ABJM Scattering Amplitudes and a Soft Theorem

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Motivation

Trnka's Lectures!

Scattering Amplitudes of 4D $\mathcal{N}$=4 SYM

give new insight of physics

⇕

How about ABJM scattering amplitudes?

(3D $\mathcal{N}$=6 Chern-Simons Matter Theory)

Basics

Scattering Amplitudes of ABJM Theory [Bargheer Loebbert Meneghelli 10]

- 3D spinor helicity

$$p^{\alpha\beta} = p_\mu (\sigma^\mu)_{\alpha\beta} = \lambda_\alpha \lambda_\beta \quad \longleftrightarrow \quad p_{\alpha\dot\beta} = p_\mu (\sigma^\mu)_{\alpha\dot\beta} = \lambda_\alpha \tilde\lambda_{\dot\beta}$$

$$(p_i + p_j)^2 = -\langle ij\rangle^2 \quad \text{with} \quad \langle ij\rangle \equiv \varepsilon^{\alpha\beta}(\lambda_i)_\alpha (\lambda_j)_\beta$$

- Scattering of matters (On shell super-fields)

$$(U(N)\times U(N))_G \times SU(4)_R$$

$$\phi : (N, \bar N, 4)\,,\ \psi : (N, \bar N, \bar 4) + (\text{c.c})$$

$$\rightarrow$$

$$\Phi(p,\eta) = \phi^4 + \eta^I \psi_I + \frac{1}{2}\varepsilon_{IJK}\eta^I\eta^J\phi^K + \frac{1}{6}\varepsilon_{IJK}\eta^I\eta^J\eta^K\psi_4$$

$$\bar\Phi(p,\eta) = \bar\psi^4 + \eta^I \bar\phi_I + \frac{1}{2}\varepsilon_{IJK}\eta^I\eta^J\bar\psi^K + \frac{1}{6}\varepsilon_{IJK}\eta^I\eta^J\eta^K\bar\phi_4$$

(note. 4D $\mathcal{N} = 4$ SYM : $\Phi = A^+ + \eta^I\chi_I + \frac{1}{2}\eta^J\eta^I\phi_{IJ} + \frac{1}{6}\varepsilon_{IJKL}\eta^I\eta^J\eta^K\chi^L + \eta^1\eta^2\eta^3\eta^4 A^-$)

$$\hat{\mathcal{A}}_{2k}(\bar\Phi_1{}^{a_1}_{\bar a_1}, \Phi_2{}^{\bar b_2}_{b_2}, \bar\Phi_3{}^{a_3}_{\bar a_3}, \ldots, \Phi_{2k}{}^{\bar b_{2k}}_{b_{2k}}) = \sum_{\sigma\in S_k,\ \bar\sigma\in\bar S_{k-1}} \mathcal{A}_{2k}(1, \sigma_1, \bar\sigma_1, \ldots, \bar\sigma_{k-1}, \sigma_k) \times \delta^{\bar b_{\sigma_1}}_{\bar a_1} \cdots \delta^{\bar b_{\sigma_k}}_{\bar a_{\bar\sigma_1}} \delta^{a_{\bar\sigma_1}}_{b_{\sigma_1}} \cdots \delta^{a_1}_{b_{\sigma_k}}$$

i denotes $\{\lambda_i, \eta_i\} \equiv \Lambda_i$

Grassmannian [Arkani-Hamed Cachazo Cheung Kaplan 09]

Orthogonal Grassmannian Formulation [Lee 11]

- ABJM scattering amplitudes are given by

$$\mathcal{L}_{2k}(\Lambda) = \int \frac{d^{k\times 2k}C}{\text{vol[GL(k)]}} \frac{\delta^{k(k+1)/2}(C\cdot C^T)\delta_B^{2k}(C\cdot\lambda)\delta_F^{3k}(C\cdot\eta)}{M_1 M_2\cdots M_{k-1}M_k}$$

$$C = \underbrace{\begin{pmatrix} c_{11} & c_{12} & \cdots\cdots & c_{12k} \\ c_{21} & c_{22} & \cdots\cdots & c_{22k} \\ \vdots & \vdots & \ddots & \vdots \\ c_{k1} & c_{k2} & \cdots\cdots & c_{2k2k} \end{pmatrix}}_{2k} \; k \qquad C \sim gC \quad \text{where} \quad g \in GL(k)$$

e.g. Det(↑) $= M_2$

# of free variables = $2k^2 - k^2 - \frac{k(k+1)}{2} - (2k-3) = \frac{(k-2)(k-3)}{2}$ (e.g. 4,6-point, no free variable)

Orthogonal Grassmannian

U-gauge [Berkovits Cherkis 04, Chin Lee YY 15]

$$OG(k,2k) \simeq SO(2k)/U(k)$$

→

$$w_m^\alpha = \lambda_{2m}^\alpha + \lambda_{2m-1}^\alpha$$
$$v^{m\alpha} = \lambda_{2m}^\alpha - \lambda_{2m-1}^\alpha$$

$$C = \begin{pmatrix} 1 & 0 & \cdots & 0 & c_{11} & \cdots & c_{1k} \\ 0 & 1 & \cdots & 0 & c_{21} & \cdots & c_{2k} \\ \vdots & \vdots & \ddots & 0 & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & c_{k1} & \cdots & c_{kk} \end{pmatrix}$$

