Sphaleron
Topology & Baryon Asymmetry

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Work in collaboration with Henry Tye

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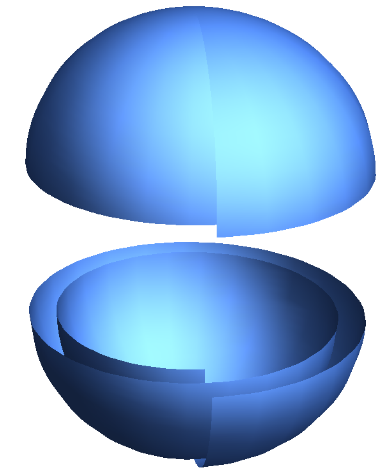
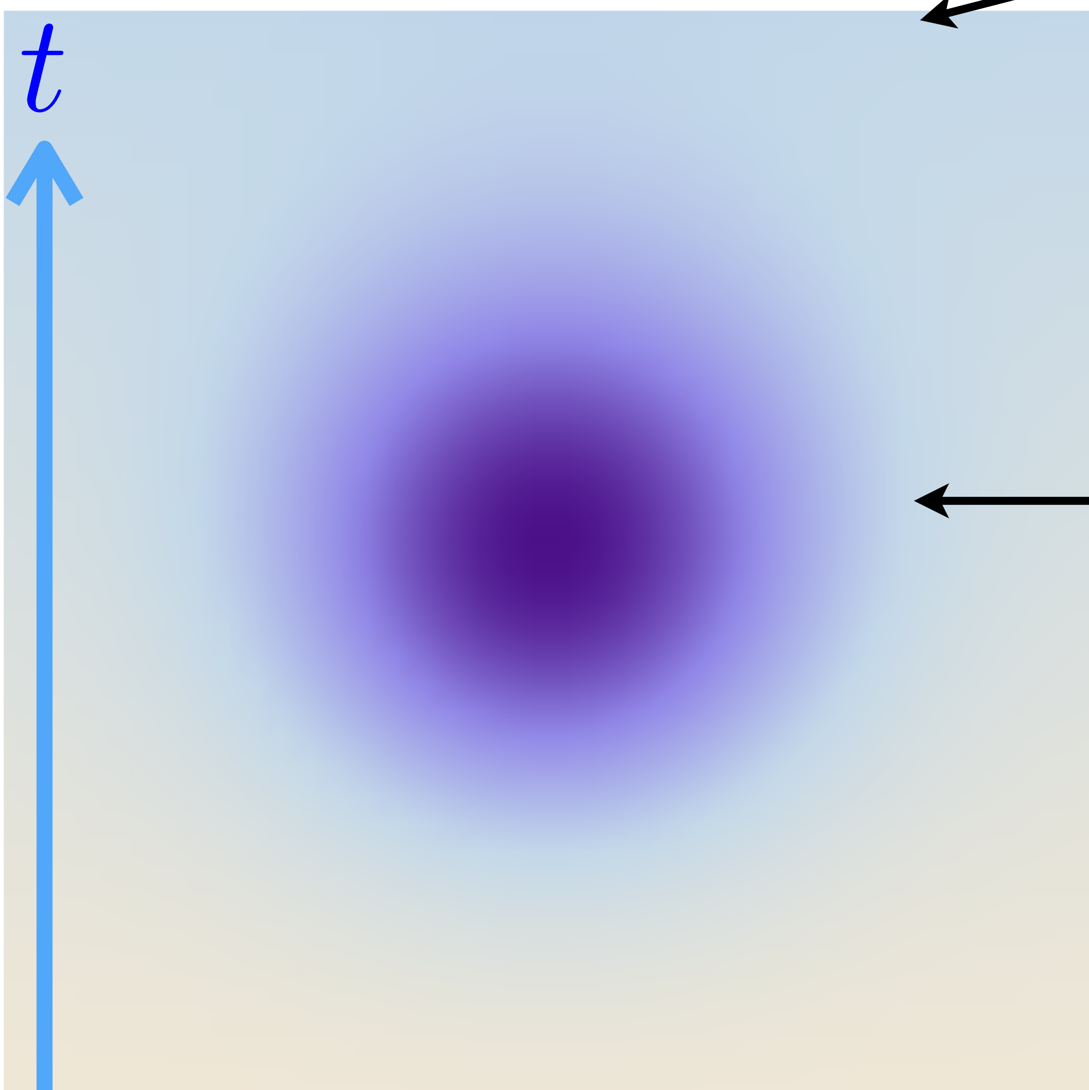
Instanton

$$N_{CS} = \frac{1}{24\pi^2} \int \mathrm{Tr}\left(U^{-1} dU\right)^3$$

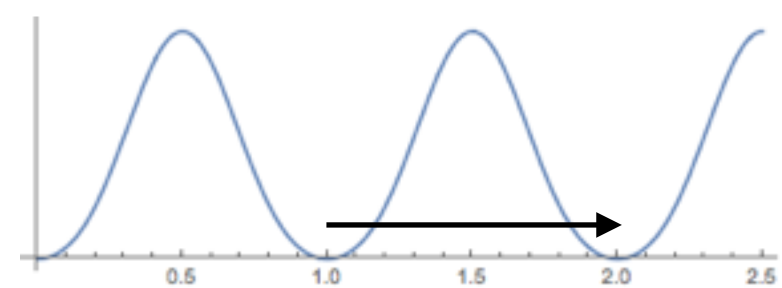
$$A_i^{(2)} = \frac{i}{g} U^{(2)-1} \partial_i U^{(2)}$$

