Quantum Entanglement of Excited States by Heavy Local Operators in Large-c CFT at Finite Temperature and the Scrambling Time

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How Fast is A Quantum System Scrambled (Mixed up) ?

Any small subsystems become (nearly) maximal entangled

($|A| <$ half) → The system is "Scrambled"

How fast are small perturbations to the highly mixed system relaxed ?

→ "Scrambling Time"

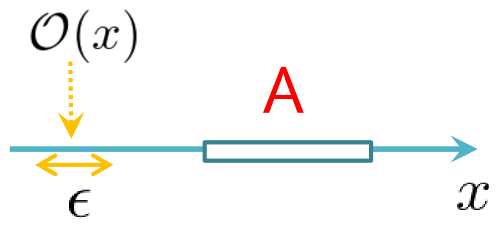
Explicit study in the case of Large-c CFT (& its holographic dual)

Motivations

Entanglement Entropy (EE) for excited states by local operators

$$S_A = -\mathrm{Tr}\rho_A \log \rho_A$$

$$\rho = \mathcal{O}(x)|\Psi\rangle\langle\Psi|\mathcal{O}^\dagger(x)$$
$$\rho_A = \mathrm{Tr}_{A^c}\ \rho$$



$t = -t_\omega$ Excite

$t = 0$ Observe S_A

→ **Can we characterize the local operators from EE ?**

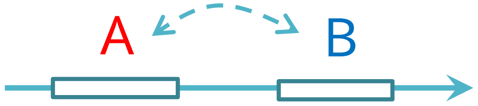
Free scalar	[Nozaki-Numasawa-Takayanagi, Nozaki 14]
RCFT	[He-Numasawa-Takayanagi-KW 14]
Large-N	[Caputa-Nozaki-Takayanagi 14]
Large-c	[Asplund-Bernamonti-Galli-Hartman 14]
Finite T	[Caputa-Simon-Stikonas-Takayanagi 14]

etc...

→ **Next! Large-c & Finite T !** (Heavy local operator)

Mutual Information (MI)

$$I_{A:B} = S_A + S_B - S_{A\cup B} \geq 0$$



$$\geq \frac{(\langle \mathcal{O}_A \mathcal{O}_B \rangle - \langle \mathcal{O}_A \rangle \langle \mathcal{O}_B \rangle)^2}{2||\mathcal{O}_A||^2||\mathcal{O}_B||^2}$$

→ Upper bound for the connected 2-pt functions !

(for finite dimensional Hilbert space)

[Wolf-Verstraere-Hastings-Cirac 08]

→ **When does MI vanish ?** $I_{A:B} = 0$

Scrambling Time

[Hayden-Preskill 07] [Sekino-Susskind 08] …

Fast Scrambling

$$t^*_\omega / \beta \sim \log S < S^{1/d}$$

→ **The minimum time required for the information about the initial state to be lost**

Holographic Calculation

[Shenker-Stanford 14]

$$I_{A:B}(t^*_\omega) = 0$$

MI is useful measure.

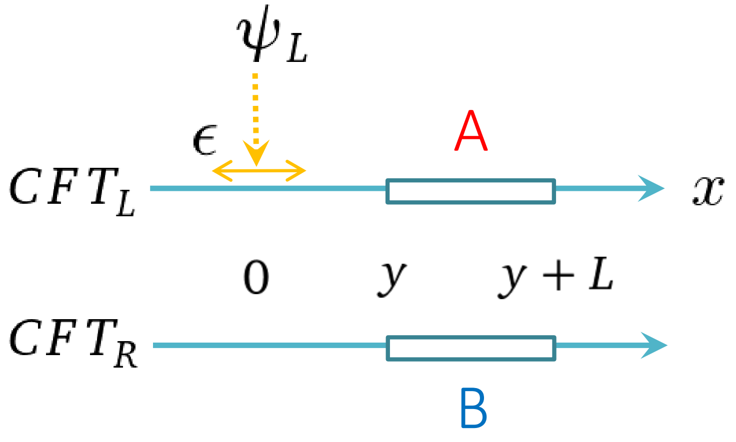
→ **How about Large-c CFT side ?**

Setup

- Large-c 2d CFT $c \to \infty$ (+ sparse spectrum)
- Thermo-field double (TFD) state

$$|TFD\rangle_\beta = \sum_n e^{-\frac{\beta}{2}E_n} |n\rangle_L |n\rangle_R$$

$\mathrm{Tr}_R |TFD\rangle\langle TFD|$ → Thermal density matrix in CFT_L



- Heavy local operator $\psi_L(x=0, t=-t_\omega)$ (primary)

$h_\psi \sim O(c)$: Heavy h_ψ / c : fixed

$t = -t_\omega$ Excite by ψ_L

$t = 0$ Observe S_A $I_{A:B}$

- Time evolution $e^{-it(H_L - H_R)}$

$|TFD\rangle_\beta \to |TFD\rangle_\beta$ Invariant ! (isometry in the dual geometry)

