A ChargeD Membrane Paradigm

Mangesh Mandlik
DTP, TIFR

Based on: 1511.03432 and
the work in progress
with S. Bhattacharyya, K. Inbasekar, U. Mehta, S. Minwalla, U. Sharma and S. Thakur

Asian Winter School 2016, OIST
Januray 9, 2016

Introduction

- Emparan, Suzuki, Tanabe (EST) and collaborators have recently noted that the classical dynamics of black holes simplifies when D, the dimensionality of space time is taken to be large .
- The spacetime becomes effectively flat at distance of the order $\delta r = r_0/D$ from the horizon of a D dimensional RN black hole in this limit, where $r_0 =$ radius of its horizon.
- the RN black holes in large D are characterised by **two** length scales: r_0 and δr, and this separation also shows up in the spectrum of their quasinormal modes.

Introduction

- EST showed that most of the quasinormal modes are 'heavy', with frequencies $\mathcal{O}(1/\delta r)$ while the remaining are anomalously light, with frequencies $\mathcal{O}(1/r_0)$.
- For all these modes the imaginary parts are of the same order in D as their real parts.
- That means in a dynamical process the heavy modes decay much more quickly than their light counterparts. And the dynamics at late times $t >> \delta r$ is governed by a nonlinear interacting theory of the light modes which are just a handful in number.

Table of Contents

What is this membrane anyway?

- As mentioned earlier, the black hole in large D has non-flat metric only up to $\mathcal{O}(r_0/D)$ outside the horizon, while the region inside the horizon is inconsequential since it is causally disconnected from the region of interest.

- Therefore these spacetimes are shown to be in one-to-one correspondence with the configurations of a non-gravitational codimension one surface propagating in flat D dimensional space. This surface is called a **membrane**.

- The dynamical degrees of freedom of this membrane are: the shape of the membrane, a velocity field u which lies entirely on the membrane and a scalar charge density Q.

and what are these 'equations'?

- It turns out that the metric and the gauge field within distance of $\mathcal{O}(\delta r)$ outside the horizon (i.e. the **membrane region**) can be written entirely out of these quantities (shape function, u and Q) and their derivatives.
- The derivatives of the shape function that will be useful here are n (the unit normal to the membrane), K (the extrinsic curvature of the membrane) and $\mathcal{K}$ (Trace of K).

- These solutions are regular at the membrane (i.e. the horizon) upto the subleading order in $1/D$ if and only if the membrane satisfied the equations

$$\left(\frac{\nabla^2 u}{\mathcal{K}} - (1 - Q^2)\frac{\nabla \mathcal{K}}{\mathcal{K}} + u \cdot K - (1 + Q^2)(u \cdot \nabla)u\right) \cdot \mathcal{P} = 0,$$

$$\frac{\nabla^2 Q}{\mathcal{K}} - u \cdot \nabla Q + Q\left(\frac{u \cdot \nabla \mathcal{K}}{\mathcal{K}} - u \cdot K \cdot u\right) = 0,$$

where $\nabla =$ the covariant derivative on the membrane world volume,

and $\mathcal{P}_{AB} = \eta_{AB} - n_A n_B + u_A u_B$.

These are called the **'membrane equations of motion'**.

Table of Contents

Zeroth order ansatz

We know that the RN blackhole boosted with velocity u

$$ds^2 = ds^2_{flat} + \left((1+Q^2)\frac{1}{\rho^{D-3}} - Q^2\frac{1}{\rho^{2(D-3)}}\right) O_M O_N dx^M dx^N,$$

$$A_M = \frac{\sqrt{2} Q O_M}{\rho^{D-3}},$$

$$O = n - u, \quad u = \text{const}, \quad u\cdot u = -1, \quad \rho = \frac{r}{r_0},$$

$$r^2 = P_{MN}x^M x^N, \quad P_{MN} = \eta_{MN} + u_M u_N, \quad n = r_0 d\rho, \quad u\cdot n = 0.$$

Solves the Einstein-Maxwell equations. So we perturb around this solution in $1/D$.

