Exact Path Integral for 3D Quantum Gravity

Akinori Tanaka (RIKEN)

Based on

- PRL. 115 no. 16, (2015) 161304, arXiv:1504.05991 [hep-th]
- arXiv:1510.02142 [hep-th]
- arXiv:1511.07546 [hep-th]

Collaborators

Masazumi Honda (Weizmann Institute)
Norihiro Iizuka (Osaka Univ.)
Seiji Terashima (YITP Kyoto)

"Quantum gravity" partition function

Localization technique $+ \sum_{\text{geomertries}}$

Spin = 2 : $c_{\text{eff}} = 24$ (PRL. 115 no. 16, (2015) 161304, arXiv:1504.05991 [hep-th])

$$Z_{\text{grav}}(q) = \text{“}\, J(q)\, \text{”}$$

Spin ≧ 2 $c_{\text{eff}} \to \infty$ (arXiv:1510.02142 [hep-th] arXiv:1511.07546 [hep-th])

$$\tilde{Z}_{\text{grav}}(q) = Z_{\text{vac}}(q) + \sum_{\Delta=1}^{\infty} c_{\Delta}^{(k_{\text{eff}})} Z_{\text{primary}}^{\Delta}(q)$$

Vacuum character of

- Virasoro-algebra
- W-algebra

Characters of **primary** for

- Virasoro-algebra
- W-algebra

Whole story

$$\sum_{\text{geometries}} \int \mathcal{D}g_{\mu\nu}\, e^{-S_{\text{grav}}}$$

$$\Downarrow$$

$$\sum_{\text{bdry conds}} \int_{\text{bdry conds}} \mathcal{D}\mathcal{A}\, e^{-S_{CS}}$$

$$\int_{\text{bdry conds}} \mathcal{D}\mathcal{V}\, e^{-S_{SCS}}$$

$$Z_{c,d}(\tau)$$

$$\Downarrow$$

$$Z_{\text{grav}} \equiv \sum_{c\geq 0 \quad (c,d)_{GCD}=1} \left(Z_{c,d}(\tau) - s_{c,d} \right) \Longrightarrow Z_{\text{grav}}(q) = \text{“}\, J(q)\, \text{”}$$

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$$\tilde{Z}_{\text{grav}}(q) = \frac{Z_{\text{grav}}}{Z_{\text{bdry fermion}}} = Z_{\text{vac}}(q) + \sum_{\Delta=1}^{\infty} c_{\Delta}^{(k_{\text{eff}})} Z_{\text{primary}}^{\Delta}(q)$$

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$$\sum_{\text{geometries}} \int \mathcal{D}g_{\mu\nu}\, e^{-S_{\text{grav}}}$$

CS formulation

$$\sum_{\text{bdry conds}} \int_{\text{bdry conds}} \mathcal{D}\mathcal{A}\, e^{-S_{CS}}$$

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Assumptions

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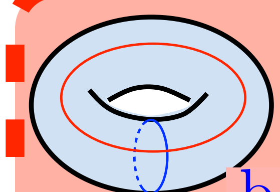

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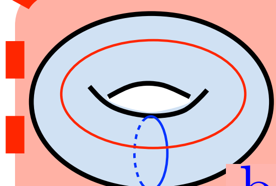

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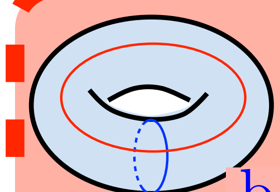

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Thank you!

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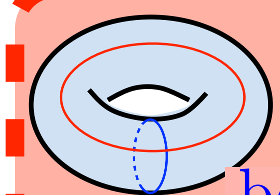

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