Primordial gravitational waves from the space-condensate inflationary model

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Based on: 1512.05072 & 1512.05062 with Seoktae Koh & Bum-Hoon Lee.

In this work:

- We consider the space-condensate inflation model as a source to the primordial GWs.
- We calculate the energy spectrum of the primordial GWs by assuming the abrupt phase transition between two consecutive regimes.
- We constrain our result with several observational upper bounds and plot CMB angular power spectrum for BB-mode.

Space-condensate inflation model:

- We start with following action motivated in the nonlinear sigma model

$$\mathcal{S} = \int d^4x\sqrt{-g}\Big[\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\delta_{ab}\partial_\mu\sigma^a\partial_\nu\sigma^b - V(\phi)\Big],$$

  where σ^a's have $SO(3)$ symmetry.

- To preserve the cosmological principles of homogeneity and isotropy, we choose following ansatz for σ^a fields along with flat FRW metric:

$$\sigma^a = \xi x^a, \quad ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j.$$

- Background EoM for $g_{\mu\nu}$ and ϕ are given by

$$H^2 = \frac{1}{3M_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V + \frac{3\xi^2}{2a^2}\right),$$

$$\dot{H} = -\frac{1}{2M_p^2}\left(\dot{\phi}^2 + \frac{\xi^2}{a^2}\right),$$

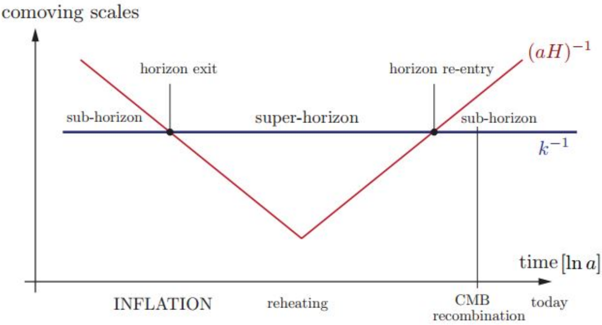
$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0.$$

Slow-roll approximation:

- With potential satisfying the slow-roll approximations: $\dot{\phi}^2/2 \ll V, \quad |\ddot{\phi}| \ll |3H\dot{\phi}|, |V_\phi|$;
- one gets the background EoM approximately,

$$H^2 \simeq \frac{V}{3M_p^2} + \frac{\xi^2}{2a^2M_p^2}, \quad 3H\dot{\phi} + V_\phi \simeq 0.$$

- There exists a phase transition and it depends on ξ values: $aH \sim k_{\rm min} \sim \frac{\xi}{M_p}$.



Slow-roll and observable parameters:

- The way to reflect these slow-roll approximations is to introduce the slow-roll parameters,

$$\epsilon_V \simeq \frac{M_p^2}{2}\left(\frac{V_\phi}{V}\right)^2\left(1-\frac{3\xi^2 M_p^2}{a^2 V}\right), \quad \eta_V \simeq \frac{V_{\phi\phi}}{V}M_p^2\left(1-\frac{3\xi^2 M_p^2}{2a^2 V}\right)$$

- The corresponding observable parameters during inflation are given by

$$\mathcal{P}_S(k) \simeq \frac{H^2}{\pi\epsilon_V M_p^2}\left(1+(6C-2)\epsilon_V - 2C\eta_V - \frac{\xi^2}{\epsilon_V M_p^2 k^2}\right)$$

$$\mathcal{P}_T(k) \simeq \frac{16H^2}{\pi M_p^2}\left(1+(2C-2)\epsilon - \frac{37\xi^2}{12M_p^2 k^2}\right)$$

$$n_S - 1 \simeq 2\eta_V - 6\epsilon_V + \frac{2\xi^2}{\epsilon_V k^2 M_p^2}$$

$$n_T \simeq -2\epsilon_V + \frac{31\xi^2}{6k^2 M_p^2}$$

$$r \simeq 16\epsilon_V + \frac{16\xi^2}{k^2 M_p^2}$$

$$\alpha_S = -\frac{4\xi^2}{\epsilon_V k^2 M_p^2}, \quad \alpha_T = -\frac{31\xi^2}{3k^2 M_p^2}$$