Metric and gauge field upto subleading order

$$h_{MN} = F(\rho) O_M O_N + H^{(T)}_{MN} + 2 O_{(M} H^{(V)}_{N)} + H^{(S)} O_M O_N + H^{(Tr)} \mathcal{P}_{MN}$$

$$A_M = \sqrt{2} Q \ \rho^{-(D-3)} \ O_M + \left(A^{(S)} O_M + A^{(V)}_M \right),$$

where

$$F(\rho) = \left[(1+Q^2)\rho^{-(D-3)} - Q^2 \rho^{-2(D-3)} \right],$$

$$\mathcal{P}_{MN} = \eta_{MN} - O_M n_N - O_N n_M + O_M O_N,$$

$$\mathcal{P}^{MN} H^{(V)}_N = \mathcal{P}^{MN} A^{(V)}_N = 0, \quad \mathcal{P}^{MN} H^{(T)}_{MQ} = 0, \quad \mathcal{P}^{MN} H^{(T)}_{MN} = 0,$$

where

$$
\begin{aligned}
A^{(V)}_M = & -\left(\frac{2\sqrt{2}}{D}\right) Q\rho^{-D}\Big[D(\rho-1)(\mathfrak{V}_{(1)} - Q^2\ \mathfrak{V}_{(2)}) \\
& - Q^2[1+\log(1-\rho^{-D}Q^2)]\mathfrak{V}_{(2)}\Big]_M + \mathcal{O}\left(\frac{1}{D}\right)^2, \\
A^{(S)} = & \left(\frac{1}{D}\right)\Big[\sqrt{2}\ Q\ D(\rho-1)\ \rho^{-D}\mathfrak{S}_{(1)} \\
& + 2\sqrt{2}\left(\frac{Q^3}{1-Q^2}\right)\rho^{-D}\ \Upsilon_A(\rho)\ \mathfrak{S}_{(2)}\Big] + \mathcal{O}\left(\frac{1}{D}\right)^2.
\end{aligned}
$$

$$H^{(T)}_{MN} = \left(\frac{2}{D}\right) \log(1 - Q^2 \rho^{-D}) \ \mathfrak{T}_{MN} + \mathcal{O}\left(\frac{1}{D}\right)^2,$$

$$\begin{aligned} H^{(V)}_{M} = &\left\{ Q^2 \left[(F(\rho) - \rho^{-(D-3)}) + (F(\rho) - 1) \log(1 - Q^2 \rho^{-D}) \right] \mathfrak{V}_{(2)M} \right. \\ &\left. - D(\rho - 1) F(\rho) \ [\mathfrak{V}_{(1)} - Q^2 \ \mathfrak{V}_{(2)}]_M \right\} \left(\frac{2}{D}\right) + \mathcal{O}\left(\frac{1}{D}\right)^2 . \end{aligned}$$

$$\begin{aligned} H^{(S)} = & -\sqrt{2} Q \ \rho^{-D} A^{(S)} + \left(\frac{1}{D}\right) \left[\rho^{-(D-3)} - F(\rho) \right] \\ & + \left(\frac{2}{D}\right) \rho^{-D} \left[Q^2 \ D(\rho - 1) \ \mathfrak{S}_{(1)} + \Upsilon_H(\rho) \mathfrak{S}_{(2)} \right] + \mathcal{O}\left(\frac{1}{D}\right)^2 , \end{aligned}$$

$$H^{(Tr)} = \mathcal{O}\left(\frac{1}{D}\right)^3,$$

where

$$F(\rho) = \left[(1+Q^2)\rho^{-(D-3)} - Q^2\rho^{-2(D-3)}\right],$$

$$\Upsilon_A(\rho) = \int_0^{D(\rho-1)} dx \ \log(1-Q^2e^{-x}),$$

$$\Upsilon_H(\rho) = \Bigg[(\rho^D - Q^2)\log(1-Q^2\rho^{-D}) - (1-Q^2)\log(1-Q^2) + Q^2\left(\frac{1+Q^2}{1-Q^2}\right)\Upsilon_A(\rho)\Bigg].$$

Membrane equations as dynamical equations

- Total number of dynamical variables on the membrane are $D-1$.
- Total number of membrane equations is also $D-1$.
- Hence these equations define an initial value problem for these variables which can be solved for the dynamics of the membrane and hence for the corresponding black hole solution.
- It can be easily shown that the RN black hole is an exact solution of the membrane equations of motion.

Table of Contents

Light quasinormal modes of an RN black hole

- Now we perturb the membrane equations about the RN black hole solution.

- Solutions of the linearized equations have the frequencies

$$\omega_l^r = \frac{-i(l-1) \pm \sqrt{(l-1)(1-lQ_0^2)}}{1+Q_0^2},$$

$$\omega_l^Q = -il,$$

$$\omega_l^v = \frac{-i(l-2)}{1+Q_0^2},$$

- where Q_0 is the charge density parameter for the RN black hole and l is the spherical harmonic number .

