Inflation in String Theory

Gary Shiu

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- **String Cosmology:** a broad & fast-growing subject, impossible to give a comprehensive review in 4 lectures!
- Focus on Inflation in String Theory, leaving many interesting topics (de Sitter vacua, big bang singularity, alternatives to inflation, …) for another occasion.
- Some key issues require inputs from a quantum theory of gravity, and can be addressed with our present knowledge of string theory in a concrete way.
- Many excellent reviews on this subject (see INSPIRE). I have also some hand-written lecture notes for previous schools (e.g., Asian Winter School, 08 & 11; Florence String School, 09; Summer Institute, Mount Fuji, 10; IFT-Madrid, 14). Email me if you are interested.
- Baumann & McAllister, ``Inflation and String Theory'', Cambridge U. Press.

Why String Cosmology?

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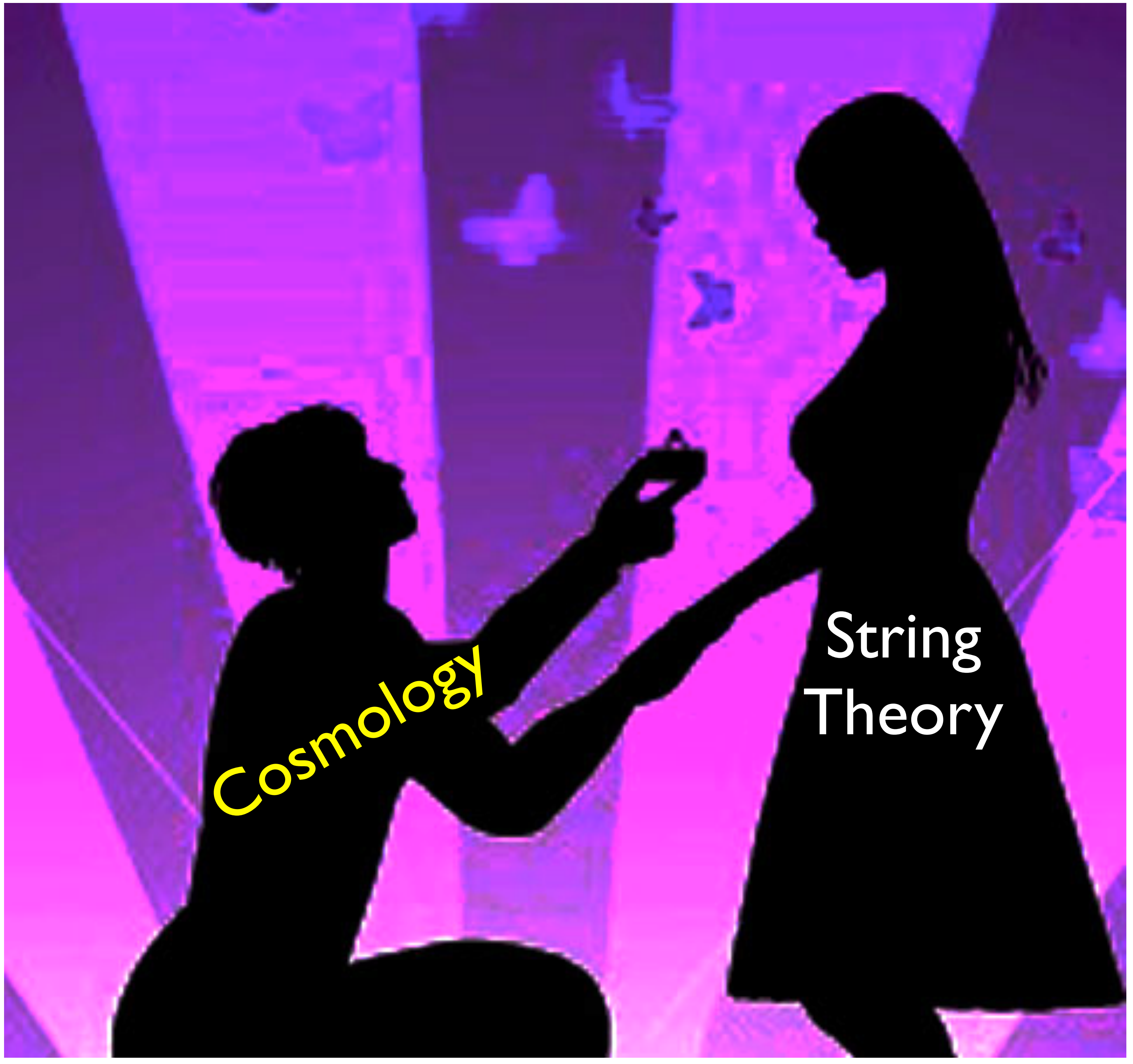

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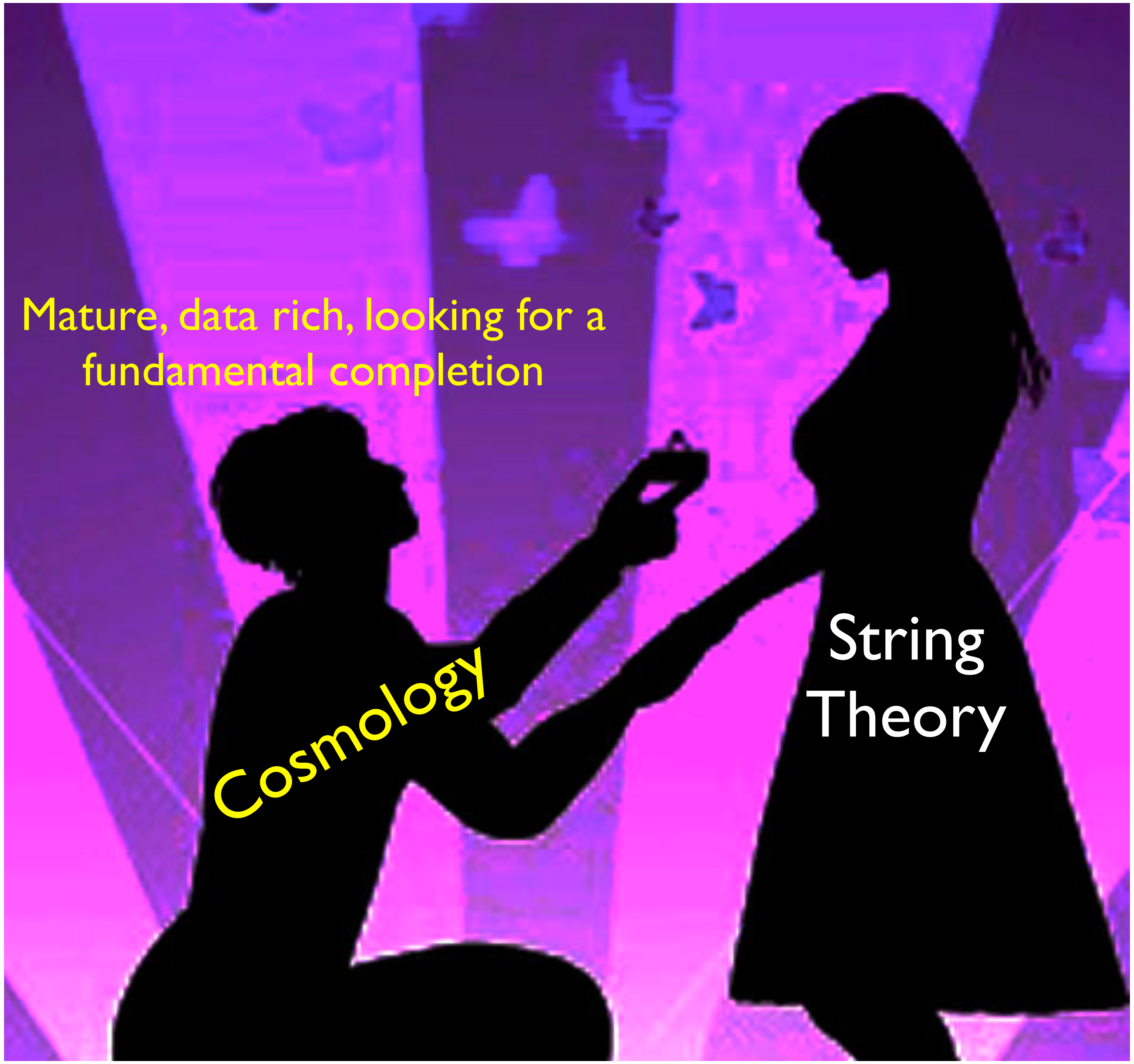

