SYMMETRIES & INFLATION

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- This talk is based on work done with **Prof. Sandip Trivedi** and **Nilay Kundu**.
- **Main References:**

1. N. Kundu, A. Shukla, and S. P. Trivedi, "*Constraints from Conformal Symmetry on the Three Point Scalar Correlator in Inflation*." **arXiv:1410.2606**
2. N. Kundu, A. Shukla, and S. P. Trivedi, "*Ward Identities for Scale and Special Conformal Transformations in Inflation*." **arXiv:1507.06017**

Also see Prof. Shiu's talks in this school.

- **Other Important References:**

A. J. M. Maldacena, "*Non-Gaussian features of primordial fluctuations in single field inflationary models*." **astro-ph/0210603**

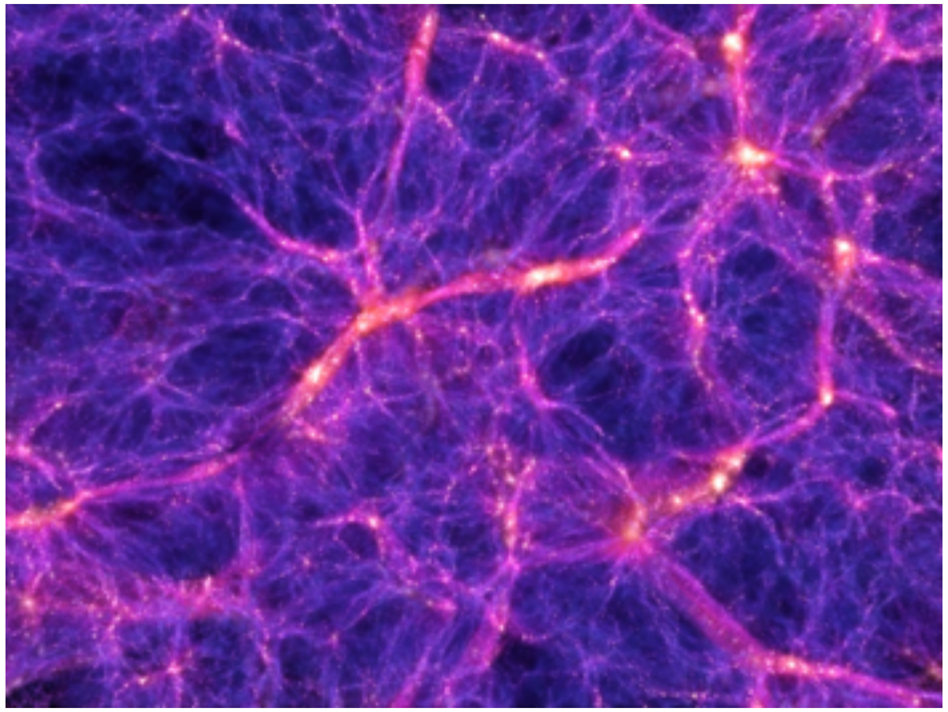
B. J. M. Maldacena and G. L. Pimentel, "*On graviton non-Gaussianities during inflation*." **arXiv:1104.2846**

C. I. Mata, S. Raju, and S. P. Trivedi, "*CMB from CFT*." **arXiv:1211.5482**

D. A. Ghosh, N. Kundu, S. Raju and S. P. Trivedi, "*Conformal Invariance and the Four Point Scalar Correlator in Slow-Roll Inflation*." **arXiv:1401.1426**