Gravitational waves:

- GWs are described by tensor perturbation: $ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$
- $|h_{ij}| \ll 1$, we obtain linearized Einstein equation: $h''_{\lambda,k} + 2\mathcal{H}h'_{\lambda,k} + k^2 h_{\lambda,k} = 0$.
- The strength of GWs is characterized by their energy spectrum: $\Omega_{GW}(k) = \frac{1}{\rho_{\rm crit}} \frac{d\rho_{GW}}{d\ln k}$
- The energy density of GWs background is: $\rho_{GW} = \frac{M_p^2}{32\pi}\int k^2 P_T(k) d\ln k$.
- The power spectrum of GWs observed today is: $P_T(k) \equiv \frac{32k^3}{\pi M_p^2}\sum_\lambda \langle h^\dagger_{\lambda,\,k} h_{\lambda,\,k}\rangle$.
- The tensor power spectrum observed today can relate to that of the inflationary one by the transfer function: $P_T = \mathcal{T}^2(k)\mathcal{P}_T(k)$

- The energy spectrum of the primordial GWs becomes:

$$h_0^2\Omega_{GW}(k) = \frac{h_0^2 k^2}{12 H_0^2}\mathcal{T}^2(k)\mathcal{P}_T(k)\,.$$

where $\mathcal{T}(k)$ reflects the damping effect of the GWs.

- For simplicity, we consider the cosmic expansion as the only damping effect

$$\mathcal{T}^2(k) = \frac{9}{k^4\tau_0^4}\left(\frac{\Omega_m}{\Omega_\Lambda}\right)^2\left[1+\frac{4}{3}\left(\frac{k}{k_{eq}}\right)+\frac{5}{2}\left(\frac{k}{k_{eq}}\right)^2\right],$$

$$\mathcal{P}_T(k) = \mathcal{P}_T(k_*)\left(\frac{k}{k_*}\right)^{n_T+\frac{\alpha_T}{2}\ln(k/k_*)}.$$

- As a result, we obtain:

$$h_0^2\Omega_{GW} \simeq \frac{3h_0^2}{4H_0^2\tau_0^4}\left(\frac{\Omega_m}{\Omega_\Lambda}\right)^2\frac{1}{k^2}\left[1+\frac{4}{3}\left(\frac{k}{k_{eq}}\right)+\frac{5}{2}\left(\frac{k}{k_{eq}}\right)^2\right]\mathcal{P}_S(k_*)r\left(\frac{k}{k_*}\right)^{n_T+\frac{\alpha_T}{2}\ln(k/k_*)}$$

Numerical results:

$$h_0^2\Omega_{GW} = \frac{3h_0^2}{16\pi^2 H_0^2 \tau_0^4 f^2}\left(\frac{\Omega_m}{\Omega_\Lambda}\right)^2\left[1+\frac{4}{3}\left(\frac{f}{f_{eq}}\right)+\frac{5}{2}\left(\frac{f}{f_{eq}}\right)^2\right]\mathcal{P}_S(k_*)r\left(\frac{f}{f_*}\right)^{-\frac{r}{8}+\frac{43\xi^2}{24\pi^2 f^2}-\frac{31\xi^2}{24\pi^2 f^2}\ln\left(\frac{f}{f_*}\right)}$$

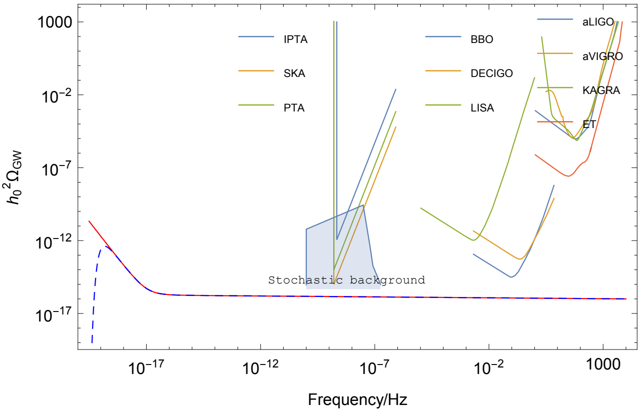
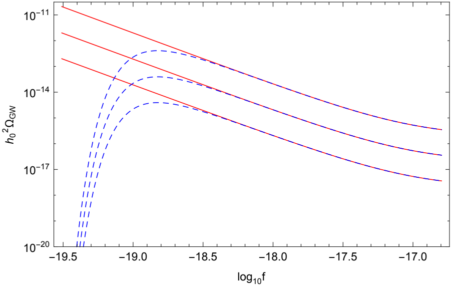
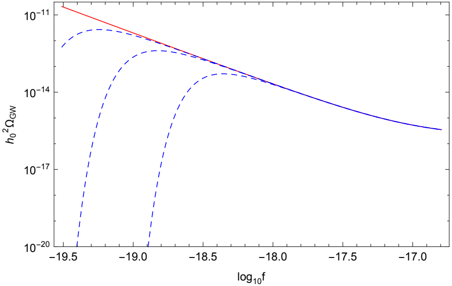