$\psi_L \to e^{-itH_L} \psi_L e^{+itH_L}$ →

$$\rho = \mathcal{N} \cdot \psi_L(-t_\omega) |TFD\rangle_\beta \langle TFD|_\beta \, \psi_L^\dagger(-t_\omega)$$

Strategy

n replica fields

$$\Psi_L = \underbrace{\psi_L^{(1)} \cdots \psi_L^{(n)}}$$

σ_n $\tilde{\sigma}_n$ Twist operators

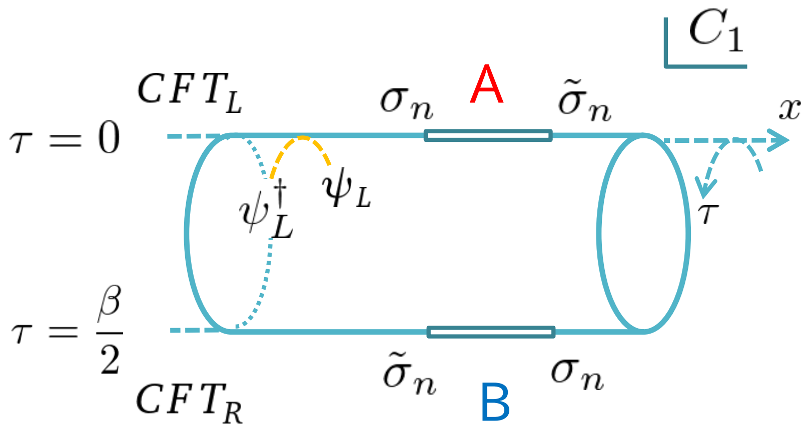
- Replica method

First compute this!

$$S_A^{(n)} = -\frac{1}{n-1} \log \mathrm{Tr}\rho_A^n \xrightarrow{n \to 1} S_A = -\mathrm{Tr}\rho_A \log \rho_A$$

$$\mathrm{Tr}\rho_A^n = \frac{\langle \Psi_L \sigma_n^A \tilde{\sigma}_n^A \Psi_L^\dagger \rangle_{C_1}}{(\langle \psi_L \psi_L^\dagger \rangle_{C_1})^n} \qquad \mathrm{Tr}\rho_B^n = \frac{\langle \Psi_L \sigma_n^B \tilde{\sigma}_n^B \Psi_L^\dagger \rangle_{C_1}}{(\langle \psi_L \psi_L^\dagger \rangle_{C_1})^n}$$

$$\mathrm{Tr}\rho_{A\cup B}^n = \frac{\langle \Psi_L \sigma_n^A \tilde{\sigma}_n^A \sigma_n^B \tilde{\sigma}_n^B \Psi_L^\dagger \rangle_{C_1}}{(\langle \psi_L \psi_L^\dagger \rangle_{C_1})^n}$$



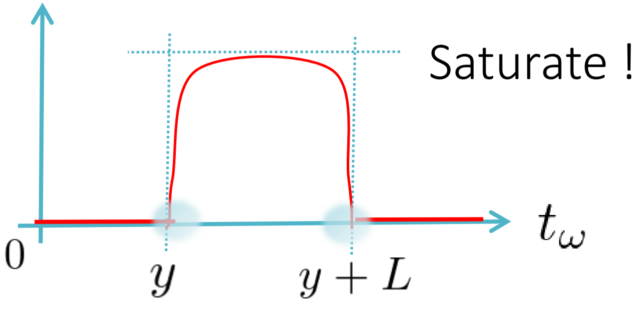
→ Main Task : Calculate the 4-pt and 6-pt functions on C_1 in the limit $n \to 1$, $c \to \infty$

- Large-c calculation ... [Zamoldchikov 87] [Fitzpatrick-Kaplan-Walters 14 15] [Asplund-Bernamonti-Galli-Hartman 14]

Results

— Entanglement Entropy

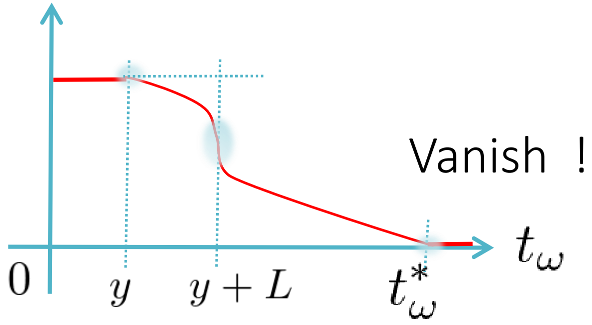
$$\Delta S_A = S_A(t_\omega) - S_A(0)$$



$(\Delta S_A)_{\max}$ depends on $\alpha_\psi = \sqrt{1 - \frac{24 h_\psi}{c}}$

— Mutual Information

$$I_{A:B} = S_A + S_B - S_{A\cup B} \geq 0$$



Monotonically decreasing

— Scrambling Time $I_{A:B}(t_\omega^*) = 0$

$$t_\omega^* \sim y + \frac{L}{2} - \frac{\beta}{2\pi}\log\left(\frac{\sin \pi\alpha_\psi}{\alpha_\psi}\right) + \frac{\beta}{\pi}\log\left(2\sinh\frac{\pi L}{\beta}\right)$$

(for large t_ω^*)

$\alpha_\psi \ll 1$ → $+\frac{\beta}{2\pi}\log\left(\frac{\pi S_{density}}{4E_\psi}\right)$

$$E_\psi = \frac{\pi h_\psi}{\epsilon} \qquad S_{density} = \frac{\pi c}{3\beta}$$

cf. [Roberts-Stanford 14]

(Read from 2-pt function)

Fast Scrambling Time !