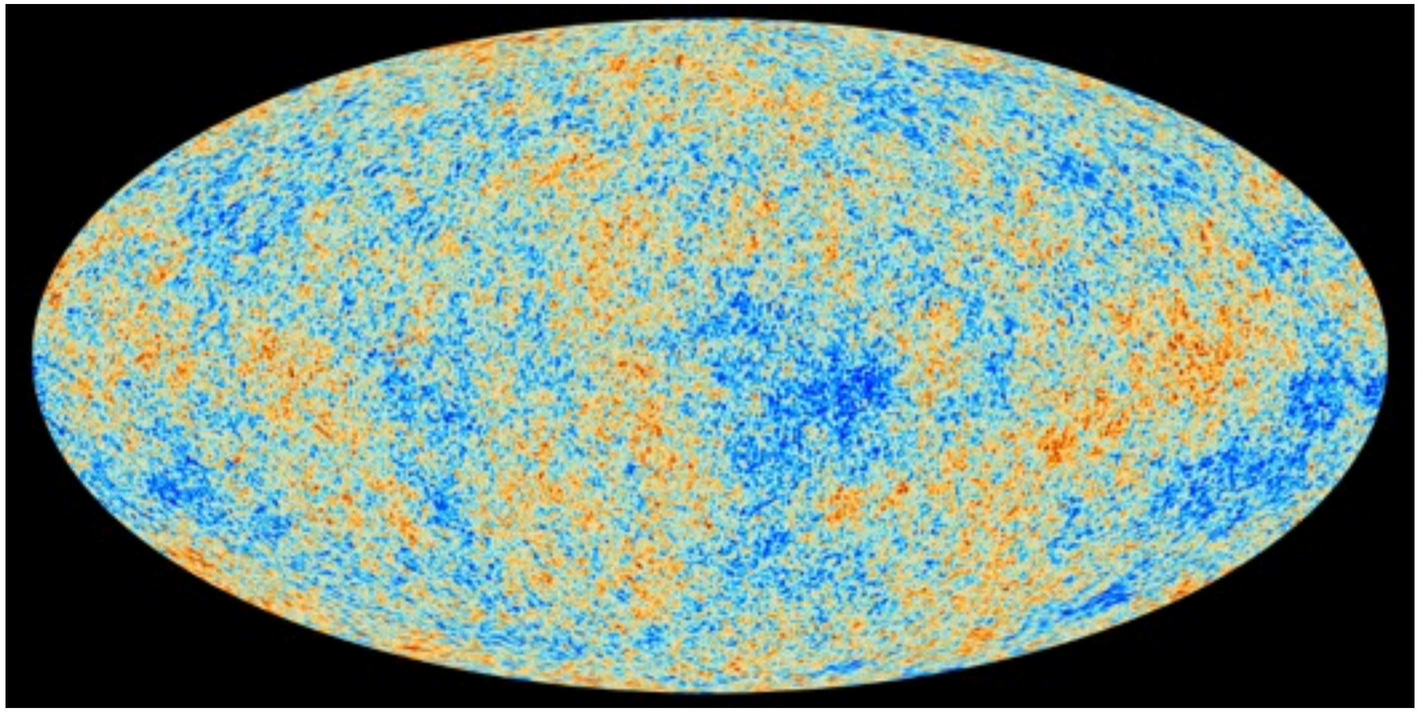
- Inflation has been very successful in explaining the large scale homogeneity and isotropy of the universe.
- It also generates perturbations which seed the growth of:

Large Scale Structure



LSS Simulation (Millenium Run)

CMB Anisotropies



Planck CMB Map

- During inflation, the universe is approximately de Sitter.

$$ds^2 = -dt^2 + e^{2Ht}(dx^2 + dy^2 + dz^2)$$

de Sitter metric

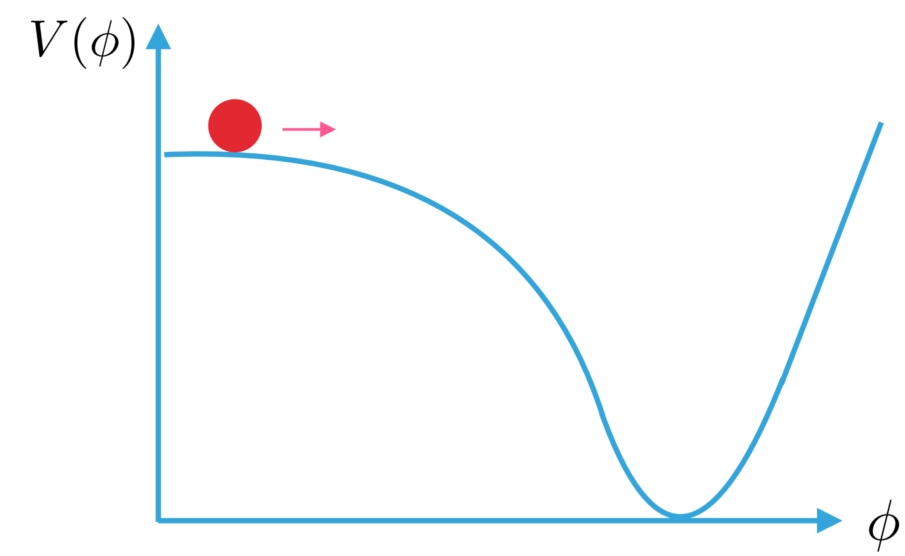
Hubble Rate

- We ask: *What constraints do the approximate de Sitter symmetries impose on the inflationary correlations functions?*

- For our analysis, we consider the single field slow roll model of inflation.

$$S = M_{Pl}^2 \int d^4x \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right) .$$

The potential is slowly varying.