$$N_{CS}^{(2)} = 2$$



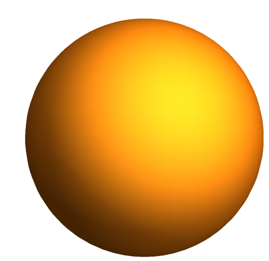


$$\mathcal{N} = \frac{g^2}{16\pi^2} \int d^4x \, \mathrm{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} = 1$$



$$A_i^{(1)} = \frac{i}{g} U^{(1)-1} \partial_i U^{(1)}$$

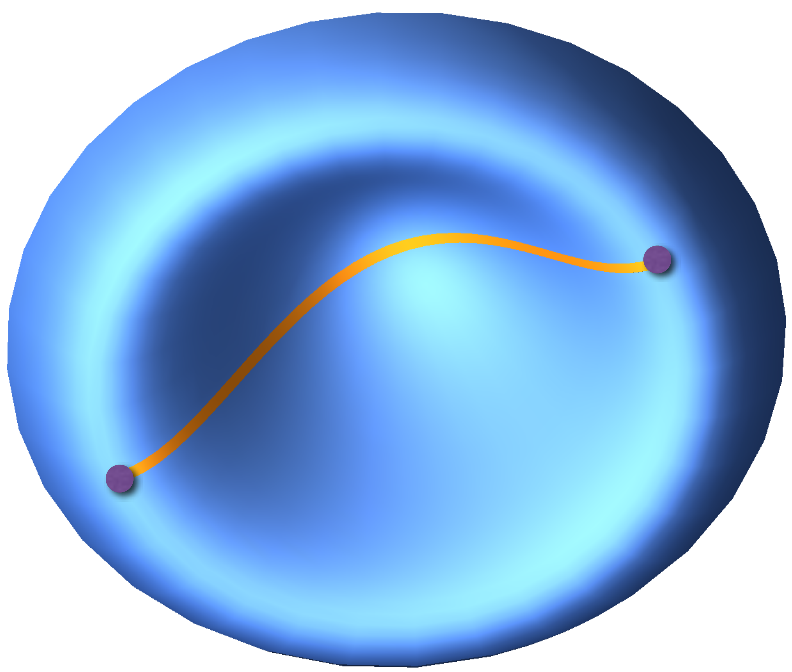
$$N_{CS}^{(1)} = 1$$



Sphaleron

$(1+1)$-D

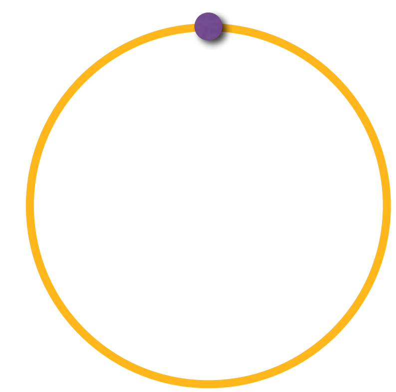