- So, for given l there are only 3 scalar (two ω_l^r and one ω_l^Q) and 1 vector (ω_l^v) quasinormal modes.
- So for every l the light quasinormal modes are finite in number, as desired.
- In the chargeless limit ($Q_0 \to 0$) these frequencies reduce to the frequencies reported in 1406.1258 and 1504.06613 for LQNM of the Schwarzschild black hole (and ω_l^Q is absent).

Table of Contents

Defining conserved currents

- Our method gives the metric and gauge field inside the membrane region.
- While outside the membrane region the gauge field and the deviation of the metric from flatness are very small, so Einstein-Maxwell equations are linear in this region.
- So for the solutions in these two regions to be consistent, the membrane region solution must produce a conserved charge current and stress tensor which produce the gauge field and the metric resp. outside the membrane region.

Charge Current

- The constraint Maxwell equations $\nabla_\mu F^{\mu\rho} = 0$ suggest that there is a conserved current on the membrane

$$J^\mu_Q = \frac{\sqrt{g}}{\sqrt{g_{red}}} F^{\mu\rho}.$$

- This current is evaluated using the metric and gauge filed listed above.

$$J = -\frac{2\mathcal{K}Q}{\rho^{D-2}}u + \frac{Q}{\rho^{D-3}}(u\cdot\nabla u)\cdot P + \frac{\mathcal{K}}{D\rho^{D-3}}dQ$$
$$-\frac{Q}{\rho^{D-3}}D(\rho-1)\left((1-Q^2)\frac{\nabla\mathcal{K}}{\mathcal{K}} + (1+Q^2)u\cdot\nabla u\right)\cdot P$$

Membrane equations from the Charge current conservation

- The conservation equation for this current on the membrane yields

$$\nabla \cdot J = \frac{1}{\rho^{D-3}} \left[\frac{\nabla^2 Q}{\mathcal{K}} - u \cdot \nabla Q - Q \left(\frac{u \cdot \nabla \mathcal{K}}{\mathcal{K}} - u \cdot K \cdot u \right) \right] - \frac{Q}{\rho^{D-3}} D(\rho - 1) \left[(1 - Q^2) \left(\frac{\nabla^2 \mathcal{K}}{\mathcal{K}^2} - \frac{u \cdot \nabla \mathcal{K}}{\mathcal{K}} \right) - (1 + Q^2) \left(\frac{u \cdot \nabla \mathcal{K}}{\mathcal{K}} - u \cdot K \cdot u \right) \right]$$

- Note that the first line is the scalar membrane equation while the rest is proportional to the divergence of the vector equation.
- Hence the membrane equations are actually demanded by the conservation of charge current.

Stress tensor and entropy current

- Similarly we believe that the membrane equations indicate the conservation of Brown-York stress tensor.

$$T^{\mu\nu} = \frac{\mathcal{K}}{D} u^\mu u^\nu + \mathcal{O}\left(\frac{1}{D}\right)$$

- And the entropy current can be defined on the membrane by Hodge dualising the area form on it. (See 0803.2526)

$$J_S^\mu = u^\mu + \mathcal{O}\left(\frac{1}{D}\right)$$

- These currents can be used to compute the charges which can be verified to satisfy the first law of BH thermodynamics.

Table of Contents

Radiation

- The charge current and the stress tensor definded on the membrane can be considered as the sources for the gravitational and Maxwell fields outside the membrane.
- These sources convoluted with the respective retarded Green's functions will presumably give the gravitational and electromagnetic radiation.
- Although the sources are of unit magnitude, the radiation is very feeble, with the magnitude of order $1/D^D$. This is a kinematical fact about large number of dimensions, and has nothing to do with other specifics of the system.

Acknowledgements

I would like to thank

- Prof. Shiraz Minwalla
- Dr. Sayantani Bhattacharyya
- Dr. Arunabha Saha
- Dr. Somyadip Thakur
- Subhajit Mazumdar
- Yogesh Dandekar
- Anandita De
- Dr. Karthik Inbasekar
- Umang Mehta
- Utkarsh Sharma

Thank you!

Major references:

- "A Charged Membrane Paradigm at Large D", 1511.03432
- Bhattacharyya et. al. 1504.06613 (It's free of charge.)
- EST: 1406.1258 etc. (Or just type 'large D' in google)
- Entropy current: 0803.2526