquad x \cdot x = \frac{1}{2}\rho^2 , \qquad y \cdot y = \frac{1}{2}\rho^2 , \qquad x \cdot y = 0$$

- Metrically, the base Y_5 is the Einstein manifold $T^{1,1}$:

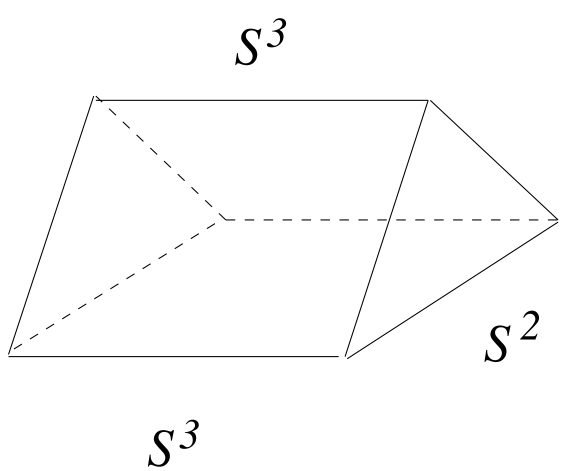
$$T^{1,1} = [SU(2) \times SU(2)]/U(1)$$

parametrized by coordinates θ_1, θ_2, Φ_1, Φ_2, Ψ (detailed form of the metric can be found e.g., in [Candelas, de la Ossa]). Isometry group SU(2) x SU(2) x U(1).

Warped Conifold

- N D3 at the tip of the conifold → warped conifold; FT dual is an N=1 SCFT worked out by [Klebanov, Witten] (more later).
- If we have N D3 at the tip and M D5 wrapping the shrinking S^2, the conifold is warped and deformed:

$$\sum_{A=1}^{4} z_A^2 = \varepsilon^2$$



- S^2 shrinks to zero size while the S^3 size remains finite:

$r_A = \sqrt{g_s M \alpha'}$ SUGRA valid if $g_s M \gg 1$

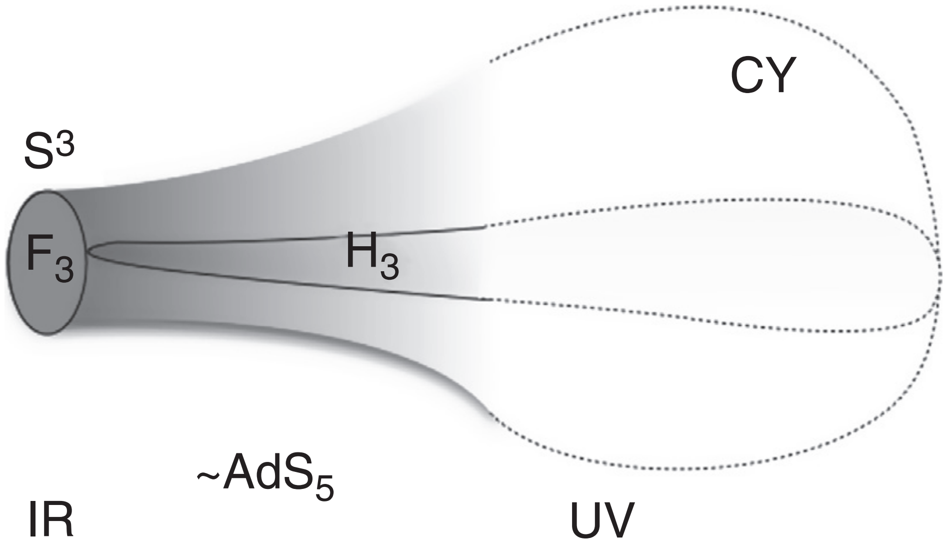
- SUGRA solution[Klebanov, Strassler] is everywhere smooth.

Warped Deformed Conifold

Alternative description in terms of fluxes:

$$\frac{1}{(2\pi)^2\alpha'}\int_A F_3 = M \qquad \text{and} \qquad \frac{1}{(2\pi)^2\alpha'}\int_B H_3 = K \qquad N \equiv MK$$

Near tip, $S^3 \times R^3$



Far from tip,

approximated by $AdS_5 \times T^{1,1}$
(up to "log corrections")

See [GS, Underwood];[GS, Underwood, Kecskemeti, Maiden] for CMB effects.

