Scaling symmetry and the generalized Smarr relation

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Talk Based on works

collaboration with B.Ahn, S.Hyun, J.Jeong, K.Kim, S.Yi

- Scaling symmetry and scalar hairy Lifshitz black holes
  1507.03574 JHEP 10:105 (2015)
- Scaling symmetry and scalar hairy rotating AdS_3 black holes
  1508.06484 to appear in PRD
- Holography without Counter Terms
  1512.09319

also related with,

- Quasi-local charges and asymptotic symmetry generators
  1403.2196 JHEP 1406:151 (2014)
- Quasi-local conserved charges and holography
  1406.7101 Phys. Rev. D 90:104016 (2014)
- Frame-independent holographic conserved charges
  1410.1312 Phys. Rev. D 91:064052 (2015)

which are generalizations of the ‘off-shell’ formalism for the conserved charges.
[Kim, Kulkarni, Yi ’13]

Introduction

BH Thermodynamics and Smarr relation

- Basic physical quantities of black holes in gravity $\Rightarrow$ Conserved charges
- The 1st law of black hole thermodynamics

$$\delta M = T_H \delta \mathcal{S}_{BH} + \Omega_H \delta J + \Phi_H \delta Q$$

Black hole entropy is also the Noether charge [Wald '93]

$$\frac{\kappa}{2\pi}\delta \mathcal{S}_{BH} = \frac{1}{16\pi G}\int_{\mathcal{H}} dx_{\mu\nu}\delta K^{\mu\nu}(\xi_H)$$

- The Smarr relation for charged Kerr sol. [Smarr '73]

$$M = 2T_H S + \Omega_H J + \Phi_H Q$$

- Scaling symmetry gives Smarr relation
  - 3 dim Einstein gravity with an minimally coupled scalar hair in AAdS geometry. [Banados, Theisen '05]
  - We generalize this formulation.

'Scaling symmetry' of the reduced action

Ansatz

- metric : $ds^2 = -e^{2A(r)} f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\mathbf{x}^2_{D-2}$
- for matter fields : $\mathcal{A}_\mu = \mathcal{A}_\mu(r),\ \varphi = \varphi(r), \cdots$

Reduced action

$$I_{\text{red}} = \frac{1}{16\pi G} \int dr d\mathbf{x}\, L(r, \Psi)\,, \qquad \Psi \equiv (A, f, \mathcal{A}_\mu, \varphi, \cdots)$$

$$\delta I_{\text{red}} = \frac{1}{16\pi G} \int dr d\mathbf{x} \left[\mathcal{E}_\Psi \delta\Psi + \Theta^r (\delta\Psi)' \right], \qquad {}' \equiv \frac{\partial}{\partial r}$$

Consider the transformation, $\delta_\sigma \Psi = \sigma(\omega\Psi - r\Psi')\,,$ s.t.

$$\delta_\sigma I_{\text{red}} = \frac{1}{16\pi G} \int dr d\mathbf{x}\, S^{r\prime}\,, \qquad S^r = -rL(r, \Psi)$$

Noether charge : $C = C(r) \equiv \frac{1}{8G} \Big[\Theta(\delta_\sigma \Psi) - S \Big]$

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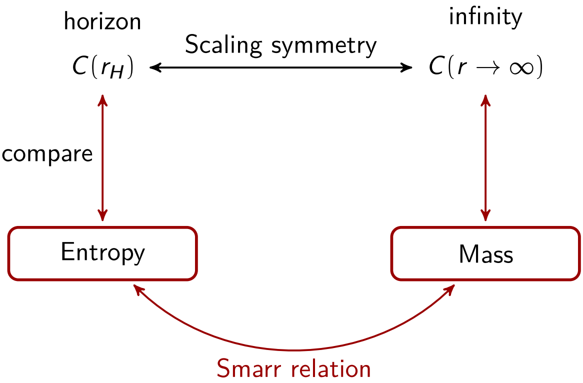
Noether charge : $\quad C = C(r) \equiv \frac{1}{8G}\left[\Theta(\delta_\sigma\Psi) - S\right]$ $\qquad$ Meaning of C?

