Partition function on higher genus Riemann surface in AdS_3/CFT_2 system

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Introduction

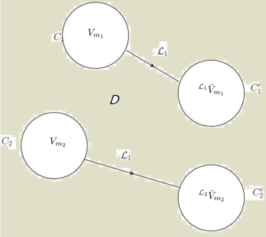
- Recent discussion for 1+1d holographic entanglement entropy revives the discussion for AdS_3/CFT_2
- We try to study the partition function on higher genus Riemann surface in AdS_3/CFT_2 system, where $c = \frac{3l}{2G}$

AdS_3/CFT_2 correspondence and partition function	
AdS_3	CFT_2
Semi-classical limit	Large c limit
Handle body partition function	Higher genus partition function for vacuum module channel
Classical on-shell action	Wyel anomaly, c^1 order
1-loop quantum correction	**c^0 order?**

- In CFT side, we try to study the c^0 order partition function on higher genus Riemann surface, and find agreement with 1-loop correction in the gravity calculation

Schottky Uniformization

- Every genus-g Riemann surface can be described by Schottky Uniformization as $\mathcal{M} = \Omega/\Gamma$, where Γ is generated by g elements $\mathcal{L}_j$ in $SL(2, C)$ (the global conformal transformation)

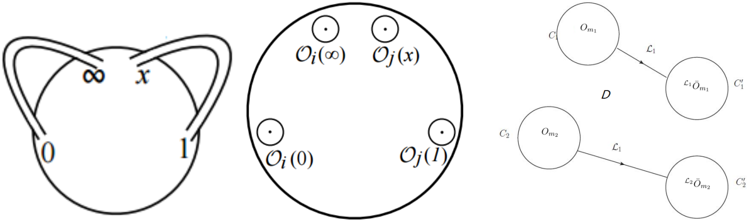


- We choose $2g$ circles C_j C_j', such that $\mathcal{L}_j$ set the inner (outer) of C_j to the outer (inner) of C_j'
- The regain outside the circles C_j and C_j' are genus-g Riemann surface

- The Schottky Uniformization can be extended into gravity and find a handle-body solution
- The 1-loop partition function for handle body solution was calculated by heat kernel method; the result only determined on Schottky group

Cutting and inserting recipe

- In conformal field theory, the higher genus partition function can be calculated by cutting and inserting a complete bases at each handle



(a) Riemann surface (b) Inserting states (c) Universal covering space

$$Z_g = \sum_{m_1, m_2, \ldots m_g} \langle {}^{\mathcal{L}_1}\bar{O}^{(1)}_{m_1} O^{(1)}_{m_1} \; {}^{\mathcal{L}_2}\bar{O}^{(2)}_{m_2} O^{(2)}_{m_2} \ldots {}^{\mathcal{L}_g}\bar{O}^{(g)}_{m_g} O^{(g)}_{m_g} \rangle \quad (0.1)$$

where $O^{(j)}_{m_j}$ is inserted in C_j and ${}^{\mathcal{L}_j}\bar{O}^{(j)}_{m_j}$ is inserted in C'_j

Large c limit of Virasoro algebra

- The Virasoro algebra is decoupled

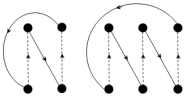
$$[\hat{L}_m, \hat{L}_n] = \delta_{m+n} + O(\frac{1}{c^{1/2}}) \tag{0.2}$$

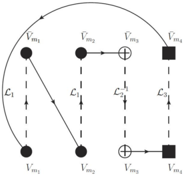
where

$$\hat{L}_m = |\frac{12}{cm(m^2-1)}|^{\frac{1}{2}} L_m \text{ for } |m| \geq 2 \tag{0.3}$$

- The vacuum module states $\prod_{m=2}^{\infty} \hat{L}_{-m}^{r_m} \mid 0\rangle$ are orthogonal to each other directly (in leading order of $\frac{1}{c}$ expansion)
- The correlation function is free: it is captured by a series two point functions' products
- Denote $\hat{L}_{-m} \mid 0\rangle$ as one particle state, (with vertex operator $\partial^{m-2} T(z)$);
- $\prod_j \hat{L}_{-m_j} \mid 0\rangle$ as multi-particle state $(\prod_j : \partial^{m_j-2} T(z) :)$

Diagram Rules





- The upper and lower dots denote inserting state in one handle; the number of dots denote the state's particle number; each dot is a $\partial^{m-2}T(z)$
- The dashed lines denote summation over m
- The solid line denotes the contraction of two stress tensor.
- A closed contour with dashed and solid lines is called a link

- We find a one-to-one correspondence between link and Schottky group element and classify all of the diagrams
- We calculate the partition function and find agreement with gravity's result

Thanks for your attention!