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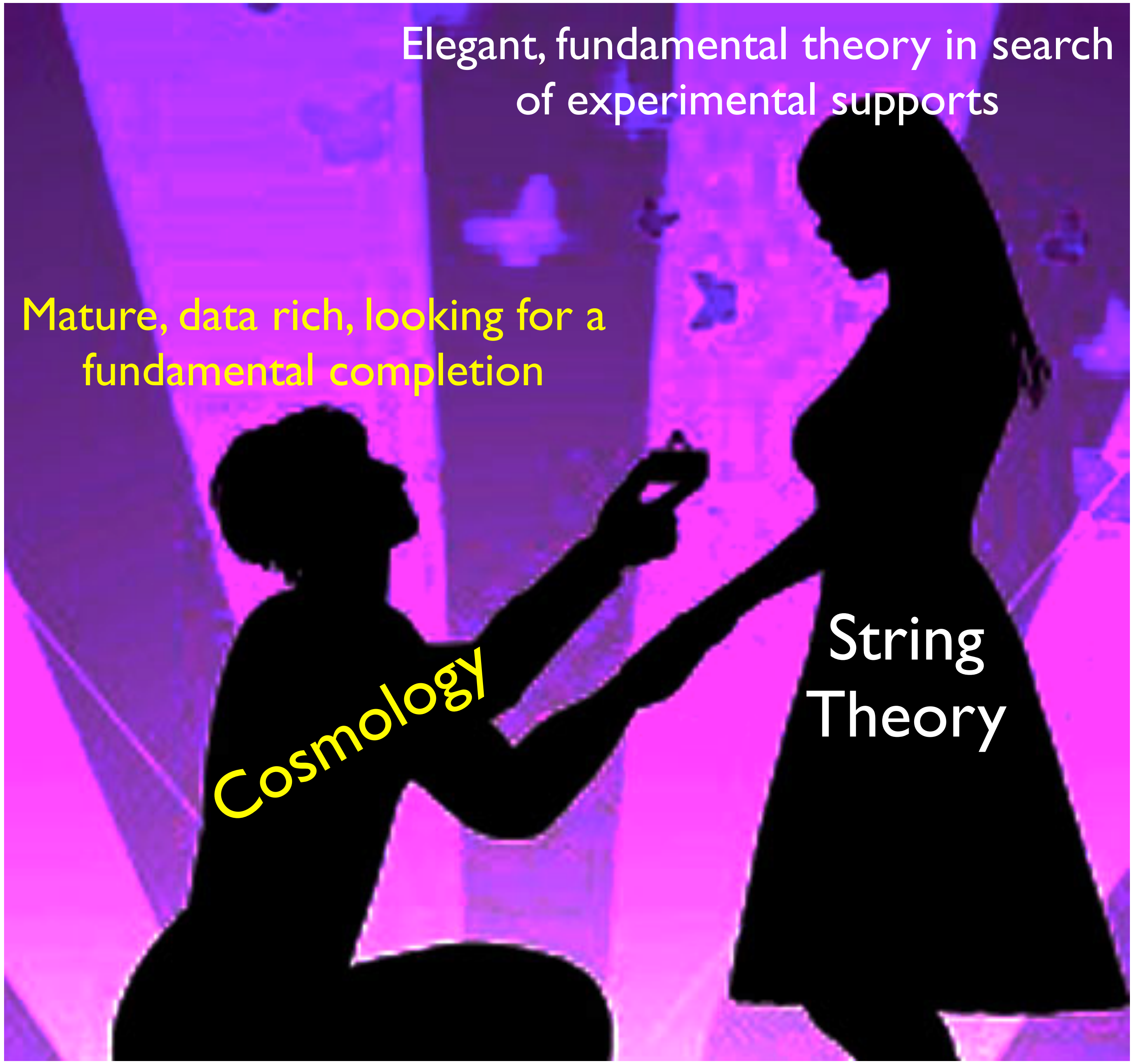


Outline

- **Lecture 1:** Motivations for Inflation in String Theory; Some inflationary basics; General discussions of realizing inflation in string theory.
- **Lecture 2:** Small-field inflation in String Theory (e.g., D-brane inflation)
- **Lecture 3:** Large field inflation in String Theory (e.g., Axion Mondromy)
- **Lecture 4:** Fencing in the **Swampland** (and the Weak Gravity Conjecture)

Lecture 1

Inflationary Universe

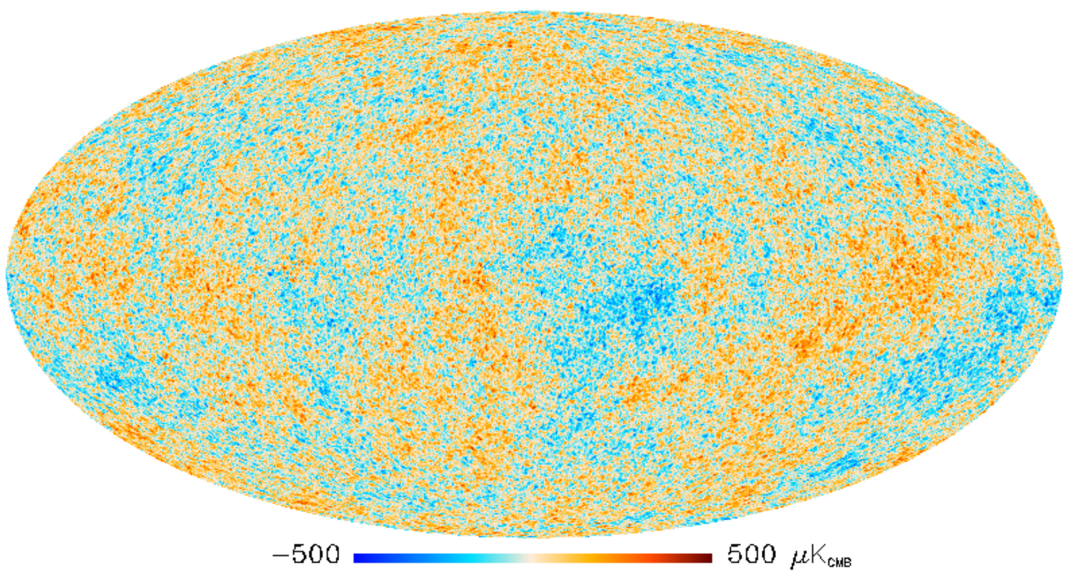
Starobinsky; Guth; Linde; Albrecht, Steinhardt; …

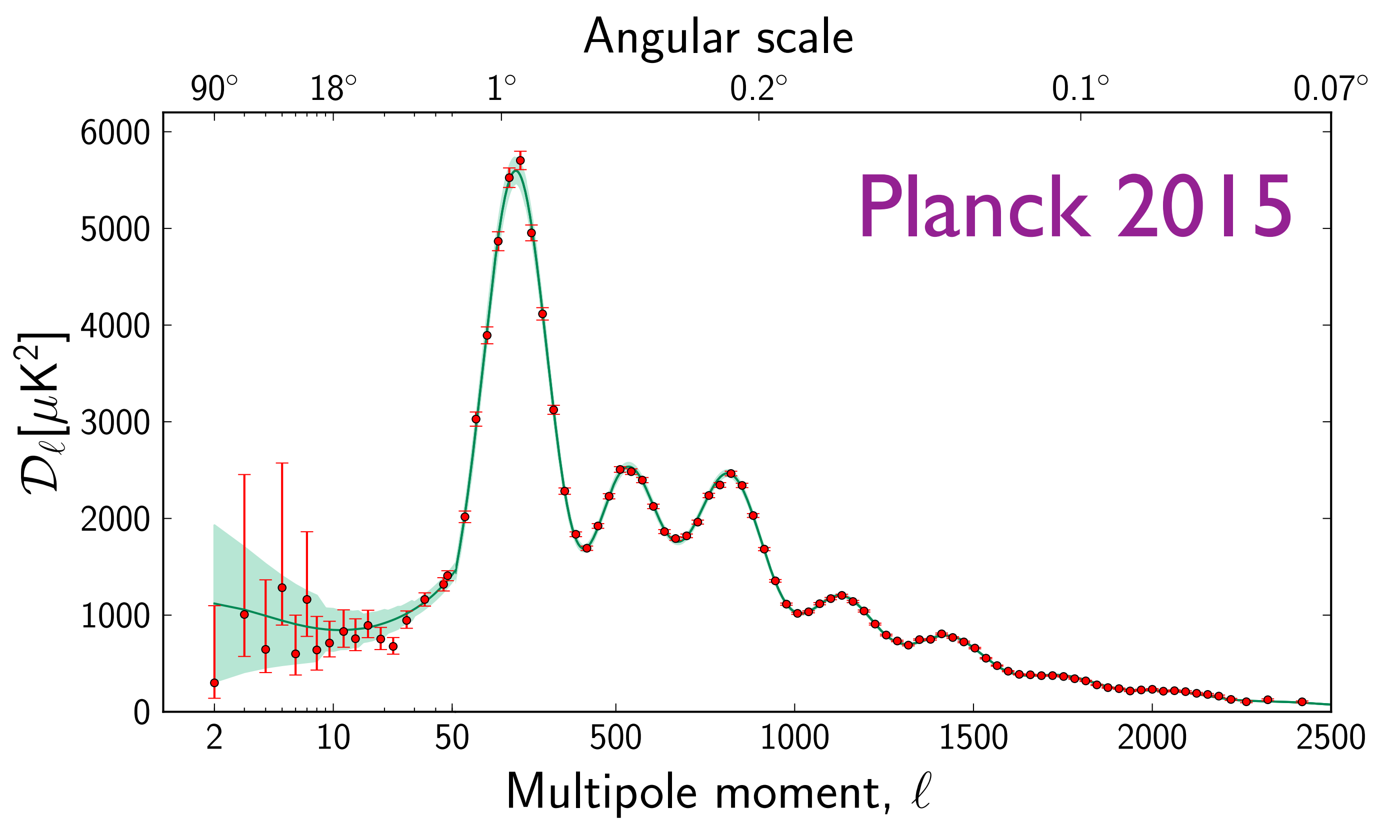
- Remarkably successful *effective theory*! (c.f., LG theory of superconductivity; Fermi's theory of the weak interaction):
  - Solves the flatness and horizon problems.
  - Provides a first principle mechanism to generate large-scale structure and CMB fluctuations.
  - Generic predictions (nearly scale-invariant, adiabatic, Gaussian primordial spectrum) in good agreement with data.