Smarr relation



$$T_H \mathcal{S}_{BH} = \frac{1}{16\pi G} \int_{\mathcal{H}} dx_{\mu\nu} \Delta K^{\mu\nu}(\xi_H)$$

$$M = \int \delta M = \frac{1}{16\pi G} \int ds \int_{\infty} dx_{\mu\nu} \Big(\delta K^{\mu\nu}(\xi_T) - 2\xi_T^{[\mu} \Theta^{\nu]}(\delta\Psi) \Big)$$

Application on Lifshitz BH

arXiv:1507.03574

1. Model : NMG coupled with a scalar field in 3 dim

$$\mathcal{L}[g,\varphi] = \eta\Big[R - 2\Lambda + \frac{1}{m^2}\Big(R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}R^2\Big)\Big] - \frac{1}{2}(\partial\varphi)^2 - \frac{\alpha}{2}R\varphi^2 - V(\varphi)$$

$$\Rightarrow C = \frac{\eta}{8G}\Big[e^A\Big(-1 + \lambda - \frac{e^{-A}}{4m^2 r}(Z' - e^A f') + \eta\frac{\alpha}{2}\varphi^2\Big)(2f - rf') + \lambda Z + e^A\Big(-2rf\lambda' + \eta r^2 f\varphi'^2\Big) + \frac{e^{-A}}{2m^2r^2}Z(2Z - rZ')\Big]$$

$$T_H\mathcal{S}_{BH} = C = (1+z)M$$

2. Model : Einstein-Maxwell-dilaton gravity in D dim

$$\mathcal{L}[g,\phi,\mathcal{A}] = R - 2\Lambda - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{\lambda\phi}\mathcal{F}^2\,, \quad \Lambda = -\frac{(D+z-2)(D+z-3)}{2}$$

$$\Rightarrow C = -\frac{r^{D-3}e^A}{8G}\Big[(D-2)\Big(2f - rf'\Big) - r^2 f\phi'^2\Big] - \frac{(D-2)}{8G}q\,a$$

$$(D-2)T_H\mathcal{S}_{BH} = C + \frac{D-2}{8G}q\,a(r_H) = (D+z-1)M$$

Modify: including spatial dependences

Ansatz

- metric : $ds^2 = -e^{2A(r,\mathbf{x})} f(r,\mathbf{x}) dt^2 + \frac{dr^2}{f(r,\mathbf{x})} + r^2 d\mathbf{x}^2_{D-2}$
- for matter fields : $\mathcal{A}_\mu = \mathcal{A}_\mu(r,\mathbf{x}),\ \varphi = \varphi(r,\mathbf{x}), \cdots$

Consider the transformation,

$$\delta_\sigma \Psi = \sigma(\omega\Psi - r\Psi')\,, \quad \delta_\sigma \Psi_{,i} = \sigma\Big((\omega+1)\Psi_{,i} - r(\Psi_{,i})'\Big)\,, \quad \delta_\sigma \alpha_n = \sigma\omega_n\alpha_n\,,$$

which gives

$$\delta_\sigma I_{\text{red}} = \frac{1}{16\pi G}\int dr d\mathbf{x}\left(\mathcal{E}_\Psi \delta_\sigma \Psi + \partial_a \Theta^a(\delta_\sigma\Psi) + \frac{\delta I_{\text{red}}}{\delta \partial_i \Psi}\left[\delta_\sigma, \partial_i\right]\Psi + \sum_n \frac{\delta I_{\text{red}}}{\delta \alpha_n}\delta_\sigma \alpha_n\right).$$