Same as the holographic results

[Shenker-Stanford 14]

Holographic Calculation

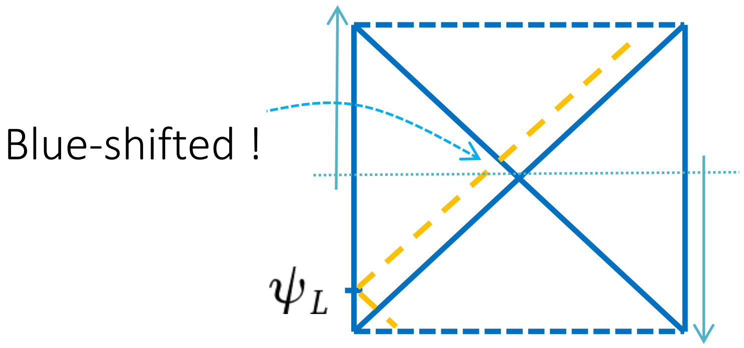
By using Ryu-Takayanagi formula,

we can also compute in the holographic model

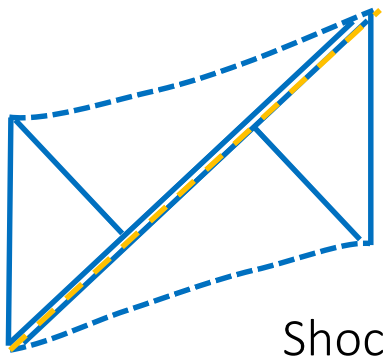
→ **Perfect matching to the Large-c 2d CFT (leading) results !**

The dual geometry:

Free falling particle in eternal BTZ BH
(including the back-reaction)



$\approx$ for large t_ω

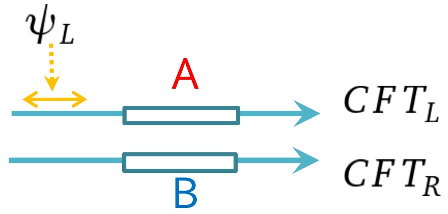


Shock wave geometry

[Nozaki-Numasawa-Takayanagi 13]
[Caputa-Simon-Stikonas-Takayanagi 14],…

[Shenker-Stanford,Susskind,
Roberts-Stanford… 13 14]

Summary & Future Directions

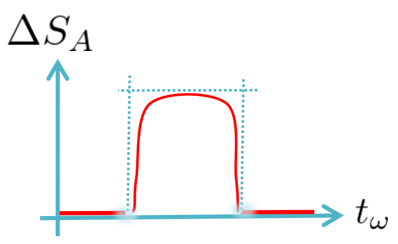


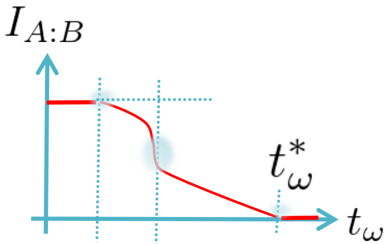
Large-c 2d CFT $c \to \infty$

Heavy local op $h_\psi \sim O(c)$

Finite T $|TFD\rangle_\beta$

$$\rho = \mathcal{N} \cdot \psi_L(-t_\omega)|TFD\rangle_\beta \langle TFD|_\beta \, \psi_L^\dagger(-t_\omega)$$





First explicit result of scrambling time in CFT side !!

cf. [Roberts-Stanford 14] (Read from 2-pt function)

Matching to the Holographic Results !!

Extension of [Shenker-Stanford 14]

$$t_\omega^* \sim f(\beta, L) + \frac{\beta}{2\pi} \log\left(\frac{\pi S_{density}}{4E_\psi}\right)$$

(for large t_ω^*)

$$E_\psi = \frac{\pi h_\psi}{\epsilon}$$

$$S_{density} = \frac{\pi c}{3\beta}$$

1/c corrections $I_{A:B}(t_\omega^*) = 0$??

Poincare recurrence(?)

Other entanglement measures

- Complexity of states
  [Susskind, Susskind-Stanford, Roberts-Stanford… 13 14]
- Quantum Information metric
  [Miyaji-Numasawa-Shiba-Takayanagi-KW15] [Lashkari-Raamdonk15]
- Tripartite Information
  [Housur-Qi-Roberts-Yoshida 15]

… etc