Inflationary Universe

Starobinsky; Guth; Linde; Albrecht, Steinhardt; …

- Standard Model of Cosmology fits data exceedingly well:





- Near future experiments (e.g., PLANCK, BICEP/KECK, ACT, PolarBeaR, SPT, SPIDER, QUEIT, Clover, EBEX, QUaD, LiteBIRD, & CMBS4) can test its predictions with higher precision.

Why String Inflation?

- **Suppose** we accept that (i) inflation solves the key problems in standard hot BB cosmology **and** (ii) provides the seed for structure formation. Are we done?

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- **Suppose** we accept that (i) inflation solves the key problems in standard hot BB cosmology **and** (ii) provides the seed for structure formation. Are we done?

- Goal of these lectures is to show that:

Inflation is sensitive to UV physics

<u>or</u> (state more positively)

Observational Cosmology (via inflation) is a powerful window into Planck scale physics!

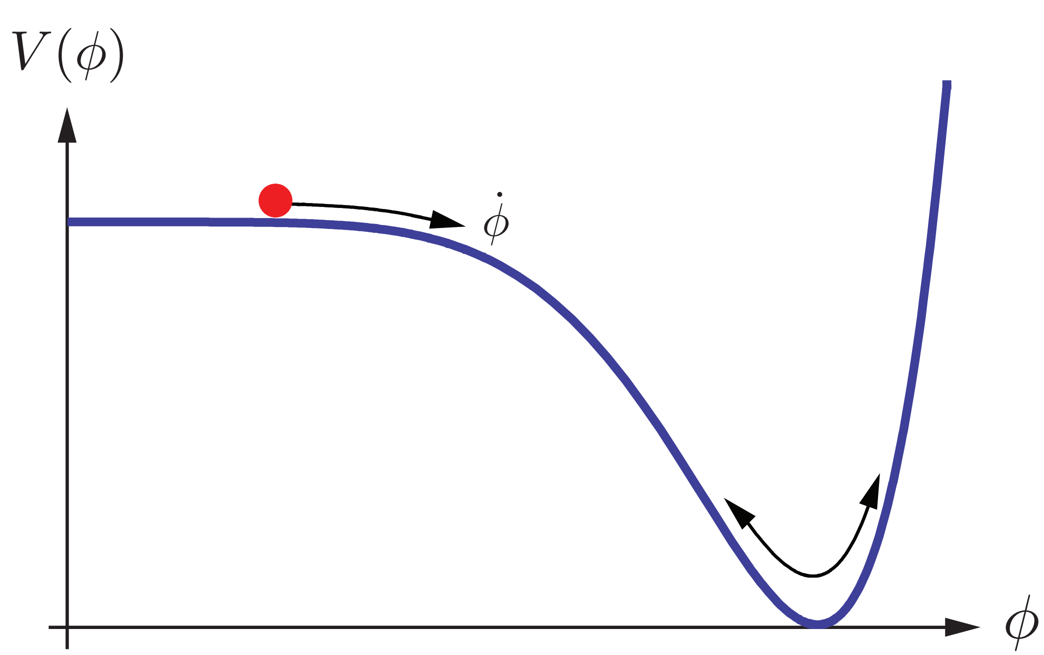
UV Sensitivity of Inflation

- Condition for inflation: $\frac{\ddot{a}}{a} > 0 \Leftrightarrow \dot{H} + H^2 > 0 \Leftrightarrow -\frac{\dot{H}}{H^2} < 1$

- Define the slow-roll parameters:

$$\begin{aligned} \epsilon_H &\equiv -\frac{\dot{H}}{H^2}, & \epsilon_H &<< 1 & &\text{inflation occurs} \\ \eta_H &\equiv \frac{\dot{\epsilon}_H}{1+\epsilon_H}, & \eta_H &<< 1 & &\text{inflation lasts} \end{aligned}$$

- Inflation is usually realized by a scalar field with a flat potential:



$$\begin{aligned} \epsilon_V &\equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 & &<< 1 \\ |\eta_V| &\equiv |M_P^2\left(\frac{V''}{V}\right)| & &<< 1 \end{aligned}$$

∃ linear map: $(\epsilon_H, \eta_H) \rightarrow (\epsilon_V, \eta_V)$

UV Sensitivity of Inflation

- The slow-roll conditions are highly sensitive to UV physics!
- Consider the inflationary Lagrangian which contains:

$$\mathcal{L} \supset \frac{1}{2}m^2\varphi^2, \qquad \begin{aligned} V'' &= m^2 + \dots \\ V &= 3H^2 M_P^2 \\ \Rightarrow \eta &= \frac{m^2}{3H^2} \end{aligned}$$

$|\eta| << 1$ if $m << H$

- **Eta Problem: why is the inflaton so light?**
- Quantum corrections tend to drive m to the cutoff $\Lambda >> H$.
- SUSY not helpful here: SUSY is broken with $m_{3/2} \sim H$, so $m_\varphi \rightarrow H$

Higher Dimensional Operators

- UV completion of GR typically involves new dofs with $M < M_P$
- **In String Theory, these include the string and KK states:**

$$M_{KK} < M_s < M_P$$

- If the inflaton has *O(1)* couplings to these heavy dofs ξ, integrating them out yields:

$$\Delta\mathcal{L}_\varphi = \frac{\mathcal{O}^\Delta(\varphi)}{M^{\Delta-4}} \text{ with } \mathcal{O} \text{ some allowed operator in the } \varphi \text{ QFT}$$

- If before considering ξ, we have

$$\mathcal{L}_\varphi = \frac{1}{2}(\partial\varphi)^2 - V(\varphi)$$

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- If before considering ξ, we have

$$\mathcal{L}_\varphi = \frac{1}{2}(\partial\varphi)^2 - V(\varphi) \quad \longrightarrow \quad \Delta\mathcal{L}_\varphi = c\, V(\varphi)\frac{\varphi^2}{M^2}$$

integrate out ξ

Higher Dimensional Operators

- If this dimension-6 Planck suppressed operator arises:

$$\Delta\eta = 2c\left(\frac{M_P}{M}\right)^2 >> 1 \text{ for } M << M_P \text{ and Wilson coefficient } c \sim \mathcal{O}(1)$$

- The above Planck-scale sensitivity is **general,** and applies to **<u>any model of inflation</u>**.

- Models with interesting next-generation observables such as large, distinctive non-Gaussianity (e.g. [Chen, Huang, Kachru, GS]) and gravity waves are **<u>even more UV sensitive!</u>**

Towards Inflation in String Theory

Moduli Problem

- Given its UV sensitivity, natural to search for inflation in string theory.
- For inflation, we need (at least) a scalar (the ***inflaton***) which
  - drives $\ddot{a} > 0$ for $\gtrsim 60$ e-folds
  - then *reheats* the universe
- Scalars are plentiful (too plentiful!) in string theory, e.g., a CY compactification comes w/ many Kahler moduli (size), complex structure moduli (shape), axio-dilaton, D-brane scalars, …
- Scalars w/ G_N -strength coupling & m $\gtrsim$ 30 TeV decay $\gtrsim$ 1 sec (recall $\Gamma \sim M^3/M_P^2$) and spoil BBN.
- If present today, these moduli lead to 5-th force.

Moduli Problem

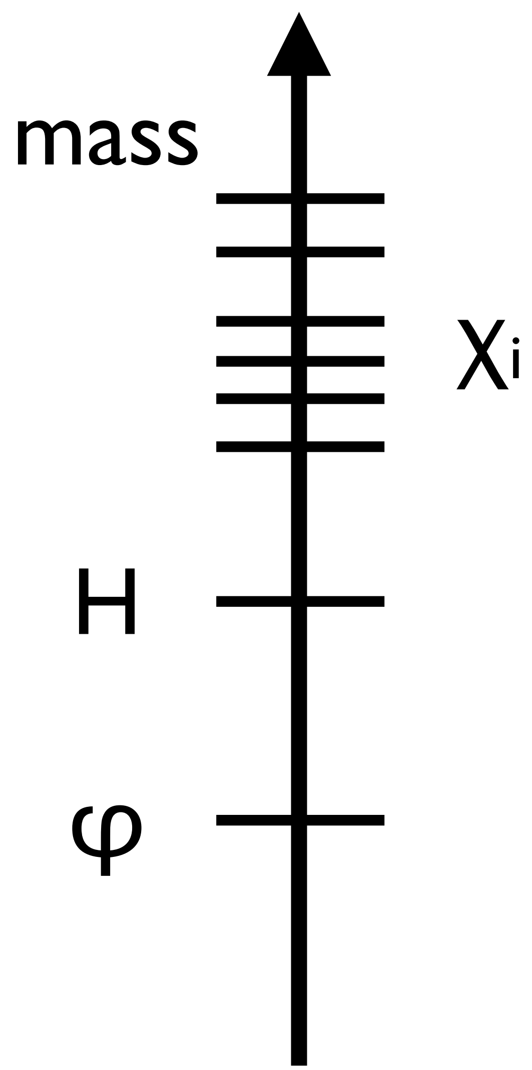
- This problem is **generic** for string theory: need moduli stabilization mechanism to give moduli masses $\gtrsim$ 30 TeV to not ruin late time cosmology.
- Flux compactification provides a class of vacua where many/most/all moduli are stabilized (more in Lecture 2).
- Realizing inflation in string theory introduces *new subtleties:*
  - inflaton is one of the moduli
  - giving other moduli masses $30\ \text{TeV} < m < H$ solves the moduli problem but they are dynamical during inflation!

