Vortex and Droplet using Non-relativistic Holography

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※Non-relativistic holography: Motivations

- Many condensed matter systems can be described by strongly interacting non-relativistic (NR) physics $\Rightarrow$ Use of holography to extract information.
- Scale invariant, Lorentz violating Lifshitz-like fixed points $t \to \lambda^z t$, $x^i \to \lambda x^i$, $(i = 1, 2, \cdots, d)$ are common in strongly interacting condensed matter systems.
  Examples:
  (i) Fermions at unitarity (having experimental realization);
  (ii) Strongly correlated electron systems, etc.
- Study of general holography principles without AdS asymptotics.

※Holographic Lifshitz superconductors

- Dual gravity models encouraged by AdS/CFT
- The model: $\mathcal{S}_g = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big(R - 2\Lambda - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}e^{b\phi}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\Big)$ [Taylor, Pang]
  $\mathcal{S}_M = \int d^4x\sqrt{-g}\Big(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |\mathcal{D}_\mu\psi|^2 - m^2|\psi|^2\Big)$[Hartnoll et al]
- Metric: $ds^2 = -r^{2z}f(r)dt^2 + \frac{dr^2}{f(r)r^2} + r^2(dx^2 + dy^2)$
- The dynamic exponent (z) dependent temperature [Pang]
- General properties have been studied by Zhang et.al.
- Consistent with mean field behavior of general holographic superconductors

※Our exploration: adding magnetic field $\mathcal{B}$

- Holographic Lifshitz Superconductors exhibits vortex and droplet solutions in presence of external magnetic field.
- Define $\epsilon = \dfrac{\mathcal{B}_c - \mathcal{B}}{\mathcal{B}_c} << 1$ and expand the fields (ψ, A_μ) in terms of ϵ.
- Solutions of EoMs with proper boundary conditions $\longrightarrow$ dependence of $\mathcal{B}_c$ on z.
- Order parameter resembles with that in Ginzburg-Landau theory $\Rightarrow$ $\boxed{\mathcal{B}_c \propto 1/\xi^2}$, ξ = superconducting coherence length.

※Vortex lattice solution

- Suitable choice of coefficients of order parameter ⇒ vortex lattice solution
- The solution: $\boxed{\sigma(\vec{x}) = \left| exp\left(\frac{-x^2}{2\xi^2}\right)\vartheta_3(\nu,\tau)\right|^2}$;

$$\vartheta_3(\nu,\tau) = \sum_{\ell=-\infty}^{+\infty} exp(2\pi i\nu\ell + i\pi\tau\ell^2) \text{ : the elliptic theta function}$$

- Vortex lattice solution is independent of z ⇒ independence on the nature of the boundary field theory
- This solution has Gaussian profile and is controlled by ξ
- Lattice structure dies out with increasing ξ ⇒ Similarity with ordinary type II superconductors

※Droplet solution

- We consider insulator/superconductor phase transition
- Lifshitz solitons (LS) realize insulating phases (alike AdS solitons)
- Compactification with appropriate b.c. generates mass gap
- Large $U(1)$ chemical potential $(\mu) \Longrightarrow$ Unstable LS to forming scalar hair $\rightarrow$ superconducting phase [Takayanagi et al]
- We choose $(4+1)$ dimensional planar LS background in polar coordinates
- Close to the critical point $(\mu \sim \mu_c,\ \psi \sim 0)$: $\boxed{A = \mu_c dt + \frac{\mathcal{B}\rho^2 d\theta}{2}}$
- Solving EoM by separating variables, $\psi(t, r, \chi, \rho) = F(t, r)H(\chi)U(\rho)$
- Droplet solution : $\boxed{U(\rho) = \exp\left(\frac{-|\mathcal{B}|\rho^2}{4}\right)}$
- Holographic droplet solution is independent of z and hence independent of the nature of boundary theory

※Critical parameters

- **Critical parameters of condensation:**
- Define $F(t,r) = e^{-i\omega t}R(r)$
- 2^{nd} order DE of $R(r)$ ($\omega = 0$, marginally stable mode) with eigenvalue $(\mu_c^2 - \mathcal{B})$
- Using Sturm-Liouville eigenvalue method

$$\boxed{\Gamma \equiv (\mu_c^2 - \mathcal{B}) = \Gamma(z, m^2)}$$

- z indeed controls the relation between the critical parameters of condensation

※Summary

z independent vortex and droplet solutions, but z dependent critical parameters of the holographic phase transition.

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※Additional slide I

- **Gauge field ansatz:** $A_\mu = (A_t, 0, A_x, A_y)$.
- Solutions are static, i.e. independent of t.
- **Boundary conditions:**

  (i) $\psi(u \to 0) \sim C_1 u^{\triangle_-} + C_2 u^{\triangle_+}$, $\quad \triangle_\pm = \frac{1}{2}\left[(z+2) \pm \sqrt{(z+2)^2 + 4m^2}\right]$,
  $m^2 > m_{lb}^2 = \frac{-(z+2)^2}{4}$

  (ii) $C_1 = 0$, $\quad \psi(u = 1)$is regular
  (iii) $A_t(\vec{x}, u \to 0) = \mu$, $\quad F_{xy}(\vec{x}, u \to 0) = \mathcal{B}$
  (iv) $A_t(u \to 1) = 0$ $\quad A_i(u = 1)$are regular

- $\mathcal{B}$ is the only tuning parameter of the theory, and μ and T are constants in the boundary theory.
- **Lifshitz soliton background metric:**
  $ds^2 = -r^2 dt^2 + r^2(dx^2 + dy^2) + \dfrac{dr^2}{r^2 f(r)} + r^{2z} f(r) d\chi^2$ (double Wick rotation of the Lifshitz black hole metric.)

※Additional slide II

- **Lifshitz-like fixed points in CMT:** Lifshitz-like fixed points appear in some strongly correlated systems (anisotropic sine-Gordon model, finite temperature multicritical points in the phase diagrams of some systems, the Rokhsar-Kivelson dimer model, some lattice models of strongly correlated electrons, etc.). The correlation functions in these systems exhibit ultralocality in space at finite temperature and fixed time which may play central role in explaining certain experimental observations. Among other non-relativistic systems, special importance has been given for systems having Schrödinger symmetry ($z = 2$). Gravity dual for these systems have been proposed. These studies are motivated mainly by experiments with fermions at unitarity and nucleon scattering experiments.
- **Vortex solution:** $\nu \equiv \frac{y-ix}{a_1\xi}$, $\tau \equiv \frac{2\pi i - a_2}{a_1^2}$; a_1 and a_2 are constants.
- In the vortex lattice the fundamental region is spanned by the following two lattice vectors:

$$\vec{v}_1 = a_1\xi\partial_y, \qquad \vec{v}_2 = \frac{2\pi\xi}{a_1}\partial_x + \frac{a_2}{a_1}\partial_y.$$

※Additional slide III

- **Droplet solution:** The insulator/superconductor phase transition is realized in the CFT language as a phase transition in which a large enough $U(1)$ chemical potential, μ, overcomes the mass gap related to the scalar field, ψ. This mechanism allows ψ to condensate above a critical value, μ_c. In fact a soliton background, which includes an extra compactified spatial direction, precisely generates this mass gap resembling an insulating phase.