'Charge function'

$$C(r) - C(r_H) = \int_{r_H}^{r} dr \int d\mathbf{x}\left[-\sum_{i=1}^{D-2}\frac{\delta I_{\text{red}}}{\delta\,\partial_i \Psi}\partial_i \Psi - \sum_n \omega_n \alpha_n \frac{\delta I_{\text{red}}}{\delta \alpha_n}\right]_{\text{on-shell}}$$

Application to rotating BH

arXiv:1508.06484

For rotating BH with one Killing vector:

- $ds^2 = -f(r,y)e^{2A(r,y)}dt^2 + \frac{dr^2}{f(r,y)} + r^2(d\theta - (r,y)dt)^2$, $\varphi = \varphi(r,y)$, $y \equiv \Omega_H t - \theta$
- $\xi_K = \frac{\partial}{\partial t} + \Omega_H \frac{\partial}{\partial \theta} \equiv \xi_T + \Omega_H \xi_R$

Conserved charges using asymptotic Killing vectors

- $M_\infty = \frac{1}{16\pi G} \int ds \int_\infty dx_{\mu\nu} \Big[\delta K^{\mu\nu}(\xi_T) - 2\xi_T^{[\mu}\Theta^{\nu]} + \sqrt{-g}\mathbf{A}^{\mu\nu} \Big]$
- $J_\infty = \frac{1}{16\pi G} \int_\infty dx_{\mu\nu} \Big[\Delta K^{\mu\nu}(\xi_R) + \int ds \sqrt{-g}\mathbf{A}^{\mu\nu} \Big]$

'Smarr-like' relation

$$M_\infty = \frac{1}{2} T_H \mathcal{S}_H + \Omega_H J_\infty - \frac{1}{32\pi G} \int drdy \left[\frac{\delta I_{\text{red}}}{\delta \dot{\Psi}} \dot{\Psi} \right]_{\text{on-shell}}$$

- issues in integrability
- 1st law still invariant
- thermodynamic stability of hairy BH

Application to holographic CMT models

arXiv:1512.09319

Thermodynamics of dual system

using grand potential for homogeneous system

$$\mathcal{W} \equiv \epsilon - \mu\mathcal{Q} - T_H\mathcal{S}$$

and expression of c

$$c(r_H) = (D-2)T_H\mathcal{S}_{BH}$$

we obtain the expression for the renormalized on-shell action

$$T_H\mathcal{I}^{\text{ren}}_{\text{on-shell}} = \mathcal{W}$$

$$= M - \mu\mathcal{Q} - \frac{1}{D-2}c(r_H)$$

$$= M - \mu\mathcal{Q} - \frac{1}{D-2}\lim_{r\to\infty}\left[c(r) + \int_{r_H}^{r} dr\left(\sum_{i=1}^{D-2}\frac{\delta I_{\text{red}}}{\delta\,\partial_i\Psi}\partial_i\Psi + \sum_n \omega_n\alpha_n\frac{\delta I_{\text{red}}}{\delta\alpha_n}\right)\right]_{\text{on-shell}}$$

without concerning counter terms.

Summary

We obtained 'charge' for scaling symmetry from reduced action. This gives:

- Smarr relation for asymptotically Lifshitz planar-BH [1507.03574]

$$T_H \mathcal{S} = \frac{D+z-2}{D-2} M$$

consistent with known results.

- Smarr-like relation for rotating BH with scalar hair [1508.06484]

$$M_\infty = \frac{1}{2} T_H \mathcal{S}_H + \Omega_H J_\infty - \frac{1}{32\pi G} \int dr dy \left[\frac{\delta I_{\text{red}}}{\delta \dot{\Psi}} \dot{\Psi} \right]_{\text{on-shell}}$$

describing thermodynamic stability.

- Smarr-like relation for CMT model [1512.09319]

$$T_H \mathcal{I}^{\text{ren}}_{\text{on-shell}} = M - \mu Q - \frac{1}{D-2} c(r_H)$$

giving renormalized on-shell action.

Thank you for listening!