$$\delta\chi \sim \frac{H}{2\pi}$$

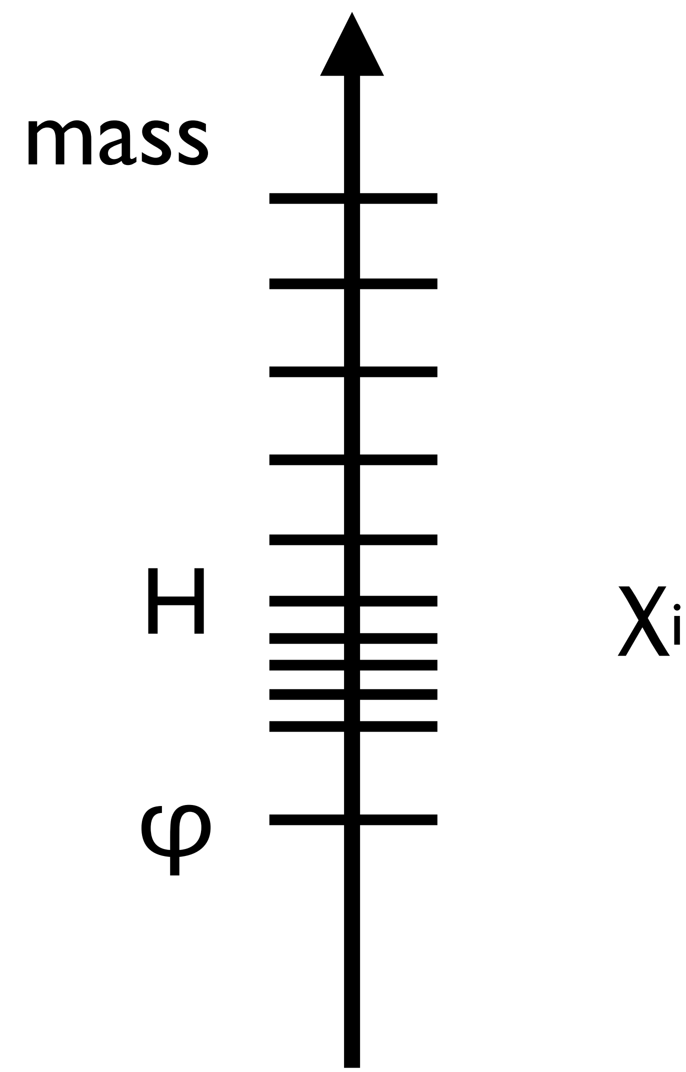
→ contribute to inflationary dynamics & curvature/entropy perturbations!

Moduli Stabilization

- Inflationary physics tied to moduli stabilization:



hard to arrange,
easier to study,
signatures straightforward



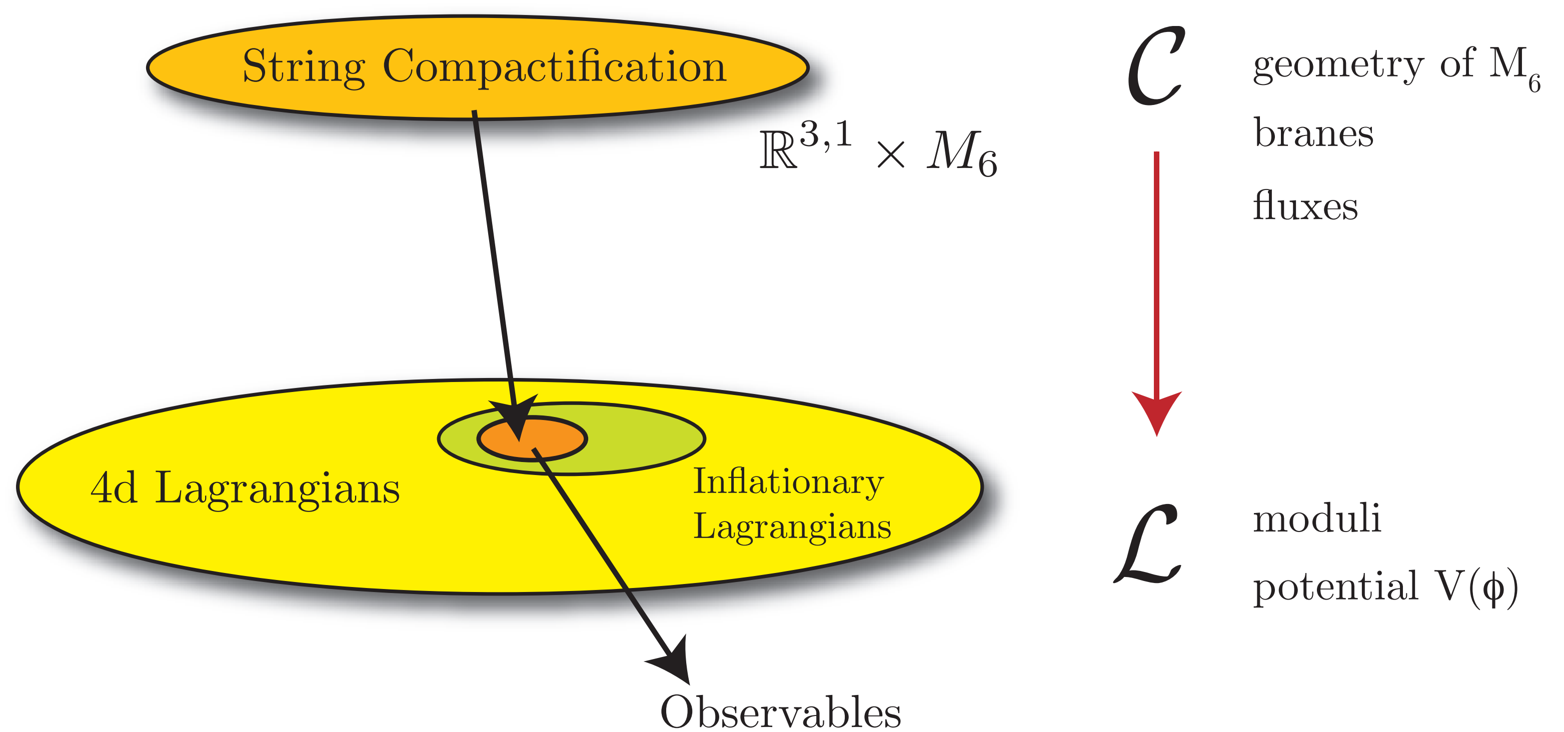
easier to arrange,
harder to study,
rich signatures (e.g., Non-Gaussianity)

Moduli Stabilization & Inflation

- Single Field Inflation: after integrating out χ_i, the EFT contains only 1 scalar φ with m< H **and** $V(\varphi)$ satisfying $\epsilon_V, |\eta_V| << 1$.
- Challenge: Integrate over Planck-scale dofs ξ already gives substantial corrections to $\mathcal{L}(\varphi)$ (in particular η_V), and here …
- Moduli masses $m << M_P$ (or even $m<<M_{KK}$ if stabilized in 4D theory), integrating out χ_i can give a large correction to $\mathcal{L}(\varphi)$!
  - One must understand moduli stabilization *in detail*
  - Corrections (g_s, α', warping, backreaction, …) to the EFT, often ignored in other contexts, *can be crucial*.

Effective Action of String Compactification

- A *complete* and *reliable* dimensional reduction, almost always beyond the leading order, is needed.
- This strongly motivates us to develop tools to compute the 4D EFT arising from string compactifications!



- Will see examples illustrating this point in the coming lectures.

