SH^c Realization of W-algebras

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AGT conjecture

Alday-Gaiotto-Tachikawa (2009)

Proposal

partition function of 4D N=2 SYM

$\updownarrow$

conformal block in 2D CFT

Further conjecture (Gaiotto's conjecture)

Giotto (2009)

$$\langle G | G \rangle = Z_{\text{inst}}^{\text{pure gauge}}$$

the inner product of the so-called Gaiotto state can reproduce the partition function of pure gauge theory.

AFLT's proof of SU(2)

Alba, Fateev, Litvinov & Tarnopolskiy (2011)

$$\frac{{}_{\vec{Y}}\langle \Delta | V_\alpha | \Delta \rangle_{\vec{Y}'}}{\langle \Delta | V_\alpha | \Delta \rangle}$$

bifundamental contribution to the partition function can be worked out with orthonormal basis of U(1)xVirasoro labeled by two Young diagrams.

SH^c

inspired by AFLT

Schiffmann & Vasserot (2013)

Central extension of Spherical degenerate double affine Hecke algebra

obtained from spherical DAHA, which is thought to be able to describe 5D N=1 systems

Representation labeled by N Young diagrams

three key families of generating operators

$D_{1,l}$ — adding one box to one Young diagram

$D_{-1,l}$ — removing one box from one Young diagram

$D_{0,l}$ — extracting the product of characterizing number of all boxes

This allows an easy control over the module (states).

U(1)xW structure

expected from the AGT conjecture

We can assign to each Young diagram a parameter a_l.

When these parameters satisfying some generic condition, it is proved that

- the module is irreducible

rep. of W without null states

- representation with N Young diagrams is isomorphic to the highest rep. of U(1)xW_N

Schiffmann & Vasserot (2013)

For instance, we can always see the identification of U(1)xVirasoro holds

$$\alpha_l = \frac{1}{\sqrt{\beta^{l-1}E_0}} D_{-l,0}\,, \quad \alpha_{-l} = \frac{1}{\sqrt{\beta^{l-1}E_0}} D_{l,0}\,, \quad \text{for } l > 0\,, \quad \alpha_0 = \frac{E_1}{\sqrt{\beta E_0}}\,,$$

$$L_l = \frac{1}{l\sqrt{\beta^l}} D_{-l,1} + \frac{(1-l)}{2} E_0^{3/2} \left(\frac{1}{\sqrt{\beta}} - \sqrt{\beta}\right) \alpha_l\,,$$

$$L_{-l} = \frac{1}{l\sqrt{\beta^l}} D_{l,1} + \frac{(1-l)}{2} E_0^{3/2} \left(\frac{1}{\sqrt{\beta}} - \sqrt{\beta}\right) \alpha_{-l}\,,$$

$$L_0 = \frac{1}{2\beta} E_2 = D_{0,1} + \frac{1}{2\beta} \left(c_2 + c_1(1-c_0)\xi + \frac{1}{6} c_0(c_0-1)(c_0-2)\xi^2\right)\,,$$

with central charge

$$c = 1 + (N-1)\left(1 - \left(\sqrt{\beta} - \frac{1}{\sqrt{\beta}}\right)(N^2+N)\right)$$

U(1)

W_N

minimal model realization

When the generic condition does not hold, the rep. becomes reducible (we have subspace generated by singular vectors)

There is no rigorous proof, but we have strong evidence that at parameters corresponding to minimal models, the rep. structure is again isomorphic to $U(1)\times W_N$.

Fukuda, Matsuo, Nakamura & NZ (2015)
and work in progress

- module structure
  the position of singular vectors agrees with W_N minimal model
  we can prove that N-Burge conditions are all required to restricted the module
- existence of level-rank duality
  at special values of parameters, the triality structure of SHc reduces to one trivial symmetry ($\beta \to 1/\beta$) and the level-rank duality
  a state-to-state correspondence can be established
- matching of the character formulae
  when there is level-rank duality or in the large N 't Hooft limit

a rough description of the computation of characters

The key ingredient is the N-Burge condition.

$$\lambda(x-p) \le \lambda(x)$$

requirement that states are labeled by Young diagrams

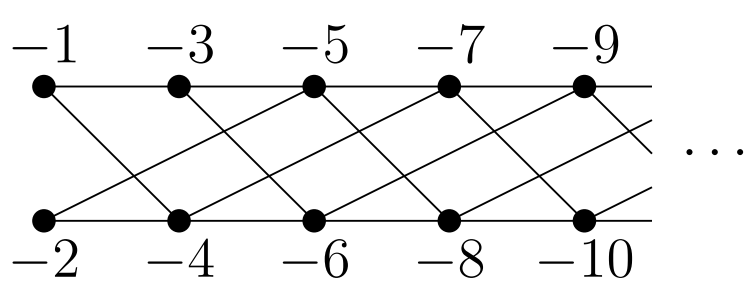
$$\lambda(x - q\tilde{n}'_i - (q-p)) \le \lambda(x) + \tilde{n}'_i$$

more nontrivial conditions

Level-rank duality

$$\lambda(x-N) \le \lambda(x)\,, \quad \lambda(x-M) \le \lambda(x)\,.$$

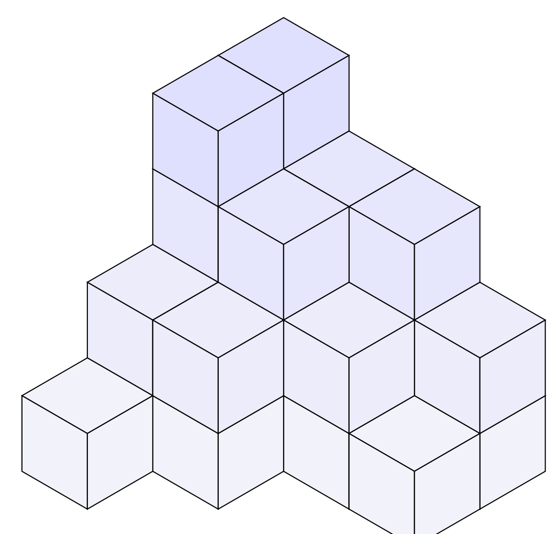
there is a dual Young-diagram picture of the same theory.



Partially ordered sets

Large N 't Hooft limit

3D analogy of Young diagram



in general, known as McMahon rep. in Mathematics

e.g. vacuum rep. (0,0) ⟷ MacMahon function

Summary

- We expect that SH^c is isomorphic to U(1)xW for <u>all</u> parameters. (Unification of all A-type W-algebras)
- In contrast to the usual W-algebras, the commutation relations are hard to establish, while it is more convenient to investigate states (their correspondence etc.) and characters. as it was constructed for such purposes
- Might also be useful in other contexts. (?) see Prochazka (2015)

  We need more evidence.

Thank you very much for your listening!