Thermalization and 2D Critical Quench

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Work done with
my supervisor Prof. Gautam Mandal,
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arXiv: 1405.6695, 1501.04580(MSS), 1512.02187
and ongoing works.

Introduction

Understanding thermalization in interacting quantum systems has been an important quest in physics. With the advent of holography, the issue of thermalization in strongly coupled QFTs have gained double importance, because of its correspondence to the problem of black hole formation.
We are concerned with 'thermalization' in 2D CFTs after a quantum quench.
The initial state is prepared by a quantum quench.
We will consider critical quenches when the final hamiltonian is critical.

Main Idea

The main idea is to use conformal symmetries, and other general principles to examine evolution of states - not every states.
In arXiv: 1501.04580(MSS), we worked with only Virosoro symmetry and perturbative resummation and compared the results with bulk duals results.
In arXiv: 1512.02187, we are examining quantum quenches more closely in QFT models.
Using ideas from arXiv: 1405.6695, we are trying to exactly find large number of thermalizing states.

History

- Thermalisation of point functions of a pure state: Calabrese and Cardy(2006) found the thermalization of correlation functions in 2D CFT starting from certain pure states.
- Definition of 'thermalization'.
- Quantum quench and preparation of initial state: They argued that quantum quench to a critical hamiltonian from the ground state of the prequench hamiltonian gives their initial state, which is a boundary state euclidean time evolved by κ_2 where κ_2 turns out to be four times the 'equivalent temperature' of the stationary state, $\beta = 4\kappa_2$. $|\Psi_0\rangle = e^{-\kappa_2 H}|B\rangle$ and $\langle\Psi_0| = \langle B|e^{-\kappa_2 H}$.

Geometry, conformal transformation

- The CFT lives in a strip of width $2\kappa_2$ with coordinates $w = \sigma + i\tau$ and $\bar{w} = \sigma - i\tau$, which can be transformed to upper half plane(UHP) using the conformal map $z = ie^{\pi w/(2\kappa_2)}$ and $\bar{z} = -ie^{\pi \bar{w}/(2\kappa_2)}$.

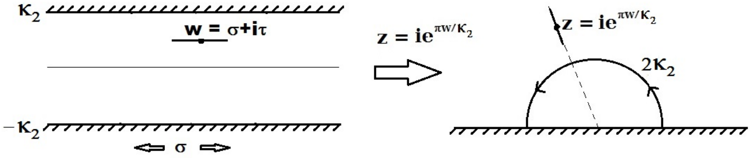

Figure: Strip to UHP transformation

- Lorentzian time evolution is obtained by Wick rotation.

Correlation functions

Scalar one point funciton with $w = 0 + i\tau$ and $\bar{w} = 0 - i\tau$,

$$\begin{aligned}\langle \phi_{h,\bar{h}}(w,\bar{w})\rangle_{strip} &= C\,[i\,Sech(2\pi t/(4\kappa_2))]^{2h} \\ &\xrightarrow{t\to\infty} C\,i^{2h}e^{-4\pi ht/(4\kappa_2)}\end{aligned}$$

Similarly 2-point functions on UHP would give 2-point or 3-point or 4-point functions on the plane depending on the field content of the operators.
Holomorphic 2-point functions(equal time) are already *thermalized* because they don't see the boundary

$$\langle \phi_h(0,t)\phi_h(r,t)\rangle = Ci^{2h}\mathrm{csch}^2\left(\frac{\pi r}{4\kappa_2}\right)$$

This is holomorphic 2-point functions in a thermal ensemble with temperature $T = 4\kappa_2$.

Thermalization function $I(t)$

The thermalization function for a spatial region A is defined as

$$I(t) = \frac{\hat{Z}_{St,Cy}(A)}{[\hat{Z}_{St,St}(A)\hat{Z}_{Cy,Cy}(A)]^{\frac{1}{2}}} = \frac{\mathrm{Tr}(\rho_{dyn,A}(t)\rho_{eqm,A}(\beta,\mu_i))}{[\mathrm{Tr}(\rho_{dyn,A}(t)^2)\mathrm{Tr}(\rho_{eqm,A}(\beta,\mu_i)^2)]^{1/2}}$$

where the dynamical reduced density matrix of 'A' is

$$\rho_{dyn,A}(t) = \mathrm{Tr}_{\bar{A}}\ \rho_{dyn}(t),\ \rho_{dyn}(t) \equiv (\exp[-iHt]|\psi_0\rangle\langle\psi_0|\exp[iHt])$$

For Calabrese-Cardy(CC) state $e^{-\kappa_2 H}|B\rangle$

$$1 - I(t) \sim -\alpha(l)e^{-8\pi ht/(4\kappa_2)}$$

Cardy's preprint and our paper, arXiv: 1501.04580(MSS)

Turning on chemical potentials

If the final theory have other conserved charges. In MSS, we proposed that the quench state(from ground state) is

$$|\psi_0\rangle = \exp[(-\kappa_2 H - \kappa_3 W_3 - \kappa_4 W_4 - ....)]|Bd\rangle$$

where W's are conserved charges of local currents. In MSS, we showed that the above state thermalizes to a Generalized Gibb's Ensemble(GGE), define by the density matrix,

$$\rho_{GGE} = \exp^{-4\kappa_2 H - 4\kappa_3 W_3 - 4\kappa_4 W_4 - \dots}$$

We are going to call $|\psi_0\rangle$, the generalized CC state.