Inflationary Basics

Inflation

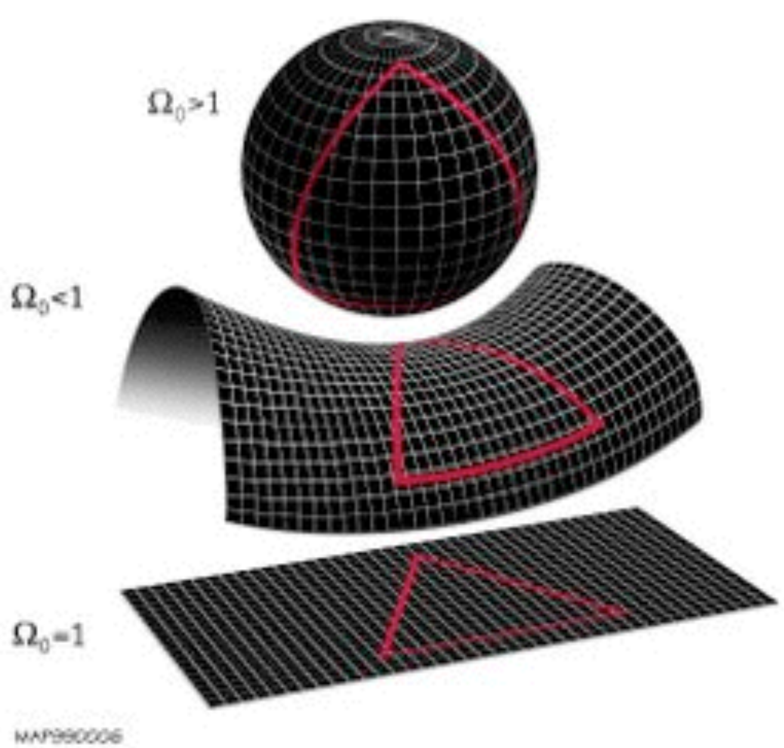
- An equivalent way of stating the condition for inflation is:

$$\frac{\ddot{a}}{a} > 0 \leftrightarrow \frac{d}{dt}(aH)^{-1} < 0$$

- A *shrinking comoving horizon* solves the flatness & horizon problems.

Flatness problem

$$1 - \frac{\rho}{\rho_{\rm cric}} \equiv 1 - \Omega(a) = -\frac{k}{(aH)^2}$$

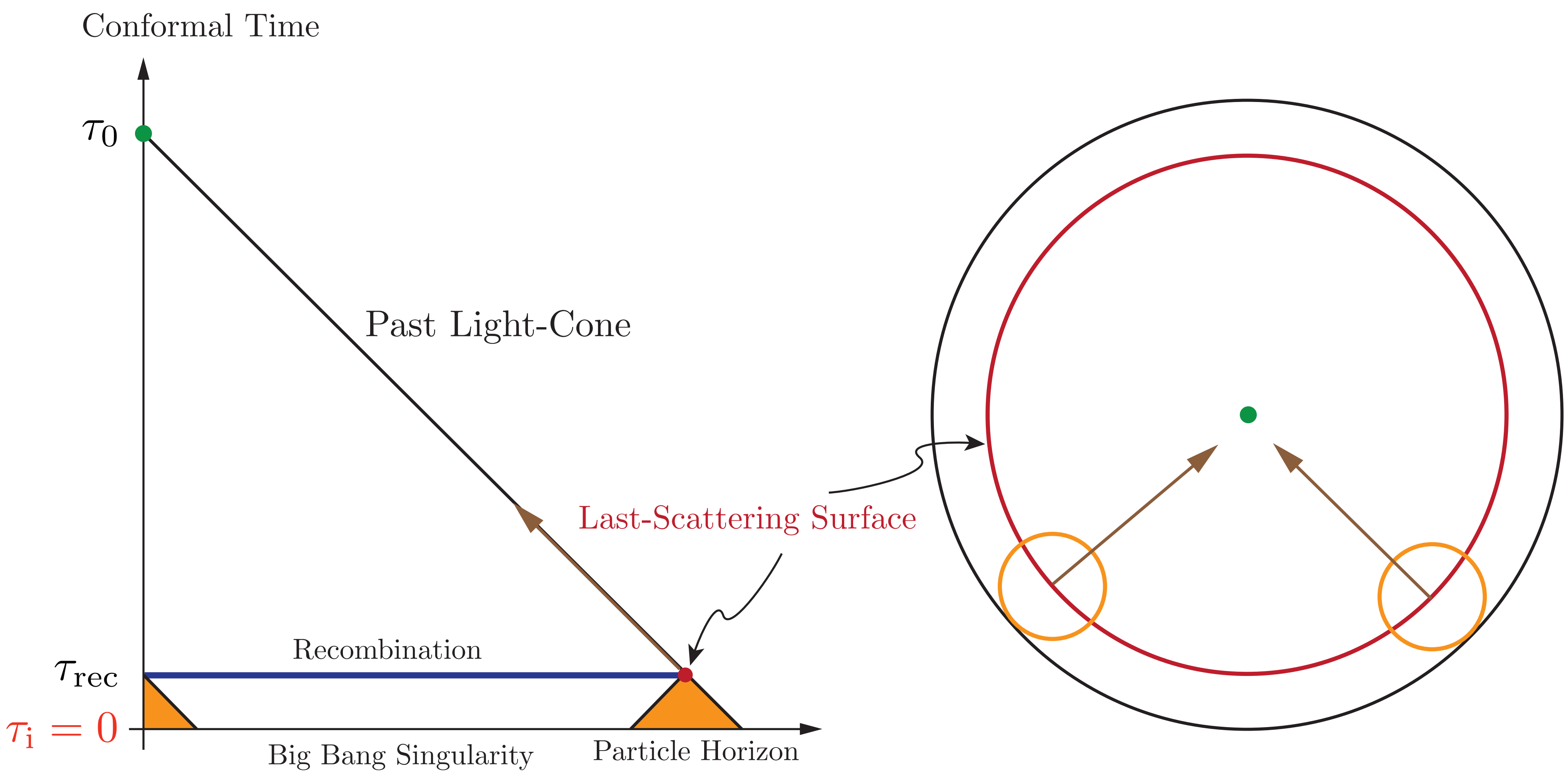


$$(aH)^{-1} \propto a^{\frac{1}{2}(1+3w)}$$

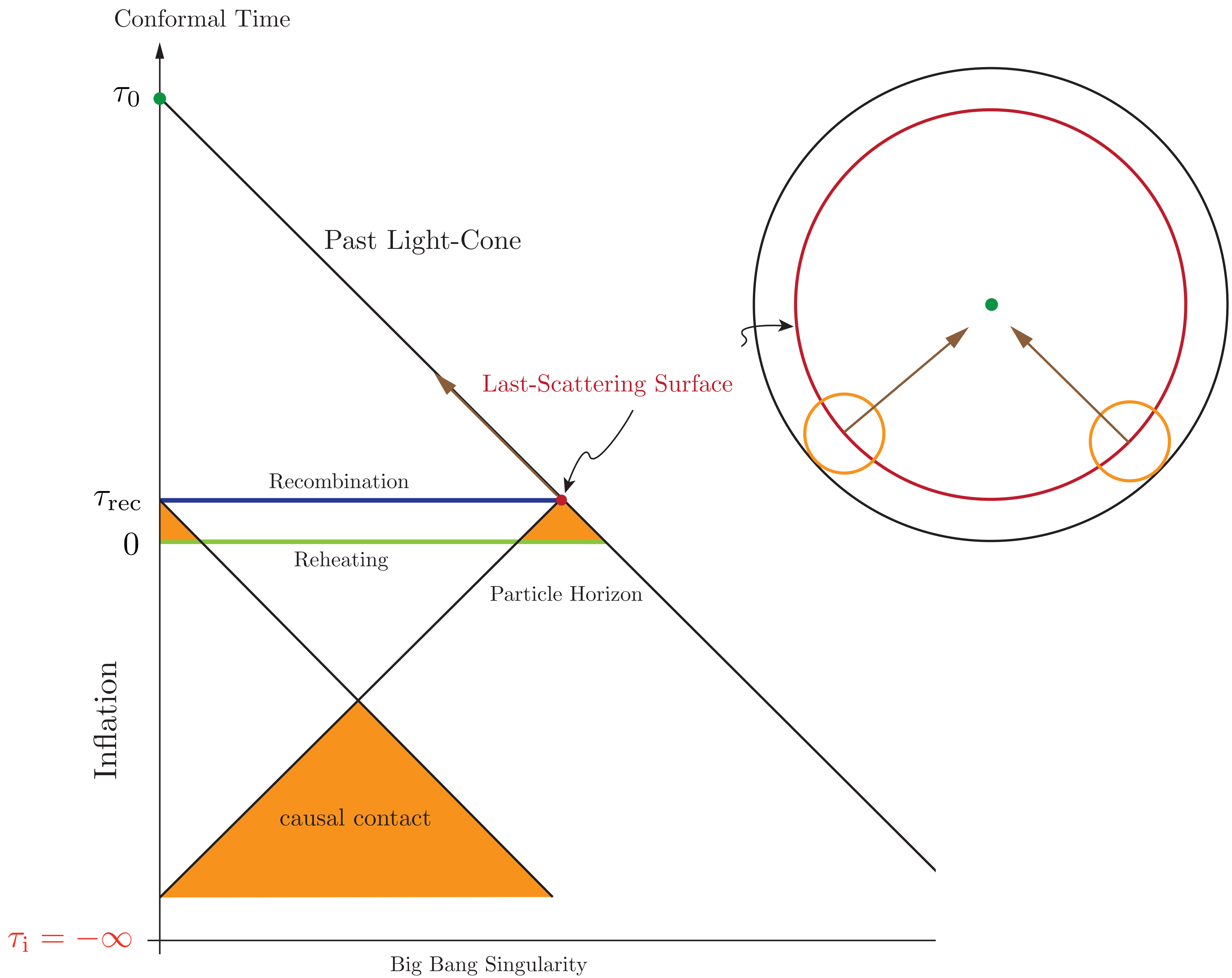
$|\Omega - 1| \sim 10^{-18}$ MeV temp. (BBN)

$|\Omega - 1| \sim 10^{-54}$ GUT temp.

