An Introduction to Brane Brick Models and *2d (0,2)* Theories

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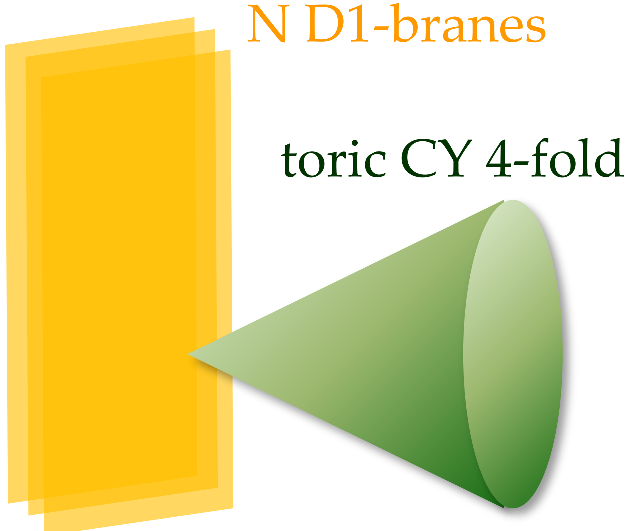
✓ arXiv:1506.03818, JHEP09 (2015) 072
with Sebastian Franco (CUNY), Sangmin Lee (SNU), Rak-Kyeong Seong (KIAS), Daisuke Yokoyama (King's College)
✓ arXiv:1510.01744 by S. Franco, S. Lee, R. Seong

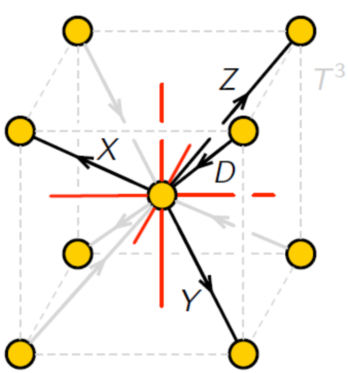
Brane construction for *2d (0,2)* theory

Worldvolume theory of D1 brane whose transverse geometry is Calabi-Yau 4-fold, which T-dual is as follows

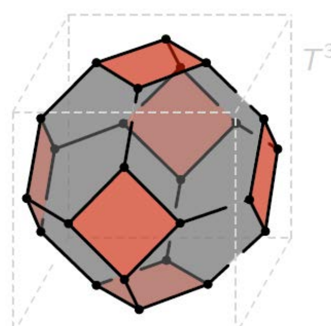


NS5 branes wrap a holomorphic cycle

	0	1	2	3	4	5	6	7	8	9
NS5	×	×	×	×	×	×	·	·	·	·
NS5′	×	×	×	×	·	·	×	×	·	·
NS5′′	×	×	·	·	×	×	×	×	·	·
D4	×	×	×	·	×	·	×	·	·	·



graph dual



holomorphic surface Σ

$(2,3) \equiv x$

$(4,5) \equiv y$

$(6,7) \equiv z$

{*arg(x), arg(y), arg(z)*} constitutes a 3 torus

J- and E-terms

2d (0,2) version of
4d superpotential which
specifies the shape of CY4.

SUSY multiplets of *2d (0,2)* theories

multiplets	superfield	component fields
vector	V	$(v_a, \chi_-, \overline{\chi}_-, D)$
chiral	Φ_{ij}	(ϕ, ψ_+)
Fermi	Λ_{ij}	(λ_-, G)

$$\overline{\mathcal{D}}_+ \Lambda = E(\Phi_i) \qquad L_J = - \int d^2y\, d\theta^+ \sum_a \left(\Lambda_a J_a(\Phi_i)|_{\bar{\theta}^+=0}\right) - h.c.$$

E-term
(introduced by modified chirality)

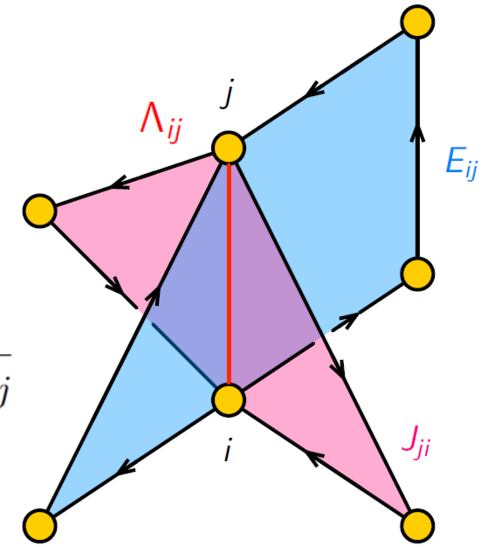
J-term
(coupled by additional term in Lagrangiain)