CFT Dual

- KW CFT has U(N)xU(N) gauge group & matter fields:

	$U(N)$	$U(N)$	$SU(2)$	$SU(2)$	$U(1)_R$
A_1, A_2	$\mathbf{N}$	$\overline{\mathbf{N}}$	$\mathbf{2}$	$\mathbf{1}$	$\frac{1}{2}$
B_1, B_2	$\overline{\mathbf{N}}$	$\mathbf{N}$	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$

w/ a superpotential (see [Klebanov,Witten] for details)

$$W_{\text{tree}} = \lambda \epsilon^{ik} \epsilon^{j\ell} A_i B_j A_k B_\ell$$

- Isometry becomes SU(2)xSU(2) global symmetries and $U(1)_R$
- Moduli space of vacua = {D=0}/U(1) is the coset space $T^{1,1}$:

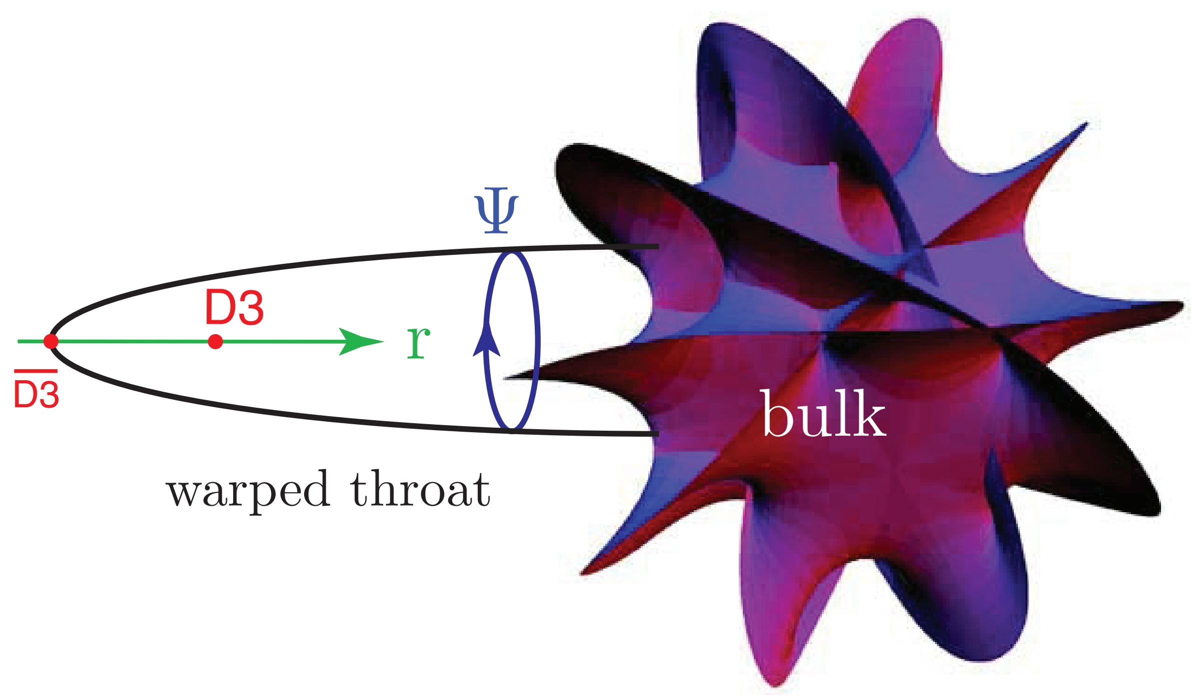
$$D = |A_1|^2 + |A_2|^2 - |B_1|^2 - |B_2|^2 - \xi_{FI} = 0$$

$$\xi_{FI} \neq 0$$

(resolved conifold)

Our Strategy

- Non-compactification gives $G_N = 0$ (not useful)
- General SU(3) holonomy manifold is intractable (need metric)
- **Middle ground:** inclusion of compactification effects in a ***finite throat:***