$-\infty$ $\mathbb{R}$ $+\infty$

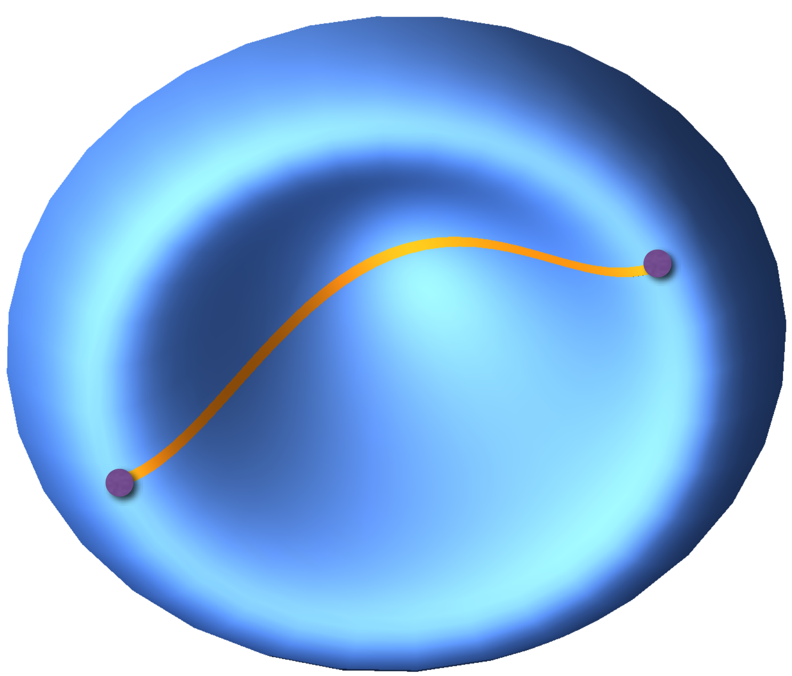


ϕ

Sphaleron

$(1+1)$-D

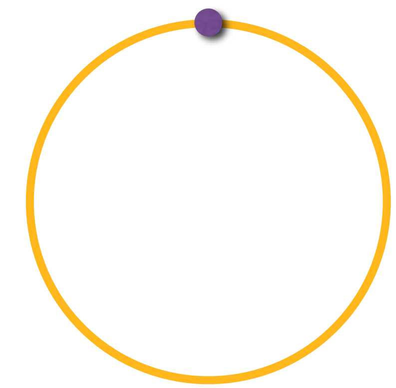




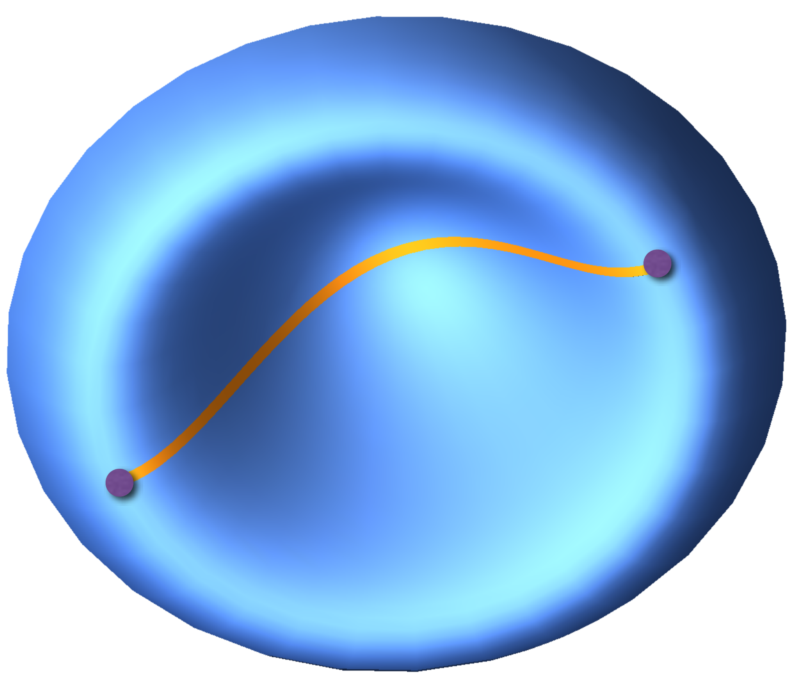
Sphaleron

$(1+1)$-D

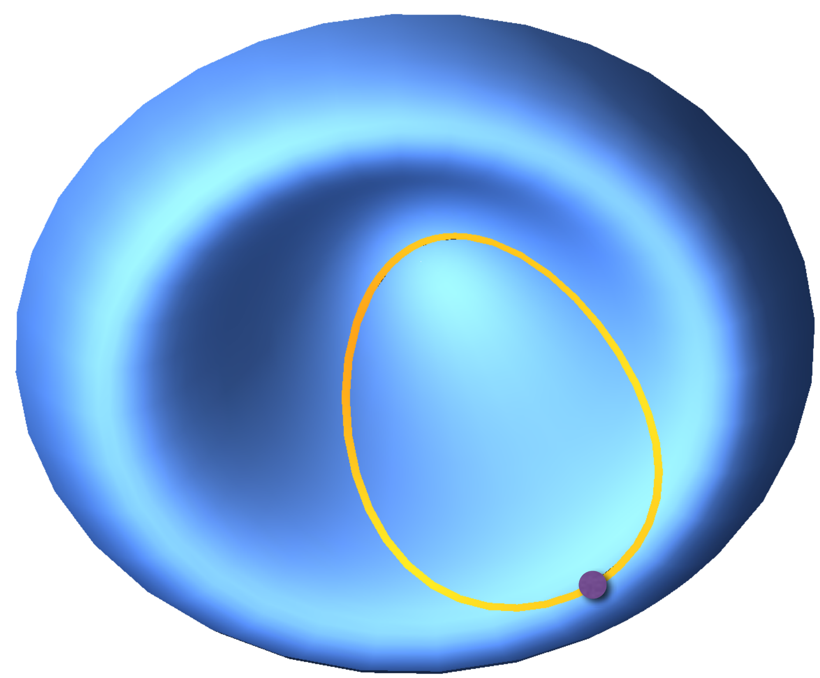
$-\infty$ $\mathbb{R}$ $+\infty$



$\mathbb{R} \cup \{\pm\infty\}$

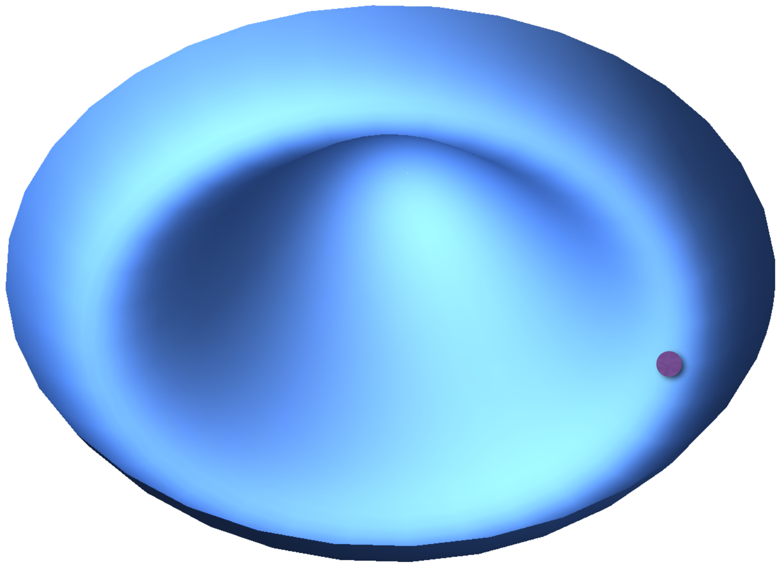


ϕ

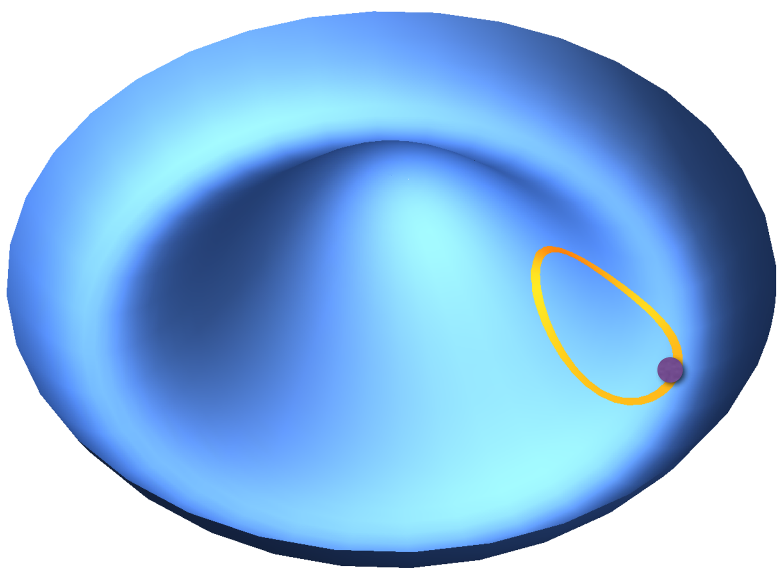