Additional two constraints on J- and E-terms:

a. SUSY condition
$$\sum_a \mathrm{tr}\left[E_a(\Phi_i) J_a(\Phi_i)\right] = 0\,.$$

b. toric condition
$$J_{ji} = J_{ji}^+ - J_{ji}^- \;,\; E_{ij} = E_{ij}^+ - E_{ij}^-$$

difference of monomials



Dimension counting of moduli space

Classical mesonic moduli space of four dimension

quotient by toric J, E-terms reduces to $n_F - 3$

$$\mathcal{M}^{mesonic} = (\mathbb{C}[\Phi_{ij}]/\langle J_{ij} = 0, E_{ij} = 0\rangle)//U(1)^{G-1}$$

freely generated space with dimension n_χ

$U(1)^{G-1}/U(1)_{overall}$
gauge invariance

$$n_\chi - (n_F - 3) - (G - 1) = 4$$

Gauge anomaly cancellation

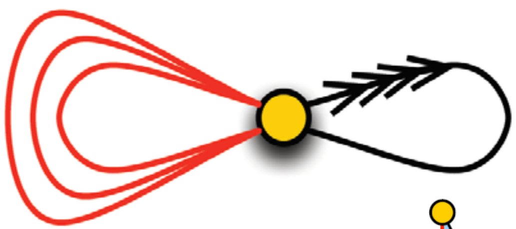
Examples of non-compact Calabi-Yau 4-fold :

a. $\mathbb{C}^4$ No non-trivial relation *(4-0-0=4)*

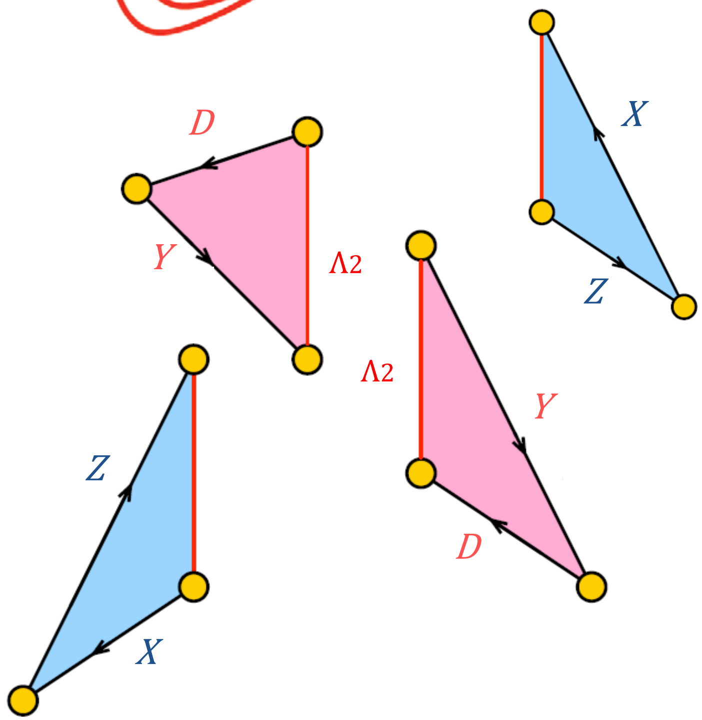
b. D3 $\mathbb{C}[A_1, A_2, A_3, B_1, B_2]/\langle A_1A_2A_3 = B_1B_2\rangle$ *(5-1=4)*

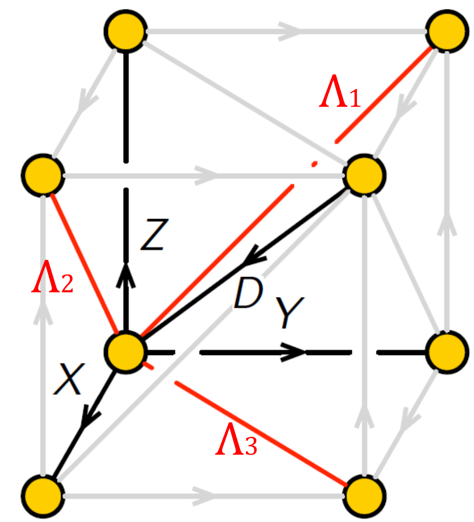
Periodic quiver

Gluing J- and E-plaquettes,
we can make a periodic quiver diagram in a 3-torus



	J	E
$\Lambda^{(1)}$:	$Y \cdot Z - Z \cdot Y = 0$	$D \cdot X - X \cdot D = 0$
$\Lambda^{(2)}$:	$Z \cdot X - X \cdot Z = 0$	$D \cdot Y - Y \cdot D = 0$
$\Lambda^{(3)}$:	$X \cdot Y - Y \cdot X = 0$	$D \cdot Z - Z \cdot D = 0$