Horizon problem



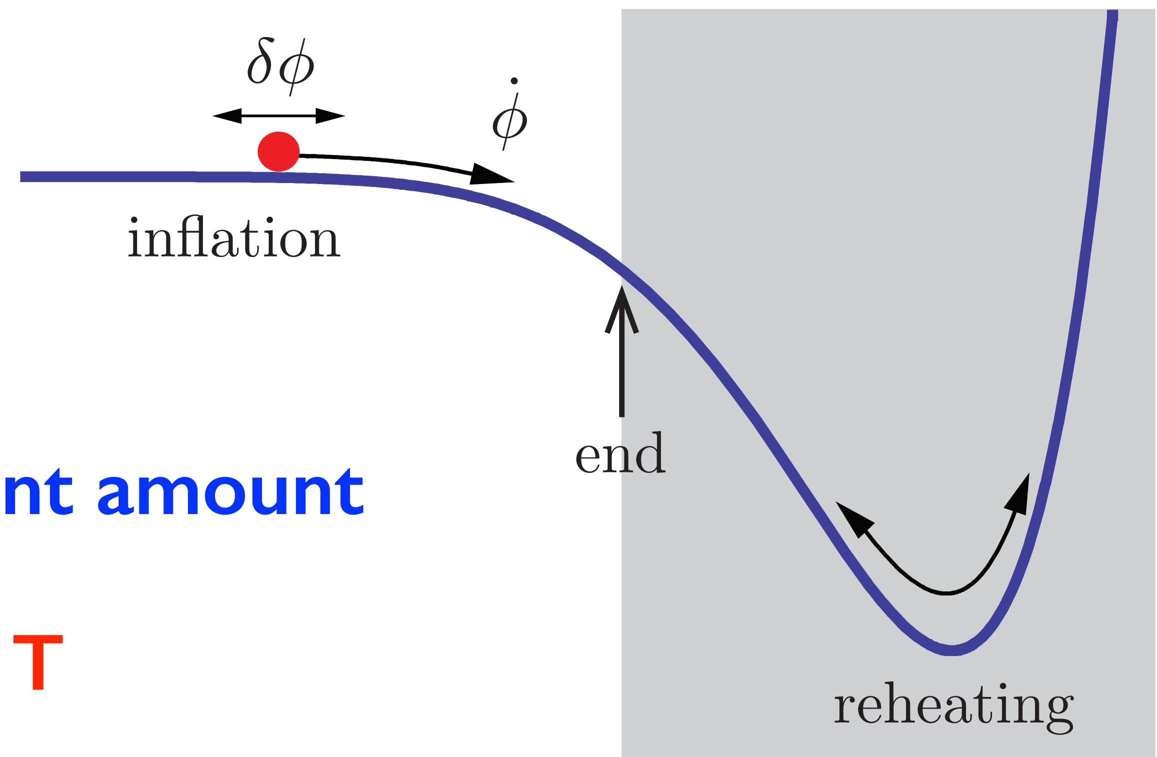
Horizon problem



Cosmological Perturbations

- A shrinking comoving horizon also leads to a prediction!
- The inflaton φ governs ϱ and the end of inflation (clock):

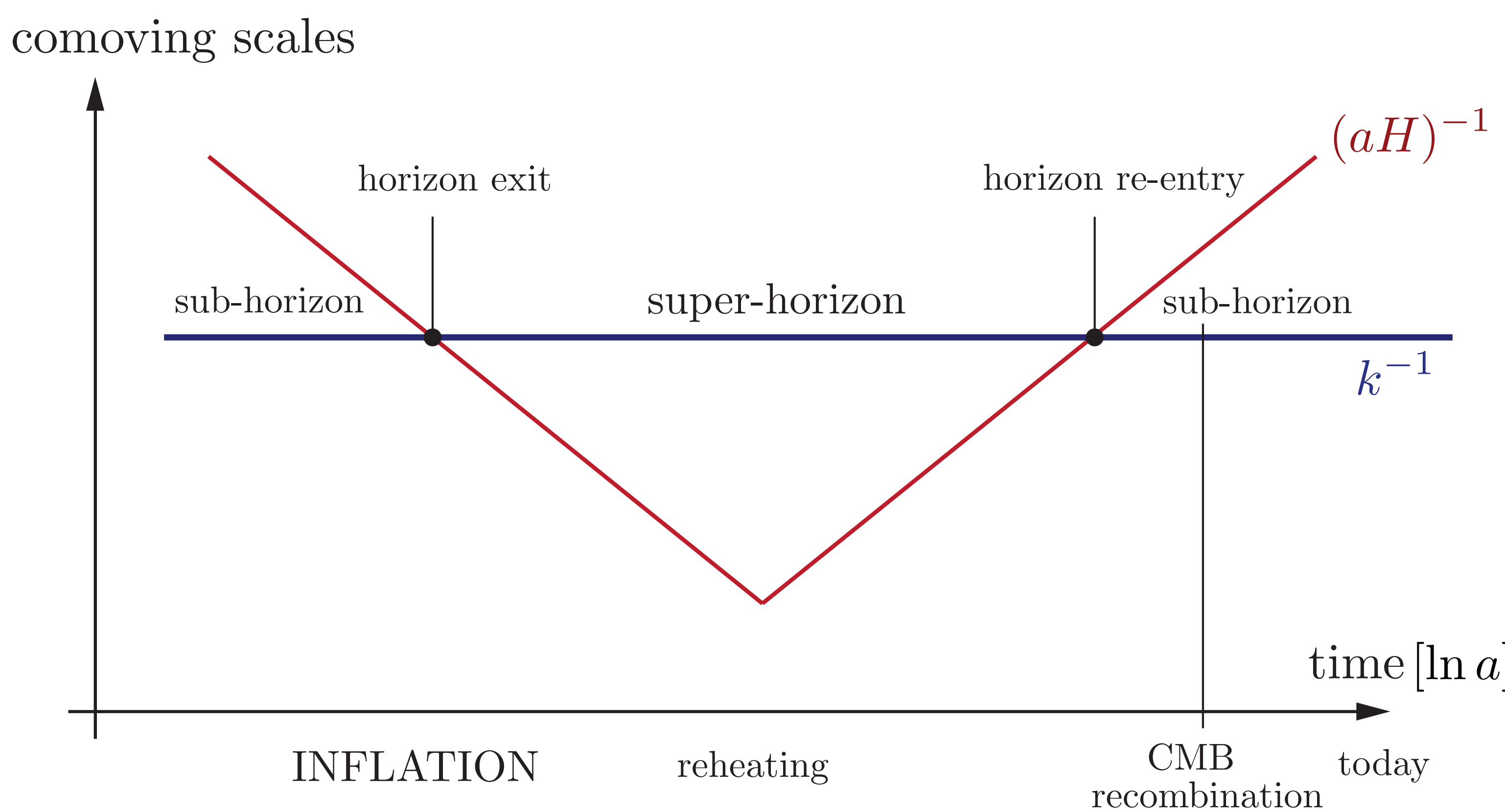
QM ⇒ $\varphi_{classical} + \delta\varphi$ (**x**,t)



different x inflate by different amount

⇒ fluctuations in ϱ and T

Cosmological Perturbations



- Cosmological perturbations amount to linearizing fluctuations:

$$\begin{aligned} \varphi(t,\vec{x}) &= \overline{\varphi}(t) + \delta\varphi(t,\vec{x}) \\ g_{\mu\nu}(t,\vec{x}) &= \overline{g}_{\mu\nu}(t) + \delta g_{\mu,\nu}(t,\vec{x}) \end{aligned}$$

and linearizing the EOMs. A linear treatment is justified because observed fluctuations are small (e.g., $\Delta T/T \sim 10^{-5}$).

Gauge Choice

- Homogeneity and isotropy fixes the form of the background.
- Perturbations no longer preserve homogeneity.
- Be careful to distinguish between real and fake perturbations
  - Example 1: $\varrho(t,x) = \varrho(t)$, but we can introduce fake perturbations by a change of coordinates:

$$\tilde{t} = t + \delta(t, \vec{x}) \Rightarrow \tilde{\rho}(\tilde{t}, \vec{x}) = \rho(t(\tilde{t}, \vec{x}), \vec{x})$$

fake inhomogeneity

  - Example 2: By choosing the hypersurface of constant time to coincide w/ surface of constant energy density:

$$\delta\tilde{\rho} = 0$$

though there are real inhomogeneities

Gauge Invariant Perturbations

- Physical dofs are the **gauge invariant combinations** of matter field + metric perturbations.
- How many scalar perturbations dof?
- Naively 5: $\delta\varphi, \; \delta g_{00}, \; \delta g_{ii}, \; \delta g_{0i} = \partial_i B, \; \delta g_{ij} = \partial_i \partial_j H$
- Coordinate transf. removes 2: $t \to t + \epsilon_0, \; x_i \to x_i + \partial_i \epsilon$
- Einstein constraint equation removes another 2.
- The only remaining gauge invariant combination of scalar dofs leads to density perturbation.