ϕ, A_μ

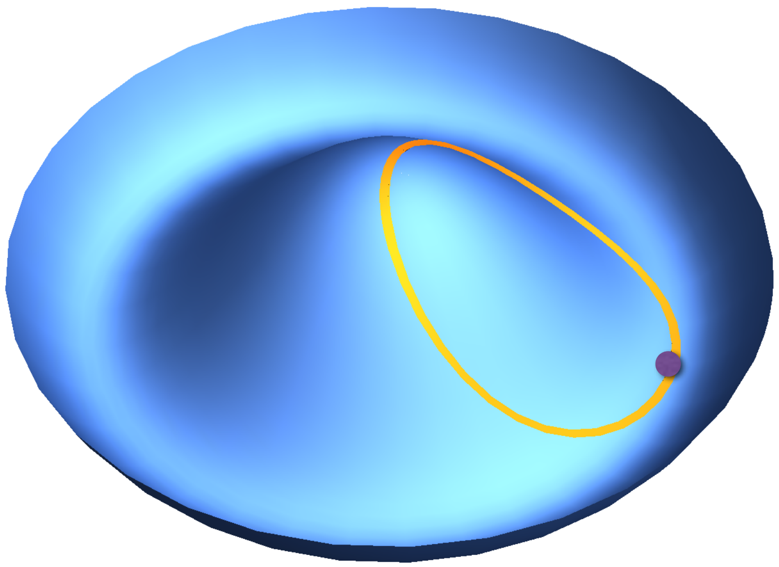
Transition through a Sphaleron



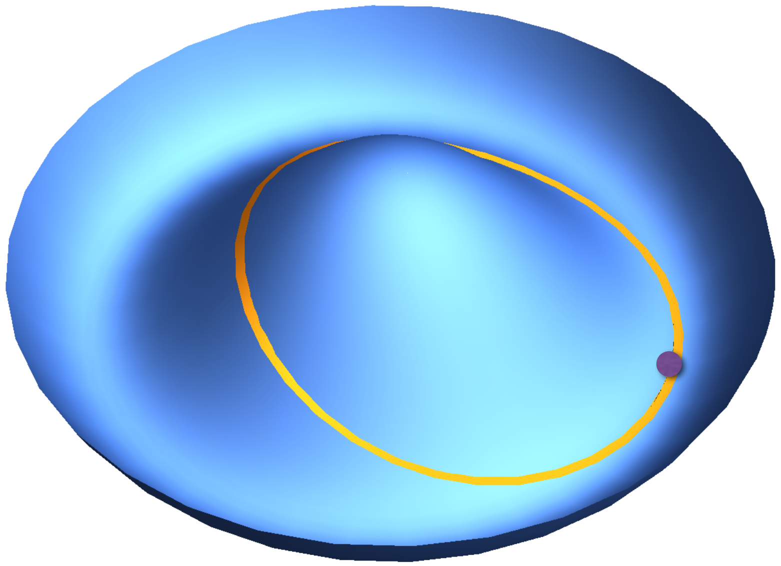
Transition through a Sphaleron



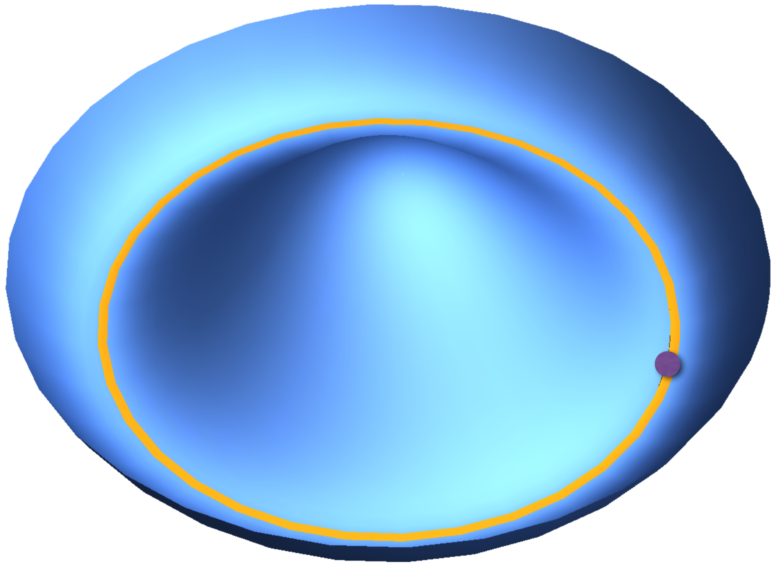
Transition through a Sphaleron



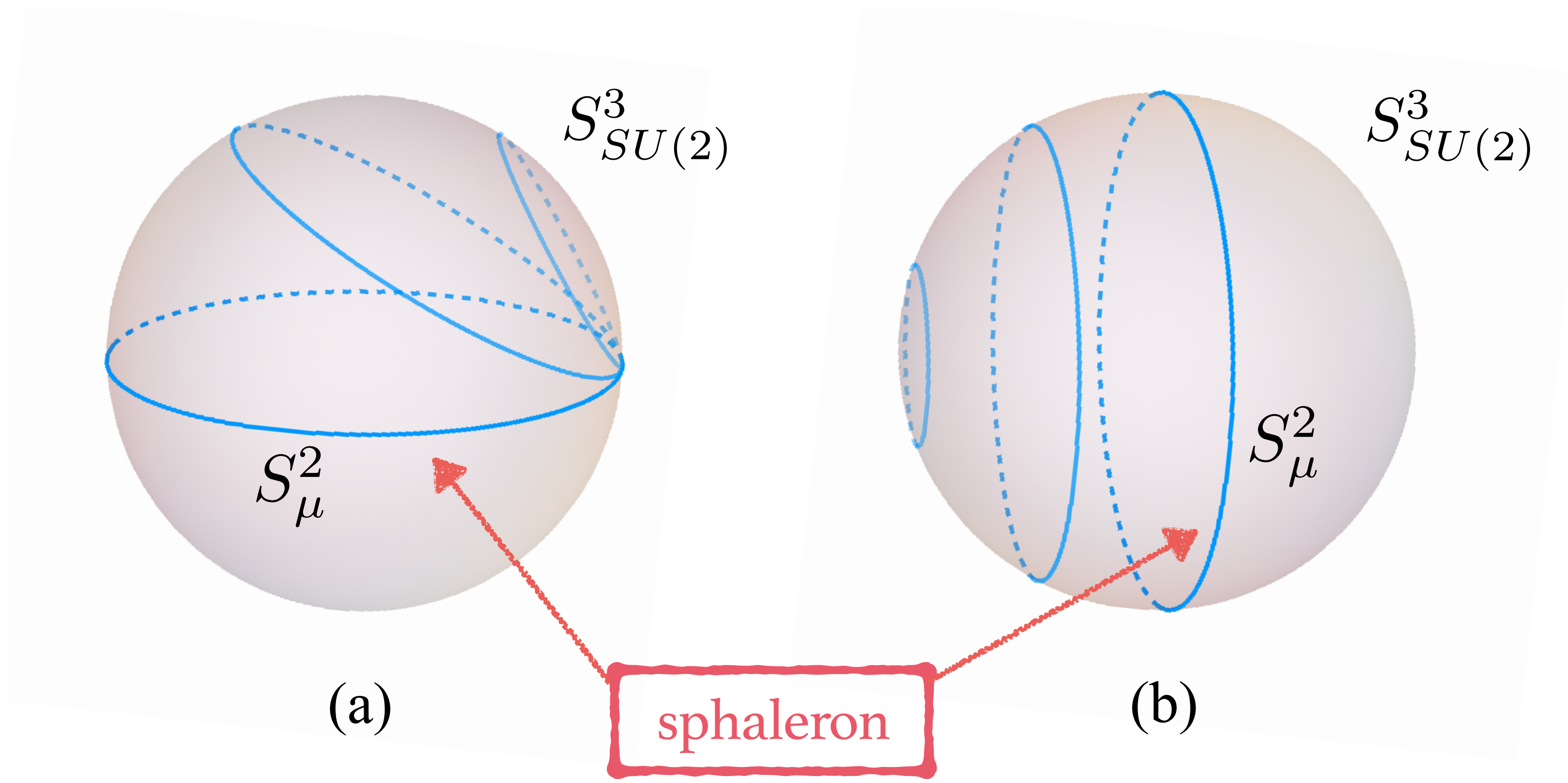
Transition through a Sphaleron



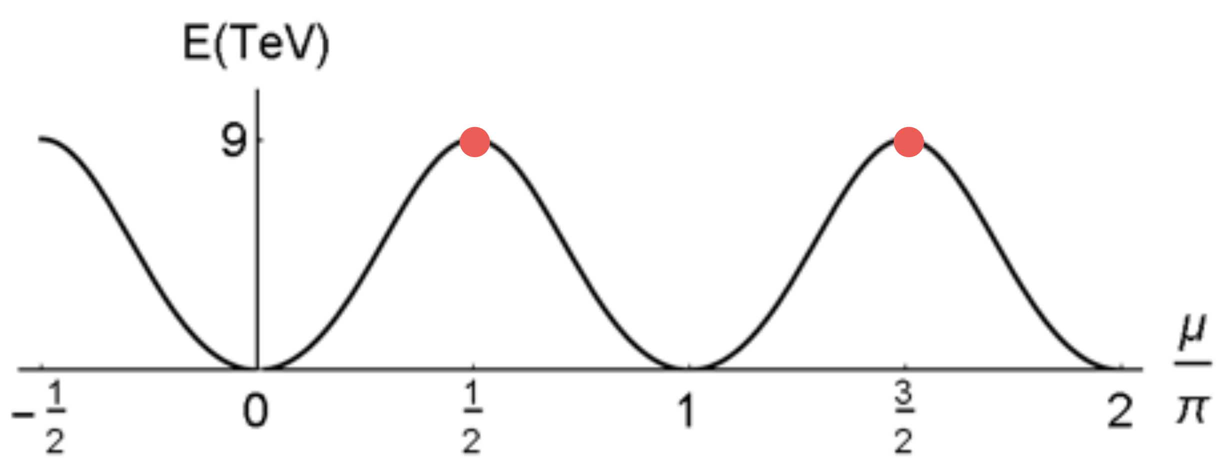
Transition through a Sphaleron