a unit cell of $\mathbb{C}^4$ model's periodic quiver

Gluing J- and E-plaquettes,
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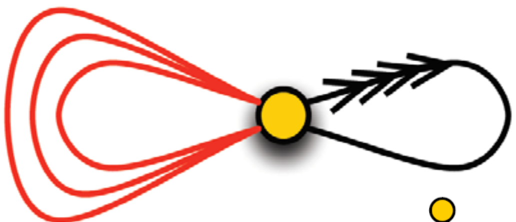

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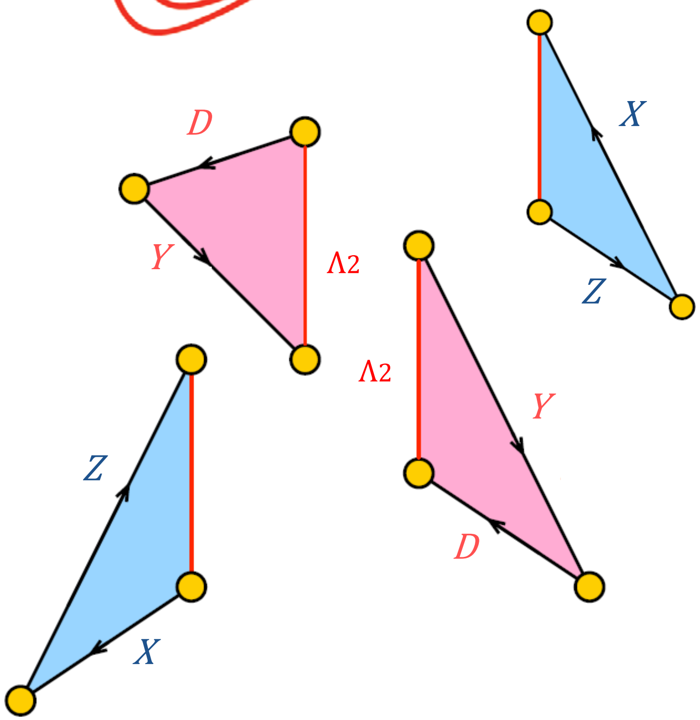
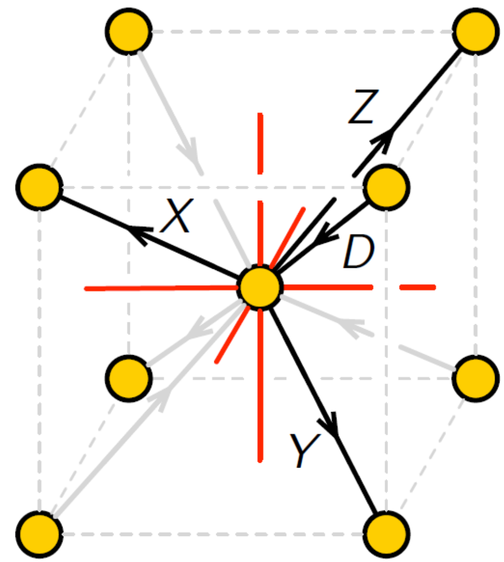


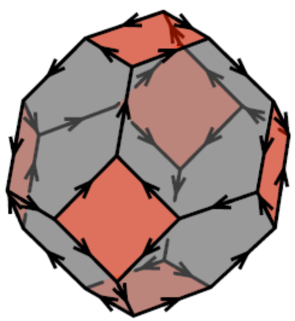


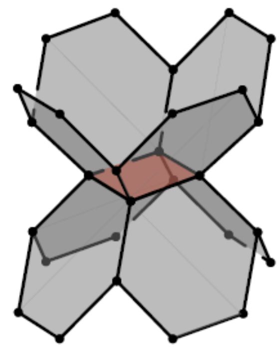
a unit cell of $\mathbb{C}^4$ model's periodic quiver
(bcc lattice)