Comoving Gauge

- An efficient approach is to
  1. Choose a good gauge
  2. Expand the action
- Fix time & space reparametrizations by using comoving gauge:

$$
\begin{aligned}
\delta\varphi &= 0 \\
\delta g_{00} &= \delta g_{0i} = 0 \\
\delta g_{ij} &= a^2(t)\ \delta_{ij}\ (1-2\mathcal{R}) + a^2(t)\ h_{ij}
\end{aligned}
$$

where $\mathcal{R}$ is a scalar, h_{ij} is transverse-traceless, i.e., $\nabla_i h^{ij} = h^i_i = 0$

- We refer to $\mathcal{R}$ as the *curvature perturbation* as comoving spatial slices φ=constant have $R^{(3)} = \frac{4}{a^2}\nabla^2\mathcal{R}$; h_{ij} ~ tensor perturbation.

Curvature Perturbations

- Expanding the action (see e.g., Maldacena, '02):

$$S_{(0)} = \int d^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right]$$

we find the 2nd order action for our (classical) variable $\mathcal{R}$

$$S_{(2)} = \int d^4x\sqrt{-g}\left[\dot{\mathcal{R}}^2 - \frac{1}{a^2}\delta^{ij}\partial_i\mathcal{R}\partial_j\mathcal{R}\right]\frac{\dot{\varphi}^2}{2H^2}$$

- Define the canonically normalized field (Mukhanov variable);

$$v \equiv \left(\frac{a\dot{\varphi}}{H}\right)\mathcal{R} \equiv z\,\mathcal{R}$$

- Transforming to conformal time adt = dτ:

$$S_{(2)} = \frac{1}{2}\int d^3x d\tau\left[z^2\mathcal{R}''^2 - (\nabla v)^2\right] = \frac{1}{2}\int d^3x d\tau\left[(v')^2 - (\nabla v)^2 + \frac{z''}{z}v^2\right] \quad \textbf{(HW)}$$

Curvature Perturbations

- Now, $z \equiv a\dot{\varphi}/H$ is background (model) dependent
- So we have a scalar w/ a time-dependent mass:

$$S_{(2)} = \int d\tau d^3x \left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu v \partial_\nu v - \frac{1}{2}m^2(\tau)v^2 \right]$$

with

$$m^2(\tau) = -\frac{z''}{z} = -\frac{H}{a\dot{\varphi}}\frac{\partial^2}{\partial\tau^2}\left(\frac{a\dot{\varphi}}{H}\right)$$

- Given a homogeneous background solution, one obtains m(τ):

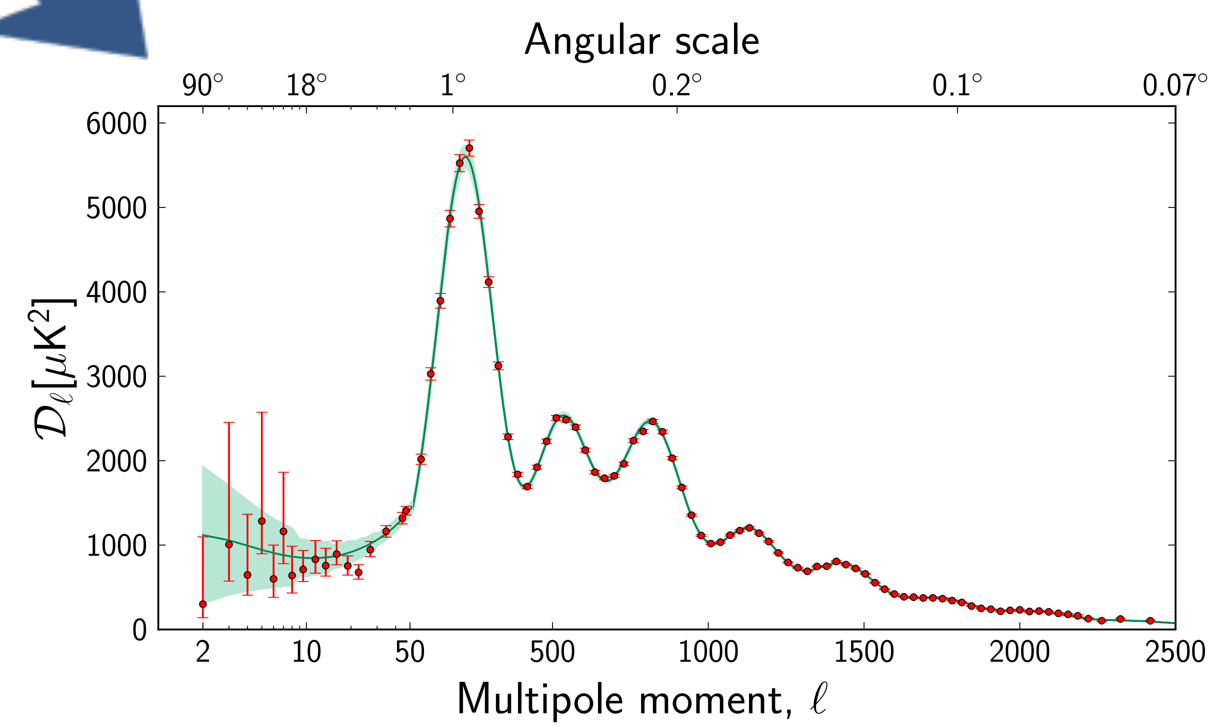
$$\{a(t), \varphi(t)\} \Rightarrow \{\dot{\varphi}(t), H(t), \tau(t)\} \Rightarrow z(\tau)$$

- Mukhanov-Sasaki equation: $v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$

Statistical properties of $\Delta T/T$ determined by that of $\mathcal{R}$!

Connecting Theory with Observations

- The scalar **power spectrum** is one of the key outputs of inflation:



$$\xi_{\mathcal{R}}(r) = \langle \mathcal{R}(\vec{x}), \mathcal{R}(\vec{x}+\vec{r}) \rangle$$

$\because$ isometry

$$P_{\mathcal{R}}(k) \equiv \int d^3 r \xi_{\mathcal{R}}(r) e^{-i\vec{k}\cdot\vec{r}}$$

- Not the only observables:

$$\langle \mathcal{R}_{\vec{k}_1} \mathcal{R}_{\vec{k}_2} \mathcal{R}_{\vec{k}_3} \rangle \neq 0$$ Non-gaussianity

$$\langle h_{ij}(\vec{k}) h_{ij}(\vec{k}') \rangle$$ Primordial GW (tensor perturbation)

Power Spectrum & Spectral Index

- Quantize the fluctuations by promoting:

$$[\hat{v}(\tau, \mathbf{x}), \hat{\pi}(\tau, \mathbf{y})] = i\delta(\mathbf{x} - \mathbf{y})$$

- Power spectrum is given by:
- Dimensionless power spectrum

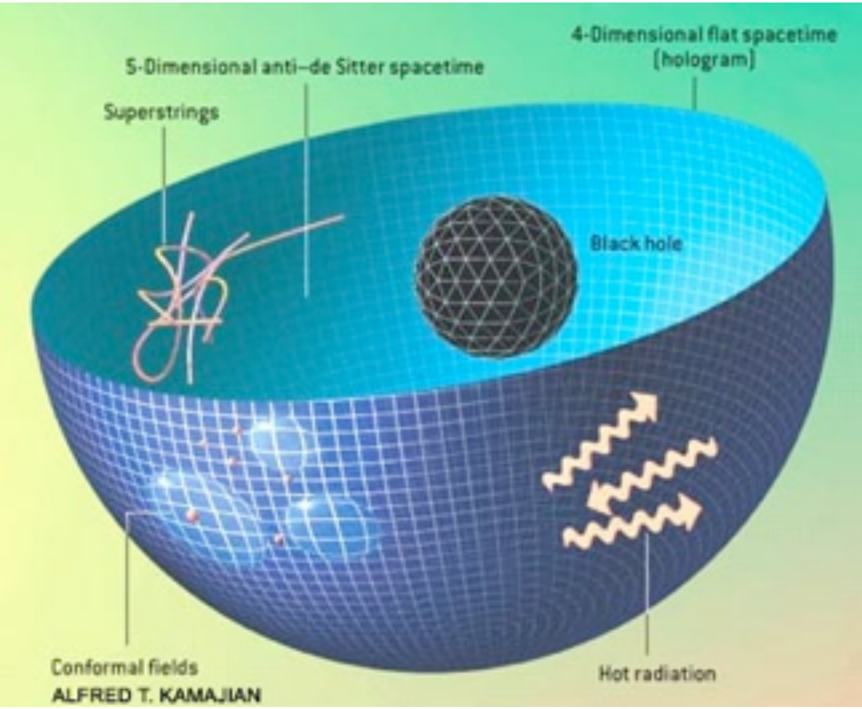
$$P_{\mathcal{R}}(k) = \frac{H^2}{2k^3}\frac{H^2}{\dot{\varphi}^2}|_{k=aH}$$

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}P_{\mathcal{R}}(k)$$

- **Spectral index:** parametrizing deviation from *scale invariance*

$$\Delta^2_{\mathcal{R}}(k) = A(k_0)\left(\frac{k}{k_0}\right)^{n_s-1} \rightarrow \frac{d}{d\ln k}\Delta^2_{\mathcal{R}}(k) = n_s - 1$$

