Primordial gravitational waves from the space-condensate inflationary model

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Based on: 1512.05072 & 1512.05062 with Seoktae Koh & Bum-Hoon Lee.
In this work:

- We consider the space-condensate inflation model as a source to the primordial GWs.

- We calculate the energy spectrum of the primordial GWs by assuming the abrupt phase transition between two consecutive regimes.

- We constrain our result with several observational upper bounds and plot CMB angular power spectrum for BB-mode.
We start with following action motivated in the nonlinear sigma model

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g^{\mu\nu} \delta_{ab} \partial_\mu \sigma^a \partial_\nu \sigma^b - V(\phi) \right], \]

where \( \sigma^a \)'s have \( SO(3) \) symmetry.

To preserve the cosmological principles of homogeneity and isotropy, we choose following ansatz for \( \sigma^a \) fields along with flat FRW metric:

\[ \sigma^a = \xi x^a, \quad ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j. \]

Background EoM for \( g_{\mu\nu} \) and \( \phi \) are given by

\[ H^2 = \frac{1}{3M_p^2} \left( \frac{1}{2} \dot{\phi}^2 + V + \frac{3\xi^2}{2a^2} \right), \]

\[ \dot{H} = -\frac{1}{2M_p^2} \left( \dot{\phi}^2 + \frac{\xi^2}{a^2} \right), \]

\[ \ddot{\phi} + 3H \dot{\phi} + V_{\phi} = 0. \]
Slow-roll approximation:

- With potential satisfying the slow-roll approximations: \( \dot{\phi}^2/2 \ll V, \ |\dot{\phi}| \ll |3H\dot{\phi}|, |V_\phi|; \)
- one gets the background EoM approximately,

\[
H^2 \simeq \frac{V}{3M_p^2} + \frac{\xi^2}{2a^2M_p^2}, \quad 3H\dot{\phi} + V_\phi \simeq 0.
\]

- There exists a phase transition and it depends on \( \xi \) values: \( aH \sim k_{\text{min}} \sim \frac{\xi}{M_p}. \)
The way to reflect these slow-roll approximations is to introduce the slow-roll parameters,
\[ \epsilon_V \simeq \frac{M_p^2}{2} \left( \frac{V_\phi}{V} \right)^2 \left( 1 - \frac{3\xi^2 M_p^2}{a^2 V} \right), \quad \eta_V \simeq \frac{V_{\phi\phi}}{V} M_p^2 \left( 1 - \frac{3\xi^2 M_p^2}{2a^2 V} \right) \]

The corresponding observable parameters during inflation are given by
\[
\mathcal{P}_S(k) \simeq \frac{H^2}{\pi \epsilon_V M_p^2} \left( 1 + (6C - 2)\epsilon_V - 2C\eta_V - \frac{\xi^2}{\epsilon_V M_p^2 k^2} \right)
\]
\[
\mathcal{P}_T(k) \simeq \frac{16H^2}{\pi M_p^2} \left( 1 + (2C - 2)\epsilon - \frac{37\xi^2}{12 M_p^2 k^2} \right)
\]
\[
n_S - 1 \simeq 2\eta_V - 6\epsilon_V + \frac{2\xi^2}{\epsilon_V k^2 M_p^2}
\]
\[
n_T \simeq -2\epsilon_V + \frac{31\xi^2}{6 k^2 M_p^2}
\]
\[
r \simeq 16\epsilon_V + \frac{16\xi^2}{k^2 M_p^2}
\]
\[
\alpha_S = -\frac{4\xi^2}{\epsilon_V k^2 M_p^2}, \quad \alpha_T = -\frac{31\xi^2}{3 k^2 M_p^2}
\]
Gravitational waves:

- GWs are described by tensor perturbation: $ds^2 = a^2(\tau) \left[ -d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j \right]$

- $|h_{ij}| \ll 1$, we obtain linearized Einstein equation: $h''_{\lambda,k} + 2H h'_{\lambda,k} + k^2 h_{\lambda,k} = 0$.

- The strength of GWs is characterized by their energy spectrum: $\Omega_{GW}(k) = \frac{1}{\rho_{\text{crit}}} \frac{d\rho_{GW}}{d \ln k}$

- The energy density of GWs background is: $\rho_{GW} = \frac{M_p^2}{32\pi} \int k^2 P_T(k) d\ln k$.

- The power spectrum of GWs observed today is: $P_T(k) \equiv \frac{32k^3}{\pi M_p^2} \sum \lambda \langle h_{\lambda,k}^\dagger h_{\lambda,k} \rangle$.

- The tensor power spectrum observed today can relate to that of the inflationary one by the transfer function: $P_T = T^2(k) P_T(k)$
The energy spectrum of the primordial GWs becomes:

\[ h_0^2 \Omega_{GW}(k) = \frac{h_0^2 k^2}{12 H_0^2} T^2(k) \mathcal{P}_T(k). \]

where \( T(k) \) reflects the damping effect of the GWs.