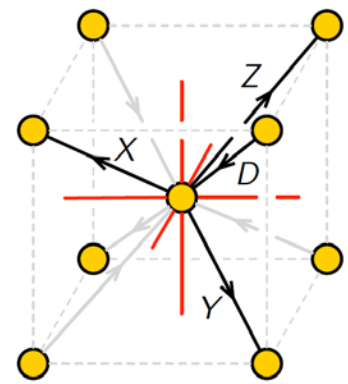
Brane brick dictionary

Brane brick ⟺ Periodic quiver

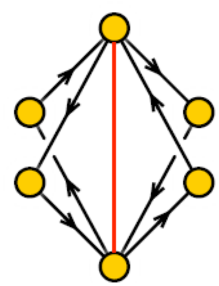
graph dual







gauge group
= bulk of brick

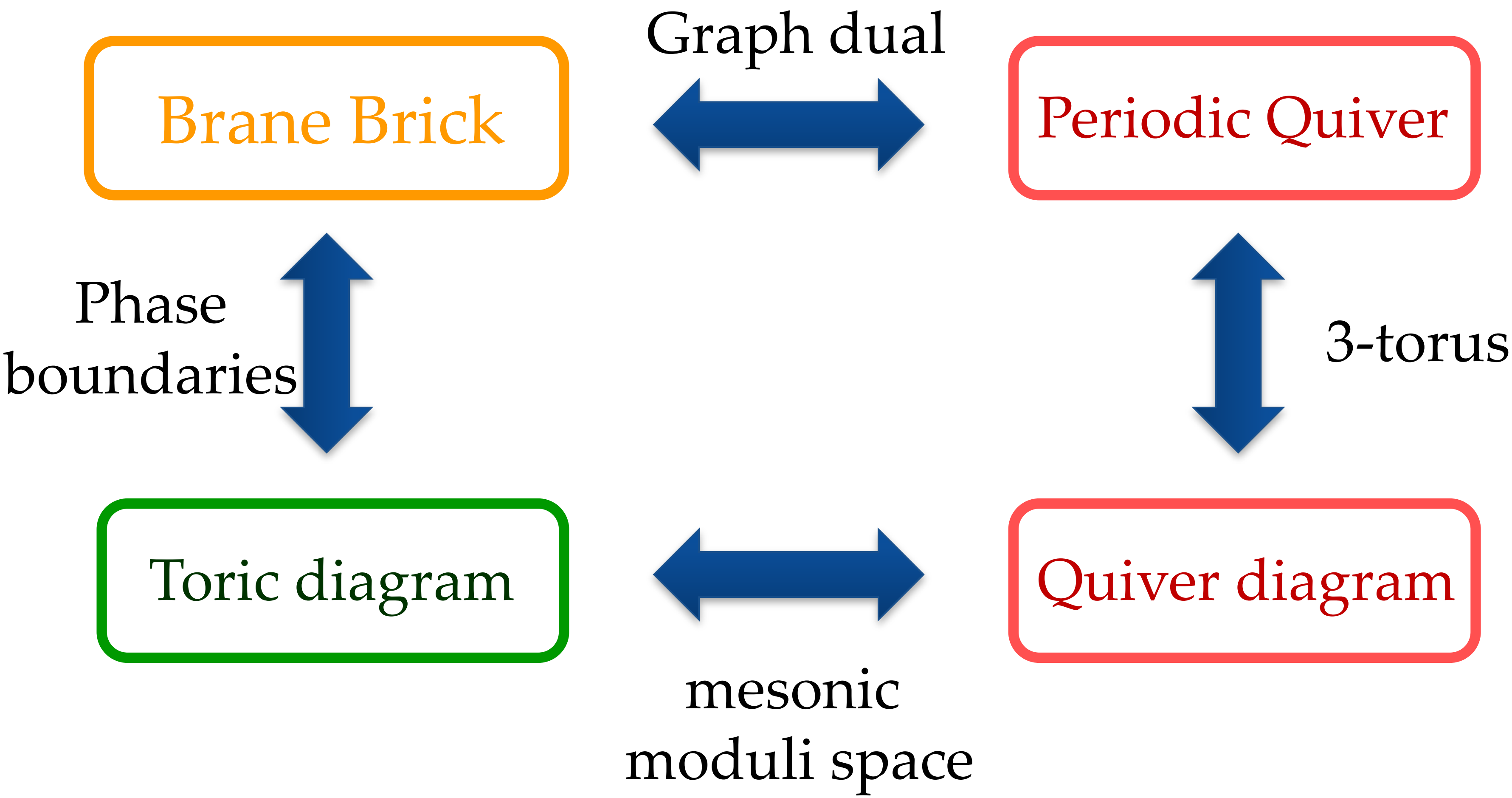


chiral fields
= oriented surfaces

Fermi fields
= unoriented surfaces

How J- and E-terms are encoded;
4-sided unoriented surface adjoining oriented ones

Summary


Graph dual
Brane Brick
Periodic Quiver
Phase
boundaries
3-torus
Toric diagram
Quiver diagram
mesonic
moduli space

Thank You