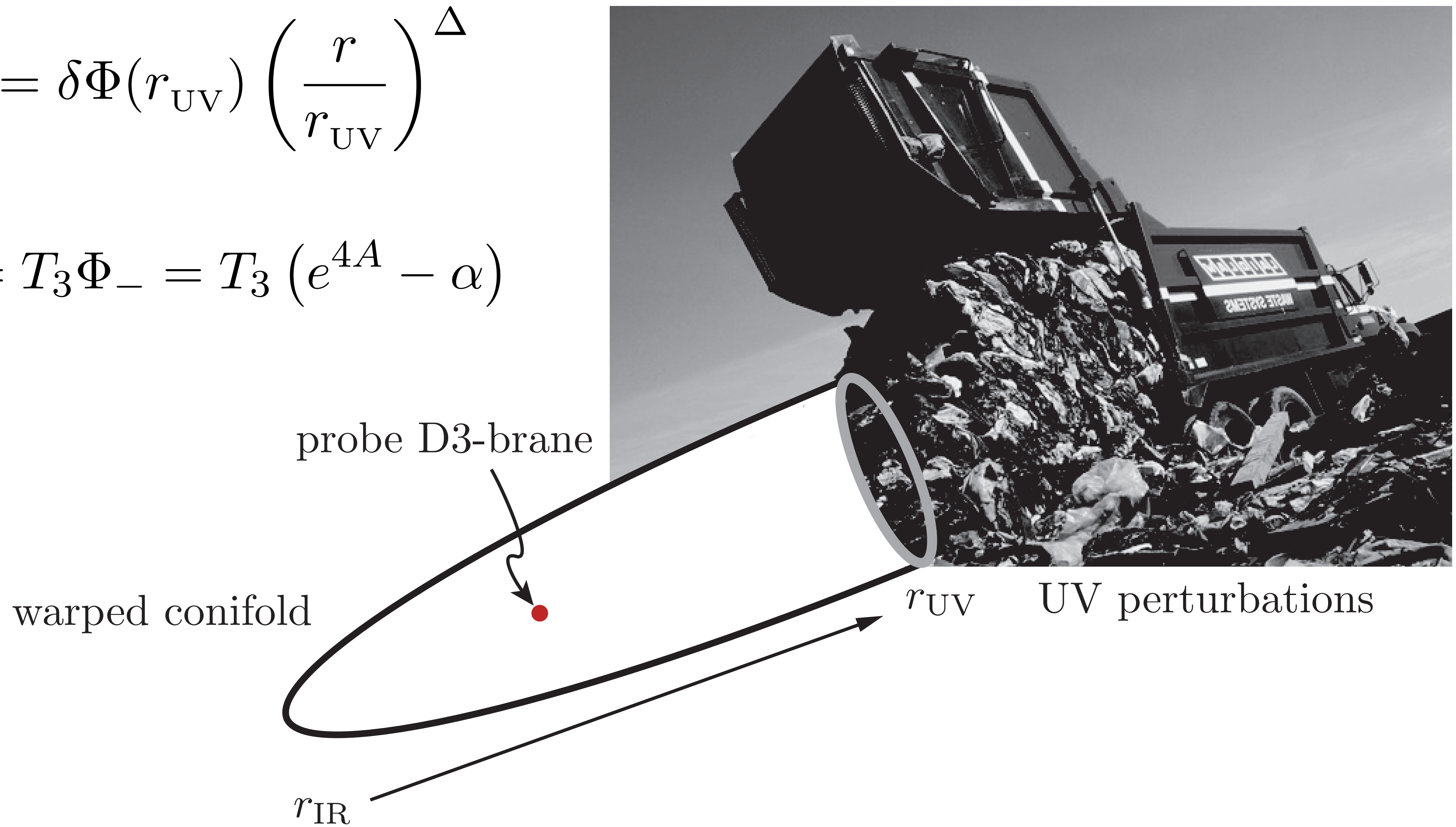


- Planck-scale effects come from the bulk of compactification ("Planck brane").