→

$$C = \begin{pmatrix} 1 & 1 & u_{12} & -u_{12} & \cdots\cdots & u_{1k} & -u_{1k} \\ -u_{12} & u_{12} & 1 & 1 & \cdots\cdots & u_{2k} & -u_{2k} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -u_{1k} & u_{1k} & -u_{2k} & u_{2k} & \cdots\cdots & 1 & 1 \end{pmatrix}$$

1. $C \cdot g \cdot C^T = 0$ $\qquad g = \text{diag}\underbrace{(-1,1,\ldots,-1,1)}_{2k}$

2. $\sum_{i=1}^{2k} C_{mi} \cdot \lambda_i = w_m + u_{mn} v^n = 0$

↓

$$\hat{u}_{mn}(z) = u^*_{mn} + \frac{1}{2(k-4)!} z^{p_1 \cdots p_{k-4}} \varepsilon_{mnp_1\cdots p_{k-4}rs} \bar{u}^{rs}_*$$

$$u^*_{mn} = \frac{1}{R}\langle w_m w_n \rangle\,, \quad \bar{u}^{mn}_* = -\frac{1}{R}\langle v^m v^n \rangle \quad \text{with} \quad R \equiv \frac{\langle w_p v^p \rangle}{2}$$

8-Point Amplitude

- 8-point amplitude reduces to a **contour integral of 'z'** $\quad (\hat{u}_{mn}(z) = u^*_{mn} + \frac{z}{2}\varepsilon_{mnpq}\bar{u}^{pq}_*)$

$$\mathcal{A}_8 = J_B J_F \delta^3(P)\delta^6(Q) \oint_{\mathcal{C}} \frac{dz}{2\pi i} \frac{\prod_5^7 (a_j z^2 + b_j z + c_j)}{M_1(z)M_2(z)M_3(z)M_4(z)} \quad \text{with} \quad M_i(z) = a_i z^2 + b_i z + c_i$$

- Contour description from **on-shell diagram** [Huang Wen 13]

$$M_2(z) = 0 \quad \text{and} \quad M_4(z) = 0 \quad (\text{or} \;\; M_1(z) = 0 \quad \text{and} \quad M_3(z) = 0)$$

$$\boxed{\mathcal{A}_8 = \delta^3(P)\delta^6(Q)(1+\pi)J\left(\frac{F(2)}{\Delta_{21}\Delta_{23}\Delta_{24}} + \frac{F(4)}{\Delta_{41}\Delta_{42}\Delta_{43}}\right)}, \quad J = 2(\frac{2}{R})^4$$

$$\Delta_{21} = -\frac{2^6}{R^4} p^2_{234}\, p^2_{678} \quad \Delta_{23} = -\frac{2^6}{R^4} p^2_{345}\, p^2_{781} \quad \Delta_{24} = \Delta_{42}$$

$$F(2) = -K_{22} J_{21}{}^{(5} J_{23}{}^{6} J_{24}{}^{7)} + \frac{3}{4} L^2{}_{(1}{}^{2}{}_{3} J_{4)2(5} L^2{}_{6}{}^{2}{}_{7)}$$

Soft Theorem

Soft Graviton & Photon [Weinberg 65, Cachazo Strominger 14, Casali 14]

- Soft limit : a graviton(photon) approaches vanishing momentum

$$M_{n+1}(p_1,\cdots,p_n,q)|_{q\to 0} = (S^{(0)}+S^{(1)}+S^{(2)})M_n(p_1,\cdots,p_n)+\mathcal{O}(q^2)$$

soft graviton $(q\to 0, \varepsilon^{\mu\nu})$

$$S^{(0)} = \sum_{a=1}^{n}\frac{\varepsilon_{\mu\nu}p_a^\mu p_a^\nu}{q\cdot p_a}\,,\quad S^{(1)} = -i\sum_{a=1}^{n}\frac{\varepsilon_{\mu\nu}p_a^\mu(q_\rho J_a^{\rho\nu})}{q\cdot p_a}\,,\quad S^{(2)} = -\frac{1}{2}\sum_{a=1}^{n}\frac{\varepsilon_{\mu\nu}(q_\rho J_a^{\rho\mu})(q_\sigma J_a^{\sigma\nu})}{q\cdot p_a}$$

Weingberg — Cachazo,Strominger — Cachazo,Strominger

- IR divergence of the theory
- Universal behavior of scattering amplitudes
- Symmetries of the theory(gauge,Lorentz,SSB,BMS,…)

ABJM Soft Theorem

ABJM Soft Theorem & Proof [Chin Lee YY 15]