- The symmetry group of 4-Dim de Sitter spacetime is SO(1,4): 10 Generators.
- The symmetry transformations are:

	No. of Generators	Transformation
Translations	3	$x^i \to x^i + \epsilon^i$
Rotations	3	$x^i \to R^i_j x^j$
Dilatation	1	$x^i \to \lambda x^i,\ t \to t - \frac{1}{H}\log(\lambda)$
SCT	3	$x^i \to x^i - 2(b_j x^j)x^i + b^i\left(\sum_j (x^j)^2 - \frac{1}{H^2}e^{-2Ht}\right),$ $t \to t + \frac{2b_j x^j}{H}$

- SO(1,4) is also the symmetry group of a 3-dim Euclidean CFT.

4D de Sitter spacetime $\overset{\text{SO(1,4)}}{\Longleftrightarrow}$ 3D Euclidean CFT

- We use this relationship at the level of symmetry groups to organise our discussion of symmetry constraints.

- The conformal (de Sitter) symmetries during inflation are not exact. Breaking is proportional to the slow roll parameters:

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \; \delta = \frac{\ddot{H}}{2H\dot{H}}, \; \epsilon = \frac{1}{2}\frac{\dot{\bar{\phi}}^2}{H^2}$$

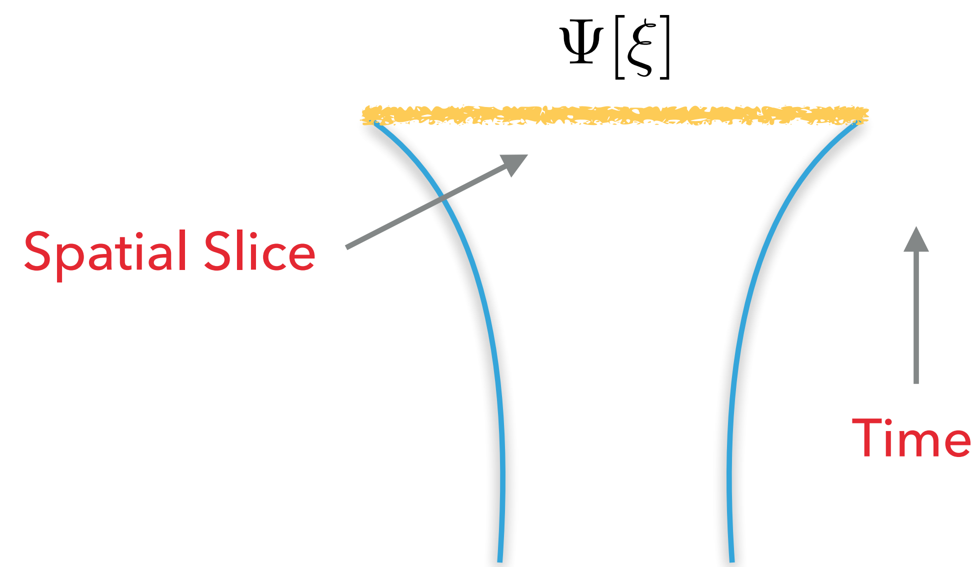
$$\epsilon_1, \delta, \epsilon \ll 1$$

- The discussion of symmetry constraints is best organised in terms of the wave function of the universe.

- The wave function is a "functional" of late time value of inflationary perturbations.

$$\Psi[\xi] = \int_{initial}^{\xi} [\mathcal{D}\xi'] \, e^{iS[\xi']}$$

Late time value of perturbations



We consider Bunch-Davies initial conditions.

- Late time $\Rightarrow$ Modes of interest have left the horizon.

- Invariance of the wave function under symmetry transformations gives us the desired constraints: the Ward identities.

- We work in the ADM formalism.
- The metric in ADM form is

$$ds^2 = -N^2 dt^2 + h_{ij}\,(dx^i + N^i dt)(dx^j + N^j dt)$$

Lapse (N), Shift (N^i)

- Fix diffeomorphism by choosing the gauge $\underbrace{N = 1, N^i = 0}_{\text{Synchronous Gauge}}$
- The form of the metric now is

$$ds^2 = -\,dt^2 + a^2(t)[(1+2\zeta)\delta_{ij} + \widehat{\gamma}_{ij}]$$

Traceless ($\widehat{\gamma}_{ij}$)

- The inflaton is: $\phi = \bar{\phi}(t) + \delta\phi$
- Remaining gauge freedom:

$t \to t + \epsilon(\mathbf{x}), x^i \to x^i + \partial_i \epsilon(\mathbf{x}) \int^t dt' \, \frac{1}{a^2(t')}$	$\delta\phi = 0$
$x^i \to x^i + v^i(\mathbf{x})$	$\partial_i \widehat{\gamma}_{ij} = 0$