UV Perturbations

$$\delta\Phi(r) = \delta\Phi(r_{\rm UV})\left(\frac{r}{r_{\rm UV}}\right)^{\Delta}$$

$$V_{D3} = T_3\Phi_- = T_3\left(e^{4A} - \alpha\right)$$



On the CFT side: $\mathcal{L}_{CFT} \to \mathcal{L}_{CFT} + \sum_i \mathcal{O}_i^{\Delta} c_i$

Two Dual Views

AdS x $T^{1,1}$

$$\delta\Phi(r) = \delta\Phi(r_{\mathrm{UV}}) \left(\frac{r}{r_{\mathrm{UV}}}\right)^{\Delta}$$

D3 postion ϕ_{D3}

KW CFT

$$\mathcal{L}_{CFT} \to \mathcal{L}_{CFT} + \sum_i \mathcal{O}_i^{\Delta} c_i$$

vev describing location on Coulomb branch

Now in EFT, we could write: $V_{renorm} + \sum_{i,\Delta_i > 4} c_i \mathcal{O}^{\Delta_i} \frac{1}{\Lambda^{\Delta_i - 4}}$

Spectroscopy of $T^{1,1}$ gives the "structure" of the potential; not the Wilson coefficients c_i

Two Dual Views

Here, the structure of operator dimensions $\{\Delta_i\}$ is not easy to get from the QFT, as the KW theory is strongly coupled.

We can obtain the spectroscopy of $\{\Delta_i\}$ in SUGRA and learn:

$\{\Delta_i\} \Leftrightarrow$ what sort of physics (fluxes, branes) in the bulk

This is a first step; next could try to understand the typical values of c_i and eventually statistics of c_i in the landscape.

Conifold Spectroscopy

D3-potential is given by Φ_-, whose solutions are given by:

$$\nabla^2\Phi_- = R_4 + \frac{g_s}{96}|\Lambda|^2 + e^{-4A}|\nabla\Phi_-|^2 + \mathcal{S}_{\rm loc}$$

$$\uparrow$$

$$\Lambda \equiv \Phi_+ G_- + \Phi_- G_+$$

The 3-form flux in turn satisfies: $\mathrm{d}\Lambda + \frac{i}{2}\frac{\mathrm{d}\tau}{\mathrm{Im}\tau} \wedge (\Lambda + \bar{\Lambda}) = 0$

Consider first: $\nabla^2\Phi_- = R_4$

In quasi-dS: $R_4 \approx 12H^2 \Rightarrow V = T_3\Phi_- = V_0 + H^2\phi^2 + \ldots$

$$\eta = M_P^2\frac{V''}{V} = \frac{2}{3} + \ldots$$

can this contribution be canceled by other sources?

Conifold Spectroscopy

Localized sources:

$$\nabla^2 \Phi_- = R_4 + \frac{g_s}{96}|\Lambda|^2 + e^{-4A}|\nabla\Phi_-|^2 + \mathcal{S}_{\rm loc}$$

$$\Downarrow$$

$$V_{\mathcal{C}}(x) = D_0 \left(1 - \frac{27}{64\pi^2}\frac{D_0}{T_3^2 r_{\rm UV}^4}\frac{1}{x^4}\right)$$

Brane Inflation [Dvali-Tye]

Warping flattens the potential

[Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi] (KKLMMT)

Conifold Spectroscopy

Bulk contributions: $$V_{\mathcal{B}}(x,\Psi) = \mu^4 \sum_{LM} c_{LM}\, x^{\Delta(L)}\, f_{LM}(\Psi)$$

After some work, one obtains in this example [Baumann et al]

$$\begin{aligned} V = V_0 \quad &+ \quad c_{ij}(\Psi)\phi^1 + a_{3/2}h_{3/2}(\Psi)\phi^{3/2} \\ &+ \quad \left[b_2 + c_2 + a_2 h_2(\Psi)\right]\phi^2 \\ &+ \quad c_{\sqrt{28}-3}\, j_{\sqrt{28}-3}(\Psi)\phi^{\sqrt{28}-3} \\ &+ \quad \ldots \end{aligned}$$

Tracing the origin: **a-terms:** fluxes; **b-terms:** effect of R_4 ; **c-terms:** harmonic parts of Φ_-

Dual CFT Operators

On the gravity side, Δ is related to the eigenvalue of Laplacian:

$$\Delta(L) \equiv -2 + \sqrt{6[J_1(J_1+1) + J_2(J_2+2) - R^2/8] + 4}$$

Δ should correspond to conformal dim. of operators in CFT.

Strongly coupled CFT, but Δ of some operators are protected.