One point function and thermalization

Considering only one chemical potential, in long time limit,

$$\begin{aligned}\langle \phi_k(w,\bar{w})\rangle^{\mu}_{str} &\sim \exp[-4\pi h t/(4\kappa_2) - (4\kappa_n)Q_n 2\pi t/(4\kappa_2)^{n-1} + ...)] \\ &\sim \exp[-4\pi h t/\beta - 2\pi t \mu_n Q_n/\beta^{n-1} + O(\mu^2))]\end{aligned}$$

Turning on other higher spin currents, $W_3, W_4, W_5, ...$, with $\hat{t} = 2\pi t/\beta$, the 1-point function on the strip becomes

$$\langle \phi(w,\bar{w})\rangle^{\vec{\mu}}_{St} = \exp(-2(\vec{\tilde{\mu}}.\vec{q})\hat{t})(1 + O(\mu\hat{t}) + O(\mu^2\hat{t}^2) + \dots)$$

And

$$I(t) = 1 - \alpha(I)\exp(-4(\vec{\tilde{\mu}}.\vec{Q})\hat{t})$$

Quench in specific models: mass quench of free bosons

EOM: Preserving spatial translational symmetry.

$$\frac{d^2\phi(k,t)}{dt^2} + (k^2 + m^2(t))\phi(k,t) = 0$$

We will always consider $\liminf m^2(t) = 0$. The in$(t \to -\infty)$ modes u_{in}'s and the out modes$(t \to \infty)$ u_{out}'s are related by a Bogoluibov transformation and the corresponding oscillators a's

$$\begin{aligned} u_{in}(k,x,t) &= \alpha(|k|)u_{out}(k,x,t) + \beta(|k|)u^{\dagger}_{out}(k,x,t) \\ a_{in}(k) &= \alpha^*(|k|)a_{out}(k) - \beta^*(|k|)a^{\dagger}_{out}(k) \end{aligned}$$

So, the 'in' ground state is

$$
\begin{aligned}
|0, in\rangle &= \exp[\sum_k \gamma(|k|) a^\dagger_{out}(k) a^\dagger_{out}(-k)]|0, out\rangle \\
&= \exp[\sum_k \kappa(k) a^\dagger_{out}(k) a_{out}(k)]|D\rangle
\end{aligned}
$$

And for critical quench or free limit of the Schrodinger equation, we have the expansion,

$$
\begin{aligned}
-\gamma(|k|) &= 1 + r_1|k| + r_2 k^2 + ... \\
\Rightarrow \kappa(|k|) &= -\kappa_2|k| - \kappa_3 k^2 - \kappa_4|k|^3 - ....
\end{aligned}
$$

Hence, we have,

$$|0, in\rangle = \exp[-\kappa_2 H - \kappa_3 \mathcal{W}_3 - \kappa_4 \mathcal{W}_4 - ...]|D\rangle$$

where $H = \sum_k |k| a^\dagger_{out}(k) a_{out}(k)$, $\mathcal{W}_3 = \sum_k k^2 a^\dagger_{out}(k) a_{out}(k)$, $\mathcal{W}_4 = \sum_k |k|^3 a^\dagger_{out}(k) a_{out}(k)$.
This is the general form of the gCC state. Indeed, this state is obtained in exactly solvable examples of mass profile. If we start from a squeezed state,

$$\Rightarrow \quad |f\rangle = \exp[\sum_k f(|k|) a^\dagger_{in}(k) a^\dagger_{in}(-k)]|0, in\rangle$$

then $\kappa(|k|)$ is modified.
Similar results are also obtained for fermions, where the effective potential becomes complex.

Exact calculations in Sudden Quench

For free scalars, in the massive-to-massless sudden limit:

$$|0, in\rangle = \quad \exp\Big[\sum_k \left(-\frac{|k|}{m_0} + \frac{|k|^3}{6m_0^3} - \frac{3|k|^5}{40m_0^5} + \ldots\right) a^\dagger_{out}(k) a_{out}(k)\Big]|D\rangle$$

Calabrese-Cardy and Cardy-Sotariadis argued that in $m_0 \to \infty$ limit, only the first term survives.

$$\begin{aligned} \langle 0, in|\partial\phi(x_1, t)\bar{\partial}\phi(x_2, t)|0, in\rangle &= -2m_0^2 K_0(m_0(r + 2t)) \\ \langle 0, in|\partial\phi(x_1, t_1)\partial\phi(x_2, t_2)|0, in\rangle &= -m_0^2\sqrt{\frac{2}{\pi}} K_2(m_0 r) \\ H &= m_0^2/(16\pi) \end{aligned}$$

These don't agree with the results of $\psi_0\rangle = e^{-\kappa_2 H}|Bd\rangle$, where $\kappa_2 = 1/m_0$.