- Double Soft Theorem

$$\mathcal{A}_{2k+2}(1,2,\cdots,2k,\varepsilon^2 p_{2k+1},\varepsilon^2 p_{2k+2})\big|_{\varepsilon\to 0} = \left(\frac{1}{\varepsilon^2}S^{(0)} + \frac{1}{\varepsilon}S^{(1)}\right)\mathcal{A}_{2k}(1,2,\cdots,2k)\,.$$

$$S^{(0)} = \frac{1}{2\langle 1,2k\rangle}\left[\frac{\delta^3(\bar\theta_{k+1})}{\alpha_+\beta_+} - \frac{\delta^3(\theta_{k+1})}{\alpha_-\beta_-}\right]$$

$$S^{(1)} = \frac{1}{2\langle 1,2k\rangle\alpha_+\beta_+}\left[\frac{1}{2}\varepsilon_{IJK}\bar\theta^I_{k+1}\bar\theta^J_{k+1}\xi^K_+ + \delta^3(\bar\theta_{k+1})\left(\beta_+R_{2k+2,1} - \alpha_+R_{2k+1,2k}\right)\right]$$
$$+ \frac{1}{2\langle 1,2k\rangle\alpha_-\beta_-}\left[\frac{1}{2}\varepsilon_{IJK}\theta^I_{k+1}\theta^J_{k+1}\xi^K_- + \delta^3(\theta_{k+1})\left(\beta_-R_{2k+2,1} + \alpha_-R_{2k+1,2k}\right)\right]$$

$$\alpha_\pm = \frac{\langle 1,2k+1\rangle \pm \langle 1,2k+2\rangle}{\langle 1,2k\rangle}$$

$$\beta_\pm = \frac{\langle 2k,2k+1\rangle \pm \langle 2k,2k+2\rangle}{\langle 1,2k\rangle}$$

$$\xi_+ = -\alpha_+\eta_{2k} + \beta_+\eta_1$$

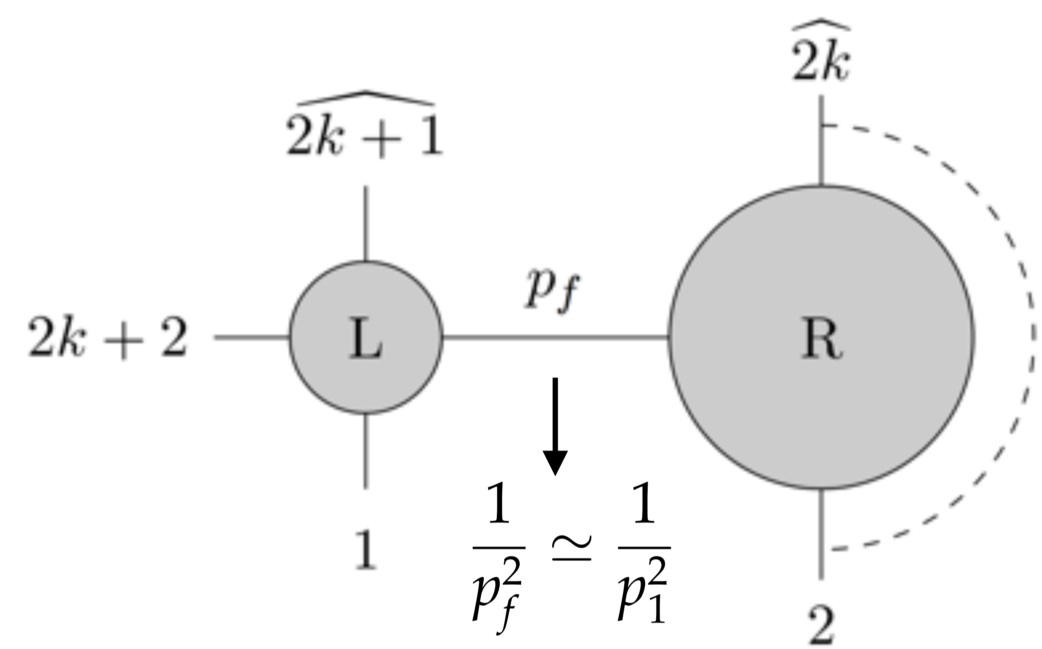
$$R_{i,j} = \eta_i\frac{\partial}{\partial\eta_j}$$

- Proof

3D recursion relation [Gang Huang Koh Lee Lipstein 10]

$$\hat\lambda_{2k} = c\lambda_{2k} + s\lambda_{2k+1}\,, \qquad \hat\eta_{2k} = c\eta_{2k} + s\eta_{2k+1}$$
$$\hat\lambda_{2k+1} = s\lambda_{2k} + c\lambda_{2k+1}\,, \qquad \hat\eta_{2k+1} = s\eta_{2k} + c\eta_{2k+1}$$

with $c = \cosh(\theta),\ s = \sinh(\theta)$



Conclusion & Future Works

We develop the systematic tool for higher point ABJM amplitudes

- F(2) & F(4) are rather complicated.
  → Simplification? or New notations?

- 10-point and higher-point amplitudes
  → Should we solve the $M_i(z_k)=0$?? (Scattering Equations & CHY formula)

Doubles soft theorem for ABJM amplitudes

- Physical understanding of the soft theorem (Strominger, BMS symmetry)

- Grassmannian proof

ABJM loop amplitudes (now 4-point 2,4-loop, 6-point 2-loop)

Amplituheron for ABJM ??

Thank you!