Chiral operators $\mathcal{O}_{3/2} \; = \; \mathrm{Tr}\,(A_i B_j) + c.c.$

Δ determined by R-charge
J = J = R/2 = 1/4

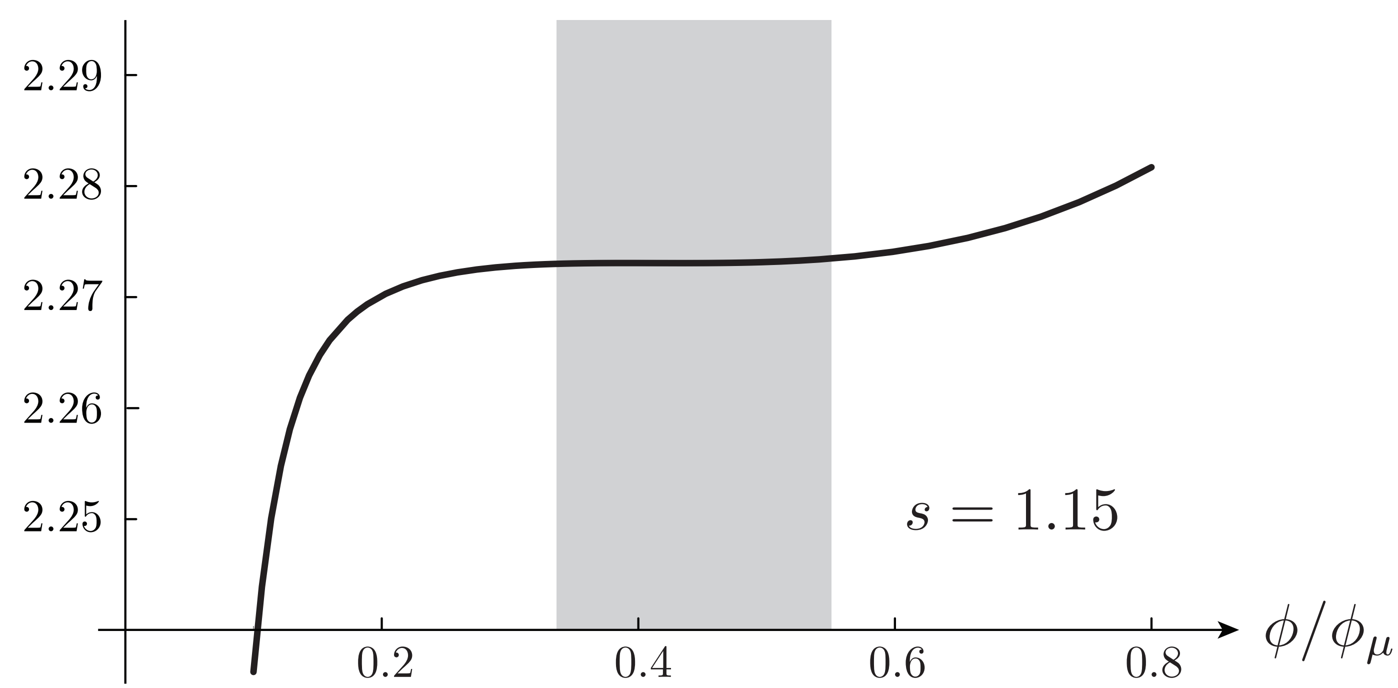
Non-chiral operators $\mathcal{O}_2 \; = \; \mathrm{Tr}\left(A_1 \bar{A}_2\right) \;,\; \mathrm{Tr}\left(A_2 \bar{A}_1\right) \;,\; \frac{1}{\sqrt{2}}\mathrm{Tr}\left(A_1 \bar{A}_1 - A_2 \bar{A}_2\right)$

same multiplet as SU(2)xSU(2) global sym, current, Δ=2

Some Features of D3-brane Inflation

- Small field inflation (undetectable r) → next lecture
- Kinetic term of DBI form: for "relativistic" motion, inflaton reaches "speed limit" (strong and distinctive NG).
- V(ϕ) receives crucial contribution from moduli stabilization, e.g., W_{NP} & bulk effects.
- Wilson coefficients depend on UV completion (string theory).
- Phenomenology:

inflection point inflation



にふぇーでーびる=Thank you!