The shear viscosity in anisotropic phases

Rickmoy Samanta

Tata Institute of Fundamental Research, Mumbai,
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Work done with Prof. Sandip Trivedi (TIFR) and Dr. Sachin Jain (Cornell University)

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- AdS-CFT has been very useful to study strongly coupled systems.
- KSS gave a bound for systems with gravity duals.

$$\frac{\eta}{s} \geq \frac{1}{4\pi}.$$

- Above result true for homogenous and isotropic phases.
- Question : What about gravitational backgrounds which correspond to anisotropic phases in the field theory?
- Result: Parametric violation of KSS bound for some viscosity components.
- Breaking of isotropy is due to an externally applied force which is translationally invariant

Example 1: Einstein gravity with one dilaton

- Action:

$$S_{bulk} = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\,\left(R + 12\Lambda - \frac{1}{2}(\partial\phi)^2\right).$$

$\phi = \rho\, z$

- At zero temperature the near horizon solution is $AdS_4 \times R$,

$$ds^2 = -\frac{4}{3}u^2 dt^2 + \frac{du^2}{\frac{4}{3}u^2} + \frac{4}{3}\; u^2(dx^2 + dy^2) + \frac{\rho^2}{8}dz^2.$$

- At small temperature, $T \ll \rho$, the geometry is that of a Schwarzschild black brane in $AdS_4 \times R$

$$-\frac{4}{3}u^2(1-\frac{\pi^2 T^2}{u^2})dt^2+\frac{1}{\frac{4}{3}u^2(1-\frac{T^2\pi^2}{u^2})}du^2+\frac{4}{3}\; u^2(dx^2+dy^2)+\frac{\rho^2}{8}dz^2.$$

Computation of viscosity

- In the limit $\omega \to 0$.

$$\eta_{\perp} = \lim_{\omega \to 0} \frac{\Pi(u,\omega)}{i\,\omega\,Z(u,\omega)}\bigg|_{u \to u_H}.$$

  - The entropy density is

$$s = \frac{1}{4G}\frac{\sqrt{-g}}{\sqrt{-g_{uu}g_{tt}}}\bigg|_{u_H}.$$

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$$\frac{\eta_{\perp}}{s} = \frac{1}{4\pi}\frac{g_{xx}}{g_{zz}}\bigg|_{u_H}.$$

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$$\frac{\eta_{\perp}}{s} = \frac{8\pi T^2}{3\rho^2},$$

Example 2: Einstein gravity with 2 dilatons

- Action:

$$S_{bulk} = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left(R + 12\Lambda - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2}(\partial\phi_2)^2 \right).$$

- $$\phi_1 = \rho_1 y, \quad \phi_2 = \rho_2 z.$$

- The zero temperature near horizon solution is now given by $AdS_3 \times R \times R$
- At small temperature, $T \ll \rho$, the geometry is that of a Schwarzschild black brane in $AdS_3 \times R \times R$
- $$\frac{\eta_{xz}}{s} = \frac{1}{4\pi} \frac{g_{xx}}{g_{zz}} \bigg|_{u_H} . \qquad (1)$$

Example 3: Einstein gravity with uniform magnetic field

- Action:

$$S = \int d^5x\sqrt{-g}(R + 12\Lambda - \frac{1}{4}F^2)$$

- The resulting value for the viscosity is given by

$$\frac{\eta_\perp}{s} = \frac{1}{4\pi}\frac{g_{xx}}{g_{zz}}$$

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$$\frac{\eta_\perp}{s} = \frac{2}{\sqrt{3}}\pi\frac{T^2}{B^2}$$

Example 4:Dilaton Axion system

- Action:

$$S_{bulk} = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\ \left(R + 12 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\alpha\phi}(\partial\chi)^2\right),$$

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$$ds^2 = R^2\left(-u^2dt^2 + \frac{du^2}{u^2} + u^2dx^2 + u^2dy^2 + \rho^2\ u^{\frac{4\alpha^2}{1+2\alpha^2}}dz^2\right),$$

$$\chi = c_1\ \rho\ z,$$

$$\phi \sim \log(u),$$

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$$\frac{\eta_\perp}{s} = \frac{1}{4\pi}\frac{g_{xx}}{g_{zz}} \sim (\frac{T}{\rho})^{\frac{2}{1+2\alpha^2}} \qquad (2)$$

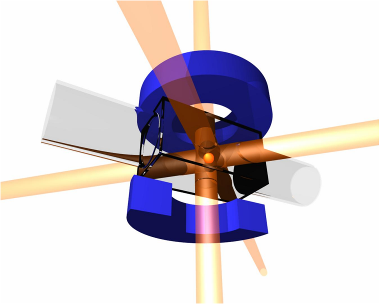
General lesson learnt from the examples

- All examples feature breaking of isotropy due to matter fields which give rise to a spatially constant driving force.
- We had a residual AdS symmetry and a corresponding Lorentz symmetry is left intact in the boundary theory.
- z be a spatial direction in the boundary theory along which there is anisotropy.
- x be a spatial direction along which the boost symmetry is left unbroken
- $$\frac{\eta_{xz}}{s} = \frac{1}{4\pi}\frac{\hat{g}_{xx}}{\hat{g}_{zz}}\Big|_{u=u_h}.$$

Dimensional reduction

- Starting from the original $D+1$ dimensional theory , go down to the AdS_{d+1} space-time.
- Different Kaluza Klein (KK) modes in the extra dimensions will not mix .
- The off diagonal components of the metric $\rightarrow$ gauge fields in the dimensionally reduced theory.
- Conductivity thus related to viscosity.
- Parametric violation of KSS understood in terms of conductivity.

Can we probe this feature in ultracold atoms/ QGP? Work in progress..



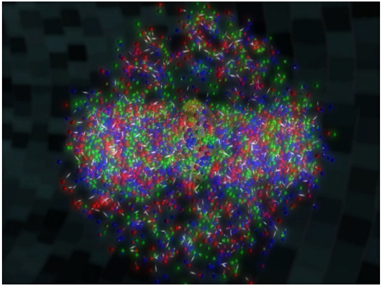


Figure : **Left :** cold atoms represented by the glowing orange ball. Six laser beams provide the cooling force necessary to slow atoms down from the background room temperature gas and quadrupole magnets for confinement. **Right:** QGP at RHIC. Photo credits : http://saaubi.people.wm.edu, RHIC

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Thank You