Standard Model Sphaleron



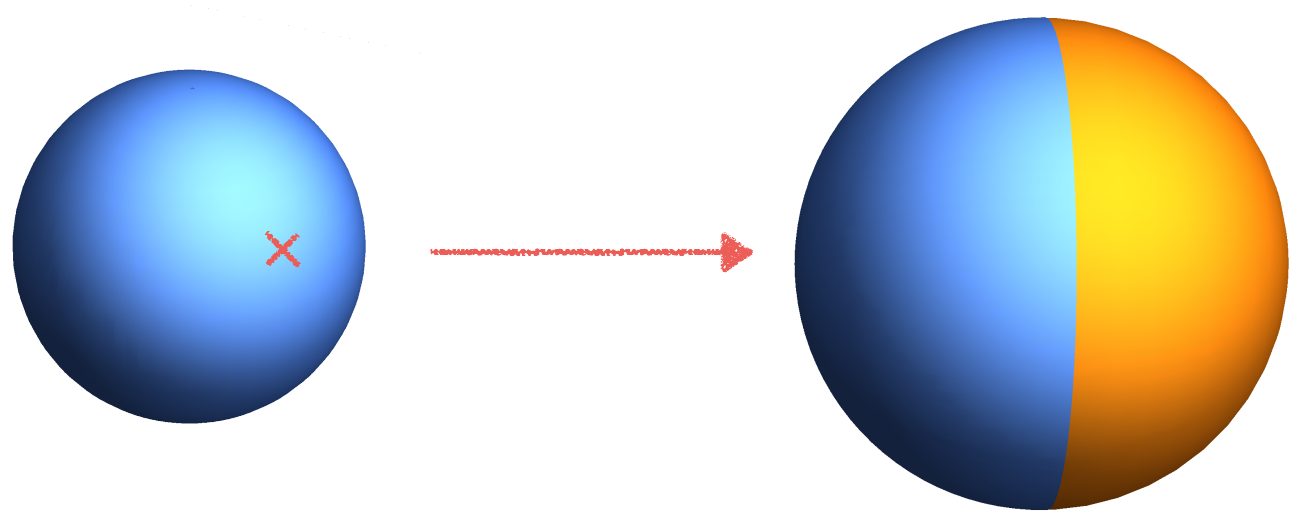
$$SO(4) \quad \Phi = \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{pmatrix} \xrightarrow{r \to \infty} v\begin{pmatrix}\hat{r}\\ 0\end{pmatrix} \longleftarrow \partial\mathbb{R}^3 = S^2$$



Topology

Vacuum	Away from vacuum	Topological number (*gauge dependent)
$A_i = \frac{i}{g} U^{-1} \partial_i U$	A_i	$N_{CS}(A) = N(t_0) = \int d^3x K^0 \big\|_{t=t_0}$ $\pi_3(S^3)$
$\hat{\Phi} = \frac{\Phi}{\|\Phi\|} \qquad \xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\mathcal{U} = \begin{pmatrix} \hat{\Phi}_1 & -\hat{\Phi}_2^* \\ \hat{\Phi}_2 & \hat{\Phi}_1^* \end{pmatrix}$	$W(\mathcal{U}) = \frac{1}{24\pi^2} \int \mathrm{Tr}(\mathcal{U} d\mathcal{U}^{-1})^3$ $\pi_3(S^3)$