Special squeezed state to give CC state

By taking special functions $f(|k|)$, we can obtain CC state in the 'out' modes. With

$$f(|k|) = 1 - \frac{2|k|}{|k| + (k^2 + m_0^2)^{1/2} tanh(\kappa_2 |k|)}$$

$$|f(|k|), in\rangle = \exp[-\kappa_2 H]|D\rangle$$

$$\langle f(k), in|\partial\phi(0,t)\bar{\partial}\phi(0,t)|f(k), in\rangle = \frac{1}{4}\frac{\pi^2}{\kappa_2^2}\,\mathrm{sech}^2\left(\frac{1}{4}\frac{\pi}{\kappa_2}(2t)\right)$$

$$\langle f(|k|), \mathrm{in}|\partial\phi(0,t)\partial\phi(r,t)|f(|k|), \mathrm{in}\rangle = \frac{1}{4}\frac{\pi^2}{\kappa_2^2}\,\mathrm{csch}^2\left(\frac{\pi r}{4\kappa_2}\right)$$

$$H = \frac{\pi}{96\kappa_2^2}$$

effective temperature $\beta = 4\kappa_2$. Subleading terms in k/m_0 are important.

gCC state with W_4 perturbation

Starting with squeezed state with

$$f(|k|) = 1 - \frac{2|k|}{|k| + \sqrt{k^2 + m^2} tanh\left(\kappa_2 |k| + \kappa_4 |k|^3\right)}$$

In terms of the out modes,

$$|f(|k|), in\rangle = \exp\left[\sum_k (-\kappa_2 |k| - \kappa_4 |k|^3) a^\dagger_{out}(k) a_{out}(k)\right] |D\rangle$$

$$\Rightarrow \qquad |f(|k|), in\rangle = \exp\left[-\kappa_2 H - \kappa_4 W_4\right] |D\rangle$$

where κ_2 and κ_4 are arbitrary.

Scalar one point function:

$$\langle \partial\phi(0,t)\bar{\partial}\phi(0,t)\rangle = \int \frac{dk}{8\pi} e^{-2ikt} k \ \mathrm{cosech}\left(2\kappa_2 k + 2k^3\kappa_4\right)$$

The three simple poles for each $n \in Z$,

$$k_1 = \frac{-2\ 6^{2/3}\kappa_2 + \sqrt[3]{6}\left(\sqrt{48\kappa_2^3 - 81\pi^2\kappa_4 n^2} + 9i\pi\sqrt{\kappa_4}n\right)^{2/3}}{6\sqrt[3]{\sqrt{3}\sqrt{\kappa_4^3\left(16\kappa_2^3 - 27\pi^2\kappa_4 n^2\right)} + 9i\pi\kappa_4^2 n}}$$

$$k_2 = \frac{4\sqrt[3]{-6}\kappa_2 + i\left(\sqrt{3}+i\right)\left(\sqrt{48\kappa_2^3 - 81\pi^2\kappa_4 n^2} + 9i\pi\sqrt{\kappa_4}n\right)^{2/3}}{2\ 6^{2/3}\sqrt[3]{\sqrt{3}\sqrt{\kappa_4^3\left(16\kappa_2^3 - 27\pi^2\kappa_4 n^2\right)} + 9i\pi\kappa_4^2 n}}$$

$$k_3 = -\frac{\sqrt[3]{-1}\left(2\sqrt[3]{-6}\kappa_2 + \left(\sqrt{48\kappa_2^3 - 81\pi^2\kappa_4 n^2} + 9i\pi\sqrt{\kappa_4}n\right)^{2/3}\right)}{6^{2/3}\sqrt{\kappa_4}\sqrt[3]{\sqrt{48\kappa_2^3 - 81\pi^2\kappa_4 n^2} + 9i\pi\sqrt{\kappa_4}n}}$$

k_1 is the κ_4 perturbative branch. Taking the leading order, which is given by the $n = \pm 1$ poles, we find the total residue

$$\langle \partial\phi(0,t)\bar{\partial}\phi(0,t)\rangle$$
$$= -\frac{\pi}{16\kappa_2^2}\left(1 + 4\pi^2\tilde{\kappa}_4 + 48\pi^4\tilde{\kappa}_4^2\right)\exp\left(-\frac{4\left(\pi + 4\pi^3\tilde{\kappa}_4 + 48\pi^5\tilde{\kappa}_4^2\right)t}{4\kappa_2}\right)$$

This agrees with MSS.

Other thermalizing states

$e^{-\kappa_2 H}|Bd\rangle$ is not unique.

- Using conformal transformations of compact support(done in arXiv: 1405.6695).
- Break conformal invariant boundary condition, $|\psi_0\rangle = e^{-\kappa_2 H}|bB\rangle$, where $|bB\rangle$ satisfies $(L_n - \bar{L}_{-n})|bB\rangle = 0$ except for finite number of integers n(ongoing works).

Thank you