Homework: $n_s - 1 = 2\eta_H - 4\epsilon_H = 2\eta_V - 6\epsilon_V$



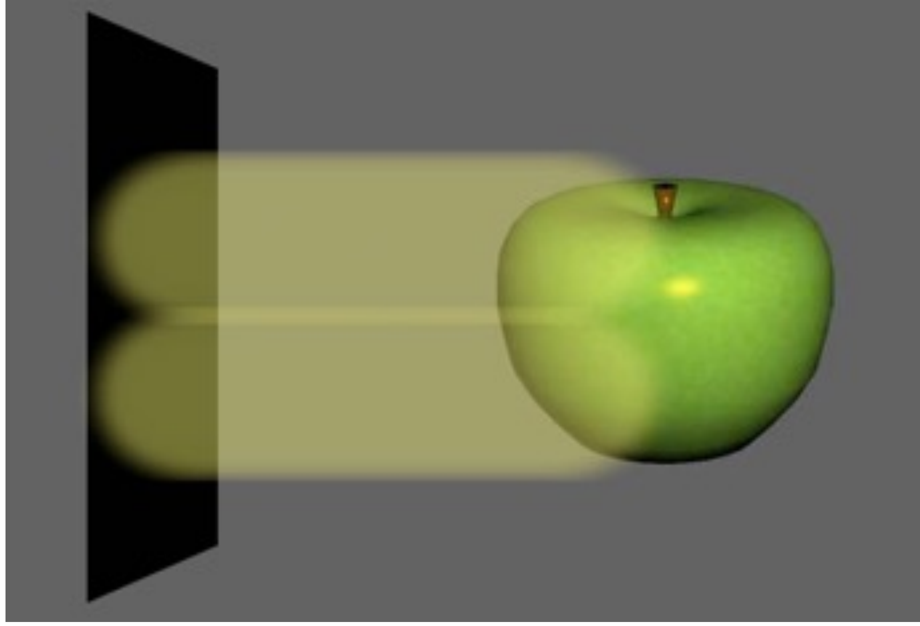
Holography?

- Is there a dS/CFT? [Strominger]
- Unlike AdS/CFT, challenge for dS/CFT include:
  - no explicit string constructions (and candidate CFT)
  - no boundary
  - no SUSY
- Pragmatic approach: some observables are fixed by symmetries
- Take as working assumption: $\Psi_{dS} = Z_{CFT}$
- Inflation is not exact dS but slightly perturbed:

$$S_{CFT} \to S_{CFT} + g \int d^d x \mathcal{O}(\vec{x})$$

- The slow-roll parameters should have their analogues in CFT.

Holography?



- The holographic direction is time:

$$ds^2 = -dt^2 + e^{2Ht} dx_d^2$$

- Introduce μ=Ha (holographic scale)

$$ds^2 = H^{-2}\left[-\mu^2 d\mu^2 + \mu^2 ds_d^2\right]$$

- The coupling constant g in the CFT is determined by φ:

$$g = \kappa\phi$$

- Beta function:

$$\beta = \frac{dg}{dln\mu} = \frac{\kappa\dot{\phi}}{H} = \sqrt{2\epsilon_H}$$

Anomalous dimension:

$$\lambda = \frac{d\beta}{dg} = \epsilon_H - \eta_H$$

Holographic Dictionary

- Back to inflationary perturbations:

$$\langle \mathcal{O}_\Delta(x)\mathcal{O}_\Delta(y)\rangle = \frac{\delta^2\Psi_{dS}[\phi]}{\delta\phi(x)\delta\phi(y)}\Big|_{\phi=0}$$

- Expand $\boldsymbol{\Psi}_{dS}$ in terms of source ϕ (justified as $\delta\rho/\rho\sim 10^{-5}$)

$$\Psi_{dS}[\phi] = e^{\frac{1}{2}\int d^3k\, d^3k' \,\langle \mathcal{O}_{\vec{k}}\mathcal{O}_{\vec{k}'}\rangle\, \phi_{\vec{k}}\phi_{\vec{k}'} + \cdots + \frac{1}{n!}\int d^3k \ldots d^3k^n\, \phi_{\vec{k}}\ldots\phi_{\vec{k}^n}\,\langle \mathcal{O}_{\vec{k}}\ldots\mathcal{O}_{\vec{k}^n}\rangle}$$

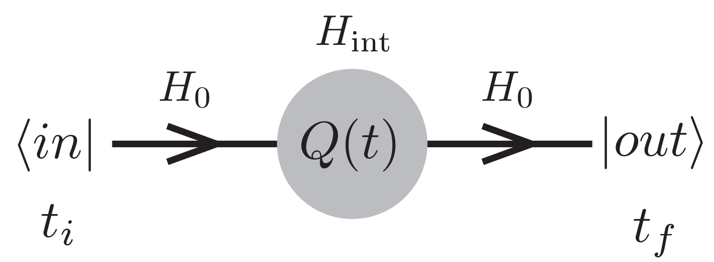
- Holographic dictionary [Maldacena, '02]:

$$\langle \phi_{\vec{k}}\phi_{-\vec{k}}\rangle' = -\frac{1}{2Re\langle \mathcal{O}_{\vec{k}}\mathcal{O}_{-\vec{k}}\rangle'} \qquad \langle \phi_{\vec{k}_1}\phi_{\vec{k}_2}\phi_{\vec{k}_3}\rangle' = \frac{2Re\langle \mathcal{O}_{\vec{k}_1}\mathcal{O}_{\vec{k}_2}\mathcal{O}_{\vec{k}_3}\rangle'}{\Pi_i\left(-2Re\langle \mathcal{O}_{\vec{k}_i}\mathcal{O}_{-\vec{k}_i}\rangle'\right)}$$

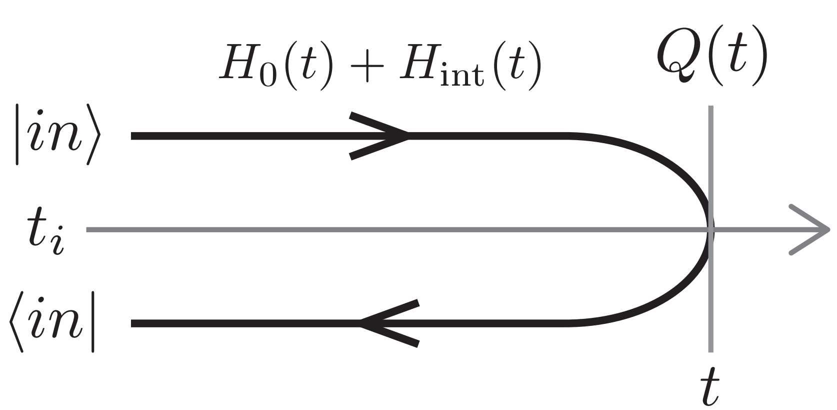
- verified for power spectrum [van der Schaar] taking into account RG flow; 3-pt function more subtle, checked only for special kinematic limit or $\varepsilon \ll \eta$ [Schalm, Shiu, van der Aalst], [Skenderis et al];[Trivedi et al]….

Non-Gaussianity

- Particle physicists compute in-out amplitude, say for the LHC:



- Cosmologists compute in-in expectation values aka **"Cosmological Collider"**



- **Sketch of calculations:** Expand the action in the comoving gauge:

$$S = S_0\left[\overline{\varphi}, \overline{g}_{\mu\nu}\right] + S_2(\mathcal{R}^2) + S_3(\mathcal{R}^3) + \dots$$

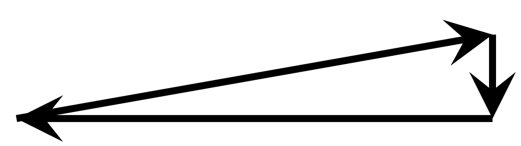
- HW: Compute NG for *general single-field inflation* [Chen, Huang, Kachru, GS]

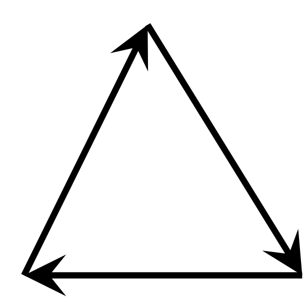
Non-Gaussianity

- Consider $S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R + P(X, \phi) \right]$ with $X \equiv -\frac{1}{2} (\partial_\mu \phi)^2$
- NG $\langle \mathcal{R}_{k1} \mathcal{R}_{k2} \mathcal{R}_{k3} \rangle$ fully worked out in [Chen, Huang, Kachru, GS]
- Slow-roll inflation corresponds to $P(x,\phi)=X-V(\phi)$ but generally:

$$P(X, \phi) = \sum c_n(\phi) \frac{X^n}{\Lambda^{4n-4}}$$

- Detectable NG if $X \gg \Lambda$ ⇒ UV completion needed!
- DBI Inflation [Silverstein, Tong]: $P(X, \phi) = \frac{\Lambda^4}{f(\phi)} \sqrt{1 - f(\phi) \frac{X}{\Lambda^4}} - V(\phi)$
- Distintive shapes:





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