$$N_{CS} = \frac{1}{2}$$



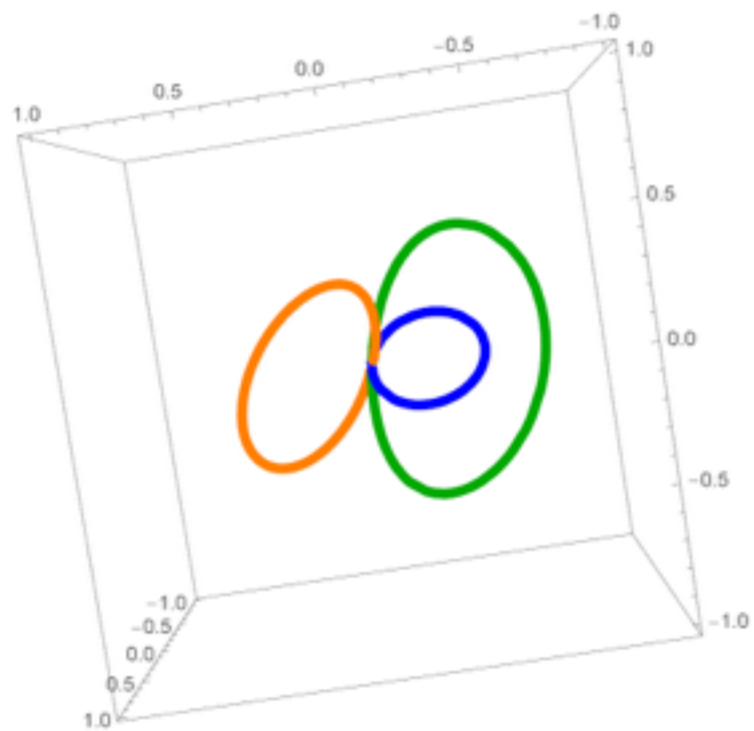
Hopf Invariant

Given any unit spinor $\hat{\Phi}$, $\hat{\Phi}^\dagger\hat{\Phi} = 1$

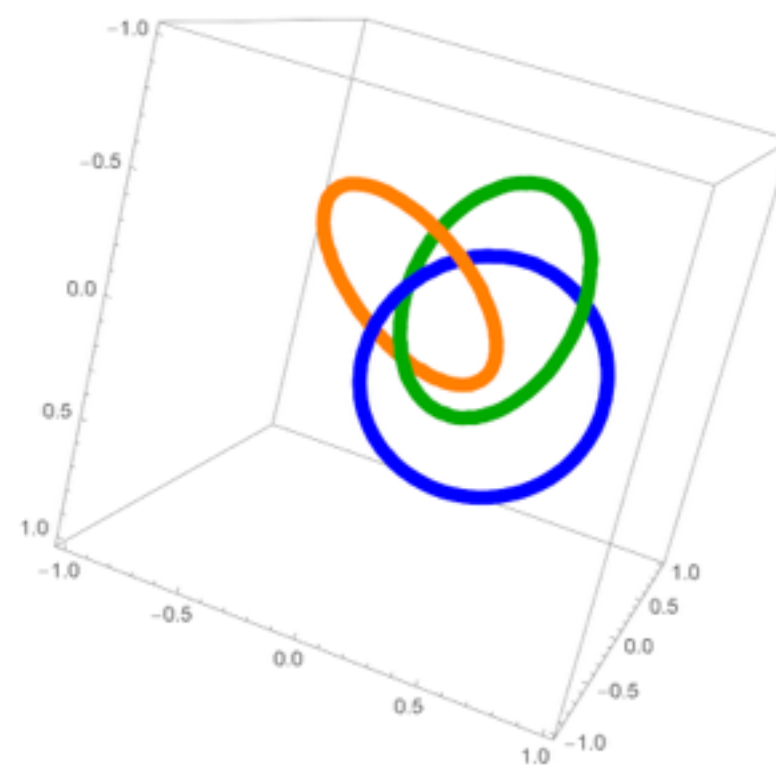
$$\eta : S^3_{\hat{\Phi}} \to S^2, \quad \hat{\Phi} \mapsto \vec{n} = \hat{\Phi}^\dagger\vec{\sigma}\hat{\Phi} \qquad \vec{n}\cdot\vec{n} = 1$$

$\eta \circ \hat{\Phi} : \mathbb{R}^3 \cup \{\infty\} \to S^2$, falls into $\pi_3(S^2)$ when the spatial infinities are identified.

$$H = \frac{1}{16\pi^2}\int a \wedge da, \qquad a = -2i\hat{\Phi}^\dagger d\hat{\Phi}$$



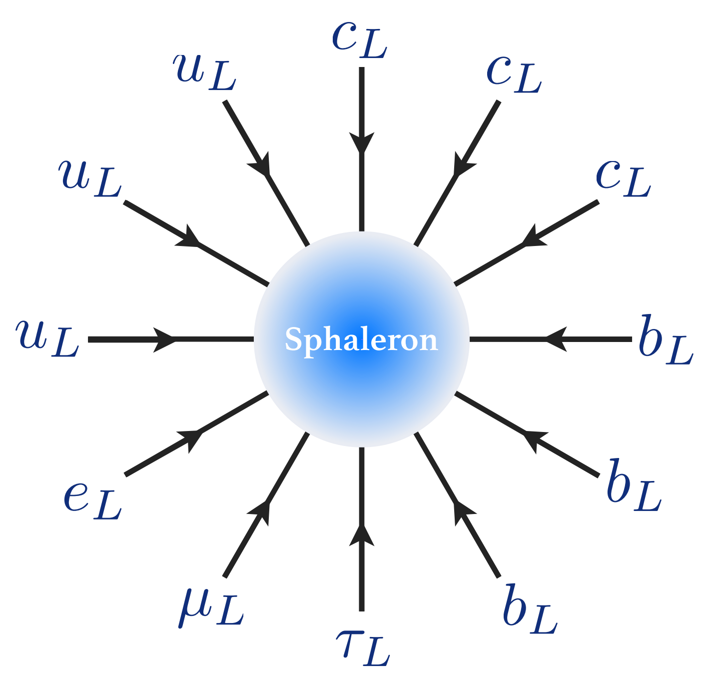
$H = \dfrac{1}{2}$, sphaleron



$H = 1$, vacuum

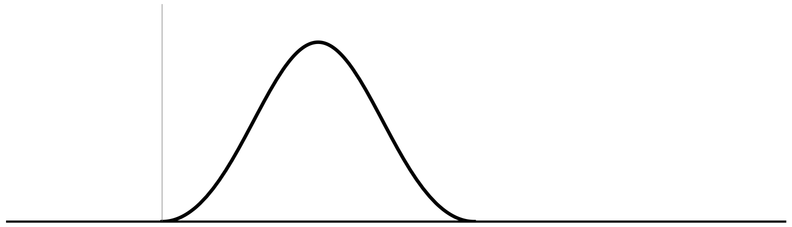
Baryon Number Violation

Due to anomaly:



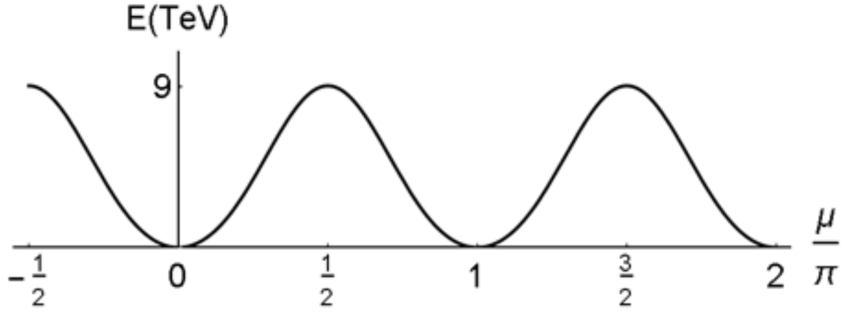
Single barrier tunnelling is highly exponentially suppressed

Espinosa (1990), Ringwald (1990), Mueller (1991), Khoze and Ringwald (1991), Zhaharov (1992)...



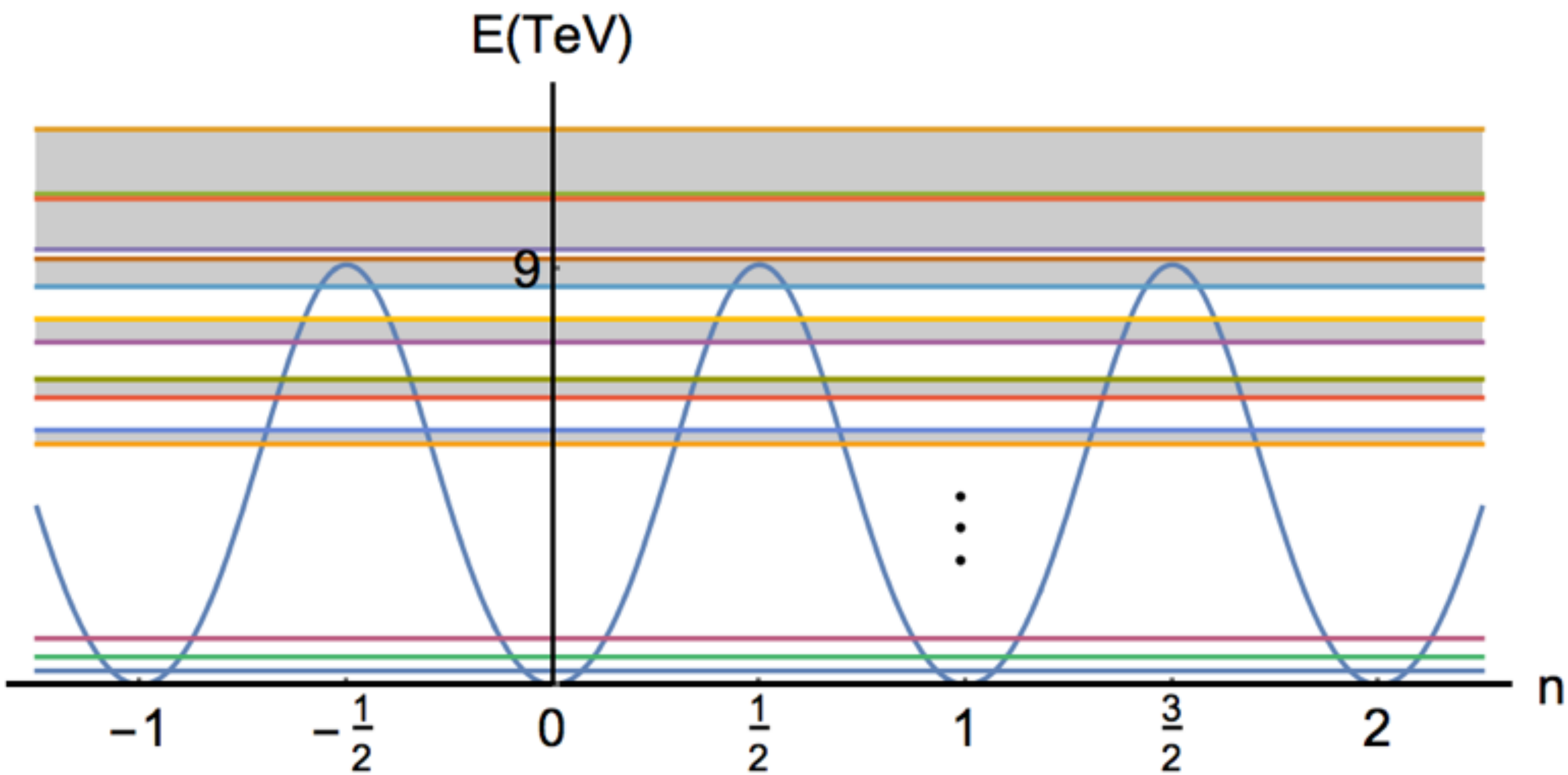
V.S.

Periodic potential & Bloch wave: tunnelling suppression maybe not there



Band Structure

Schematically:




Thank you very much!