- The perturbations are now $\zeta, \widehat{\gamma}_{ij}$

Transverse & Traceless

- The wave function can be expanded as

Correlators in a 3-dim Euclidean CFT

$$\Psi[\zeta, \widehat{\gamma}_{ij}] = \exp\Big[-\frac{1}{2}\int d^3x\, d^3y\, \zeta(\mathbf{x})\zeta(\mathbf{y})\, \langle T(\mathbf{x})T(\mathbf{y})\rangle - \frac{1}{2}\int d^3x\, d^3y\, \widehat{\gamma}_{ij}(\mathbf{x})\widehat{\gamma}_{kl}(\mathbf{y})\, \langle \widehat{T}^{ij}(\mathbf{x})\widehat{T}^{kl}(\mathbf{y})\rangle - \frac{1}{3!}\int d^3x\, d^3y\, d^3z\, \zeta(\mathbf{x})\zeta(\mathbf{y})\zeta(\mathbf{z})\, \langle T(\mathbf{x})T(\mathbf{y})T(\mathbf{z})\rangle + \cdots \Big]$$

- The invariance of the wave function under the left-over coordinate freedom gives us the desired Ward identities.

Scaling SCT

- For instance, under scaling $x^i \to x^i + \lambda x^i,\ \lambda \ll 1$ we get

$$\left(3(n-1) + \sum_{a=1}^{n} k_a \frac{\partial}{\partial k_a}\right)\langle\zeta(\mathbf{k_1})\cdots\zeta(\mathbf{k_n})\rangle' = - \frac{1}{\langle\zeta(\mathbf{k_{n+1}})\zeta(-\mathbf{k_{n+1}})\rangle'}\langle\zeta(\mathbf{k_1})\cdots\zeta(\mathbf{k_{n+1}})\rangle'\Big|_{\mathbf{k_{n+1}}\to 0}$$

Maldacena Consistency Conditions

- Added twist for SCTs: One needs to club an SCT with a *compensating field dependent spatial reparametrization* to maintain the gauge choice.

$$x^i \to x^i - 2(\mathbf{b}\cdot\mathbf{x})x^i + b^i\mathbf{x}^2 - \frac{6b^j\widehat{\gamma}_{ij}}{\partial^2}$$

arXiv:1410.2606

arXiv:1507.06017

$$\big\langle\delta(\zeta(\mathbf{k_1}))\cdots\zeta(\mathbf{k_n})\big\rangle + \cdots + \big\langle\zeta(\mathbf{k_1})\cdots\delta(\zeta(\mathbf{k_n}))\big\rangle = -2\left(\mathbf{b}\cdot\frac{\partial}{\partial\mathbf{k_{n+1}}}\right)\frac{\langle\zeta(\mathbf{k_1})\cdots\zeta(\mathbf{k_{n+1}})\rangle}{\langle\zeta(\mathbf{k_{n+1}})\zeta(-\mathbf{k_{n+1}})\rangle'}\Bigg|_{\mathbf{k_{n+1}}\to 0}$$

$$\delta(\zeta(\mathbf{k})) = \widehat{\mathcal{L}}^{\mathbf{b}}_{\mathbf{k}}\,\zeta(\mathbf{k}) + 6\,b^m k^i\int\frac{d^3\tilde{k}}{(2\pi)^3}\,\frac{1}{\tilde{k}^2}\,\zeta(\mathbf{k}-\tilde{\mathbf{k}})\,\widehat{\gamma}_{im}(\tilde{\mathbf{k}}) + 2\,b^m k^i\int\frac{d^3\tilde{k}}{(2\pi)^3}\,\frac{1}{\tilde{k}^2}\,\widehat{\gamma}_{ij}(\mathbf{k}-\tilde{\mathbf{k}})\,\widehat{\gamma}_{jm}(\tilde{\mathbf{k}})$$

$$\widehat{\mathcal{L}}^{\mathbf{b}}_{\mathbf{k}} = 2\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{k}}\right)\left(\mathbf{b}\cdot\frac{\partial}{\partial\mathbf{k}}\right) - (\mathbf{b}\cdot\mathbf{k})\left(\frac{\partial}{\partial\mathbf{k}}\cdot\frac{\partial}{\partial\mathbf{k}}\right) + 6\left(\mathbf{b}\cdot\frac{\partial}{\partial\mathbf{k}}\right)$$

- Analysis based on symmetries is model independent and robust.

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  Powerful constraints on various models of inflation

- In the future, we would like to study the Ward identities for:

- The possibility of initial states being in α-vacuum, rather then the Bunch-Davies vacuum.

- Non-canonical models of inflation like DBI etc.

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THANK YOU

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