# $2 \to 2$ scattering in supersymmetric matter Chern-Simons theories at large N

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# Scattering in CS matter theories

- In QFT, Crossing symmetry: analytic continuation of amplitudes.
- Particle-antiparticle scattering: obtained from particle-particle scattering by analytic continuation.
- Naive crossing symmetry leads to non-unitary $S$ matrices in U(N) Chern-Simons matter theories. [Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama]
- Consistency with unitarity required
  - Delta function term at forward scattering.
  - Modified crossing symmetry rules.
- Conjecture: Singlet channel $S$ matrices have the form

$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\text{naive}}(s,\theta)$$

- $\mathcal{T}^{S;\text{naive}}$: naive analytic continuation of particle-particle scattering.

# Scattering in $U(N)$ CS matter theories at large N

- Particle: fund rep of $U(N)$, Antiparticle: antifund rep of $U(N)$.

$$\text{Fundamental} \otimes \text{Fundamental} \rightarrow \text{Symm}(U_d) \oplus \text{Asymm}(U_e)$$
$$\text{Fundamental} \otimes \text{Antifundamental} \rightarrow \text{Adjoint}(T) \oplus \text{Singlet}(S)$$

- Eigenvalues of Anyonic phase operator $\nu_m = \frac{C_2(R_1)+C_2(R_2)-C_2(R_m)}{2\kappa}$

$$\nu_{Asym} \sim \nu_{Sym} \sim \nu_{Adj} \sim O\left(\frac{1}{N}\right), \nu_{Sing} \sim O(\lambda)$$

- symm, asymm and adjoint channels - non anyonic at large N.

- Scattering in the singlet channel is effectively anyonic at large N - naive crossing rules fail unitarity.

- Conjecture beyond large N: general form of $2 \rightarrow 2$ $S$ matrices in any U(N) Chern-Simons matter theory

$$\mathcal{S}(s,\theta) = 8\pi\sqrt{s}\cos(\pi\nu_m)\delta(\theta) + i\frac{\sin(\pi\nu_m)}{\pi\nu_m}\mathcal{T}(s,\theta)$$

# Universality and tests

- Delta function and modified crossing rules conjectured by Jain et al appear to be universal.
- Tests of the conjecture:
  - Unitarity of the S matrix.
  - 3d bosonization duality.
  - Non-relativistic limit gives unitary Aharanov-Bohm.
- All the tests had been explicitly verified for
  - $U(N)$ Chern-Simons coupled to fundamental bosons
  - $U(N)$ Chern-Simons coupled to fundamental fermions
- We tested the conjecture in $\mathcal{N} = 1, 2$ Supersymmetric $U(N)$ Chern-Simons matter theories. [K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama]

# Summary of results

- Results are in perfect agreement with 3d bosonization duality.
- Unitarity requires the delta function at forward scattering and crossing symmetry rules modified in exactly the same way for the singlet channel as conjectured by Jain et al.
- Non-relativistic limit of the $\mathcal{N} = 1$ $S$ matrix precisely reproduces the unitarized Aharanov-Bohm result.
- Substantial evidence for universality of the conjecture.
- $\mathcal{N} = 1$ $S$ matrix has interesting pole structure, in particular there exists a massless bound state.
- $\mathcal{N} = 2$ results obtained at special value of quartic scalar coupling.
- We find non-renormalization of pole mass and vertex for $\mathcal{N} = 2$ theory - good things happen with more susy.

# Future outlook

- Generalization to higher supersymmetry - mass deformed $\mathcal{N} = 3, 4, 5$, and mass deformed $\mathcal{N} = 6$ ABJ theory - in progress [K.I, S.Jain, S.Minwalla, S. Yokoyama]
- Effective field theory for the massless bound states of the $S$ matrix.
- Four point correlator of fields: plays a crucial role in computation of two, three point functions of gauge invariant currents - explicit computation in $\mathcal{N} = 2$ theory and possible $\mathcal{N} = 2$ generalization of Maldacena-Zhiboedov solutions - in progress [K.I, S.Jain, P.Nayak]
- Rigorous proof of delta function and modified crossing rules, generalisation to finite $N$ and $\kappa$.