Figure 1: For the dashed line $\xi = 10^{-19}$ while it is $\xi = 0$ for the solid line where $r = 10^{-1}$.

$$h_0^2\Omega_{GW} = \frac{3h_0^2}{16\pi^2 H_0^2 \tau_0^4}\left(\frac{\Omega_m}{\Omega_\Lambda}\right)^2 \frac{1}{f^2}\mathcal{P}_S(k_*)r\left(\frac{f}{f_*}\right)^{-\frac{r}{8}+\frac{43\xi^2}{24\pi^2 f^2}-\frac{31\xi^2}{24\pi^2 f^2}\ln(f/f_*)}$$



(a) $r = 10^{-1}$, 10^{-2}, & 10^{-3}; for $\xi = 10^{-19}$

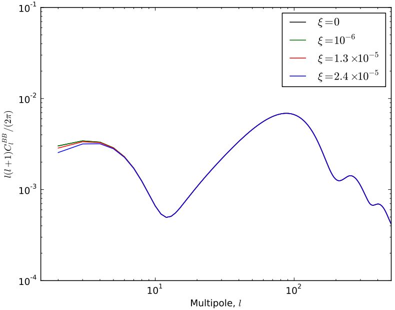


(b) $\xi = 3.1\times 10^{-20}$, 10^{-19}, & 4×10^{-19}; for $r = 10^{-1}$.

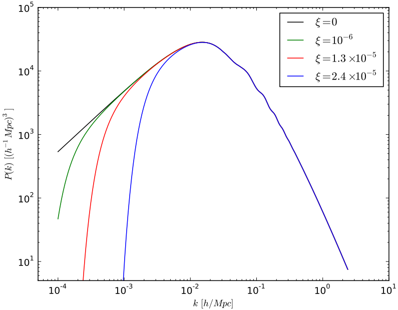
- Cosmological parameters adopted in this work are listed as follows:

$H_0 = 100h_0\text{kms}^{-1}\text{Mpc}^{-1}$	$h_0 \simeq 0.67$	$a(\tau_0) = 1$	$\tau_0 = 1.41 \times 10^4\text{Mpc}$
$k_{eq} = 0.073\Omega_m h_0^2\ \text{Mpc}^{-1}$	$\Omega_m = 0.315$	$\Omega_\Lambda = 0.685$	$f_{eq} \equiv k_{eq}/2\pi$
$\mathcal{P}_S(k_*) \simeq 2.19 \times 10^{-9}$	$k_* = 0.002\text{Mpc}^{-1}$	$f_* \equiv k_*/2\pi$	$f_* = 3.092 \times 10^{-18}\text{Hz}$

Numerical results:



(c) Angular power spectrum of the BB-mode



(d) Matter power spectrum

Conclusion:

- In this work:
  - we calculated the energy spectrum of the primordial GWs and found that it has the significant suppression in low frequency regime.
  - The suppression of the energy spectrum is sensitive to ξ parameter. As ξ increases the suppression shifts to the large frequency regime, while for decreasing ξ, our result eventually converges to that of the standard single-filed inflation model.
  - The suppression can be explained by the existence of early phase transition, prior to inflation, such that there exists the cutoff scale for the comoving wavenumber $k_{\rm min} \sim \xi/M_p$.

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Thank you for your attention!