For simplicity, we consider the cosmic expansion as the only damping effect

\[ T^2(k) = \frac{9}{k^4 \tau_0^4} \left( \frac{\Omega_m}{\Omega_\Lambda} \right)^2 \left[ 1 + \frac{4}{3} \left( \frac{k}{k_{eq}} \right) + \frac{5}{2} \left( \frac{k}{k_{eq}} \right)^2 \right], \]

\[ \mathcal{P}_T(k) = \mathcal{P}_T(k_\ast) \left( \frac{k}{k_\ast} \right)^{n_T + \frac{\alpha T}{2} \ln(k/k_\ast)}. \]

As a result, we obtain:

\[ h_0^2 \Omega_{GW} \approx \frac{3 h_0^2}{4 H_0^2 \tau_0^4} \left( \frac{\Omega_m}{\Omega_\Lambda} \right)^2 \frac{1}{k^2} \left[ 1 + \frac{4}{3} \left( \frac{k}{k_{eq}} \right) + \frac{5}{2} \left( \frac{k}{k_{eq}} \right)^2 \right] \mathcal{P}_S(k_\ast) \left( \frac{k}{k_\ast} \right)^{n_T + \frac{\alpha T}{2} \ln(k/k_\ast)}. \]
Numerical results:

\[ h_0^2 \Omega_{GW} = \frac{3h_0^2}{16\pi^2 H_0^2 \tau_0^4 f^2} \left( \frac{\Omega_m}{\Omega_\Lambda} \right)^2 \left[ 1 + \frac{4}{3} \left( \frac{f}{f_{eq}} \right) + \frac{5}{2} \left( \frac{f}{f_{eq}} \right)^2 \right] P_S(k_*) \left( \frac{f}{f_*} \right)^{-r/8} + \frac{43\xi^2}{24\pi^2 f^2} - \frac{31\xi^2}{24\pi^2 f^2} \ln \left( \frac{f}{f_*} \right) \]

**Figure 1:** For the dashed line \( \xi = 10^{-19} \) while it is \( \xi = 0 \) for the solid line where \( r = 10^{-1} \).
Numerical results:

\[ h_0^2 \Omega_{GW} = \frac{3h_0^2}{16\pi^2H_0^2\tau_0^4} \left( \frac{\Omega_m}{\Omega_\Lambda} \right)^2 \frac{1}{f^2} \mathcal{P}_S(k_*) r \left( \frac{f}{f_*} \right)^{-\frac{r}{8}} + \frac{43\xi^2}{24\pi^2f^2} - \frac{31\xi^2}{24\pi^2f^2} \ln(f/f_*) \]

(a) \( r = 10^{-1}, 10^{-2}, \& 10^{-3}; \) for \( \xi = 10^{-19} \)

(b) \( \xi = 3.1 \times 10^{-20}, 10^{-19}, \& 4 \times 10^{-19}; \) for \( r = 10^{-1} \).

- Cosmological parameters adopted in this work are listed as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0 = 100h_0 \text{km/s/Mpc}^{-1} )</td>
<td>( h_0 \approx 0.67 )</td>
</tr>
<tr>
<td>( k_{eq} = 0.073\Omega_m h_0^2 \text{Mpc}^{-1} )</td>
<td>( \Omega_m = 0.315 )</td>
</tr>
<tr>
<td>( \mathcal{P}<em>S(k</em>*) \approx 2.19 \times 10^{-9} )</td>
<td>( k_* = 0.002 \text{Mpc}^{-1} )</td>
</tr>
<tr>
<td>( \Omega_\Lambda = 0.685 )</td>
<td>( f_{eq} \equiv k_{eq}/2\pi )</td>
</tr>
<tr>
<td>( f_* \equiv k_*/2\pi )</td>
<td>( f_* = 3.092 \times 10^{-18} \text{Hz} )</td>
</tr>
</tbody>
</table>
Numerical results:

(c) Angular power spectrum of the BB-mode

(d) Matter power spectrum

\[
\frac{l(l+1)C_{BB}}{(2\pi)^{\frac{1}{2}}} = \begin{cases} 
\xi=0 \\
\xi=10^{-6} \\
\xi=1.3 \times 10^{-5} \\
\xi=2.4 \times 10^{-5} 
\end{cases}
\]

\[
P(k) = \left[ (h^{-1} \text{Mpc})^3 \right]
\]

\[
\xi = 10^{-6}, 1.3 \times 10^{-5}, 2.4 \times 10^{-5}
\]
Conclusion:

- In this work:
  - we calculated the energy spectrum of the primordial GWs and found that it has the significant suppression in low frequency regime.

  - The suppression of the energy spectrum is sensitive to $\xi$ parameter. As $\xi$ increases the suppression shifts to the large frequency regime, while for decreasing $\xi$, our result eventually converges to that of the standard single-filed inflation model.

  - The suppression can be explained by the existence of early phase transition, prior to inflation, such that there exists the cutoff scale for the comoving wavenumber $k_{\text{min}} \sim \xi/M_p$. 

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  - The suppression of the energy spectrum is sensitive to \( \xi \) parameter. As \( \xi \) increases the suppression shifts to the large frequency regime, while for decreasing \( \xi \), our result eventually converges to that of the standard single-filed inflation model.
  - The suppression can be explained by the existence of early phase transition, prior to inflation, such that there exists the cutoff scale for the comoving wavenumber \( k_{\text{min}} \sim \xi/M_p \).

Thank you for your attention!