References


- K.I, S.Jain, S.Mazumdar, S.Minwalla, V.Umesh, S.Yokoyama: Arxiv 1505.06571, *JHEP* **1510** (2015) 176
- S.Jain, M.Mandlik, S.Minwalla, T.Takimi, S.Wadia, S.Yokoyama: Arxiv 1404.6373, *JHEP* **1504** (2015) 129


**Thank You!**

# Dyson-Schwinger equations

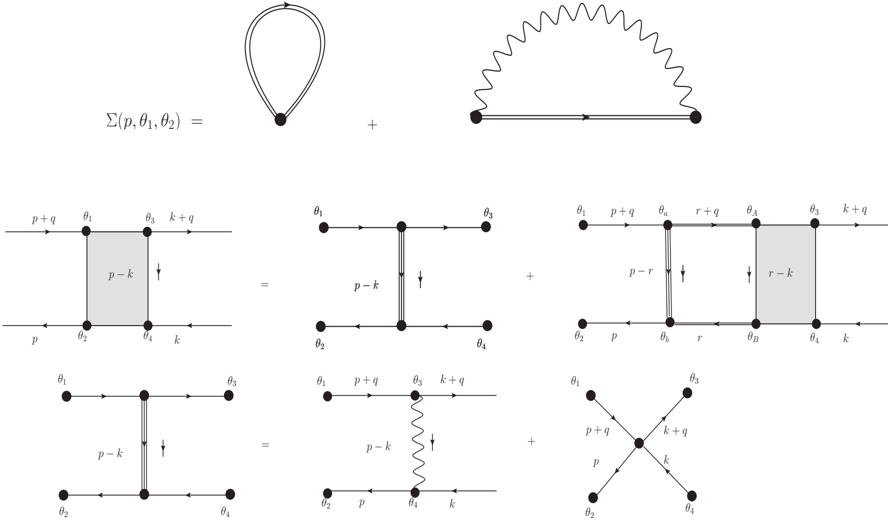


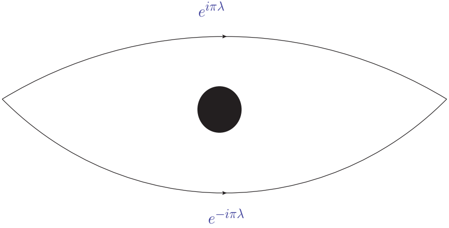

Figure: Dyson-Schwinger equation for exact propagator and exact offshell four point function.

# Nature of the conjecture: Delta function

$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\text{naive}}(s,\theta)$$

- The conjectured S matrix has a non-analytic $\delta(\theta)$ piece.

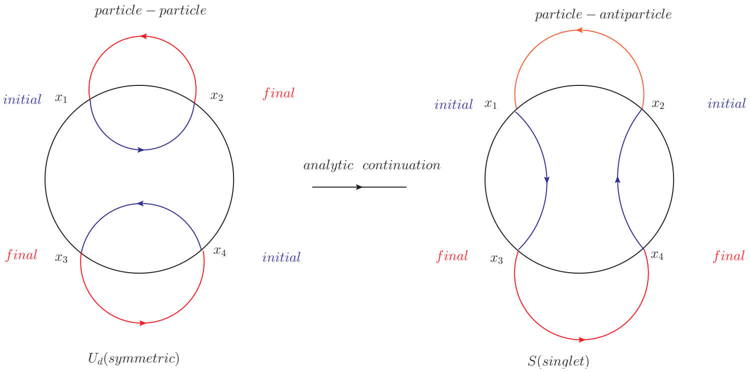
- delta function is already known to be necessary to unitarize non-relativistic Aharanov-Bohm scattering [Ruijsenaars; Bak,Jackiw,Pi].




- $\cos(\pi\lambda)$ is due to the interference of the Aharonov-Bohm phases of the wave packets.

# Modified crossing rules: A heuristic explanation

$$\mathcal{S} = 8\pi\sqrt{s}\cos(\pi\lambda)\delta(\theta) + i\frac{\sin(\pi\lambda)}{\pi\lambda}\mathcal{T}^{S;\text{naive}}(s,\theta)$$




$$T_{U_d} W_{U_d} \to T_S W_S$$

$$\frac{W_{U_d}}{W_S} = \frac{\oint \text{with 2 circular Wilson lines}}{\oint \text{with 1 circular Wilson line}} = \frac{\sin(\pi\lambda)}{\pi\lambda} \quad \text{[Witten]}$$