Light interaction with three-level systems:
Coherent Population Trapping (CPT)
Electromagnetically Induced Transparency (EIT)
Stimulated Raman Adiabatic Passage (STIRAP)

Jordi Mompart

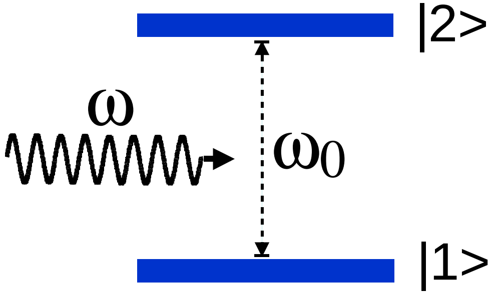
Department of Physics, Universitat Autònoma de Barcelona, Catalonia

From July 7th to July 9th (10:00 to 11:30) 2015,

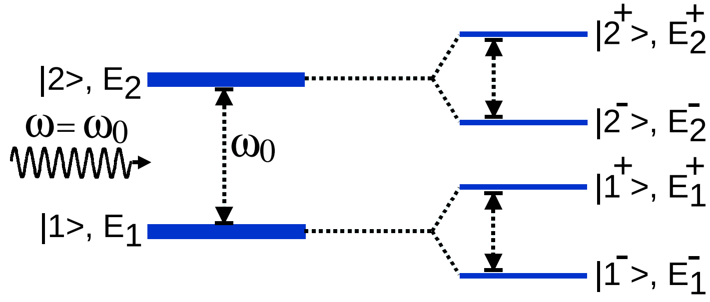
C700, Level C, Lab 3, OIST

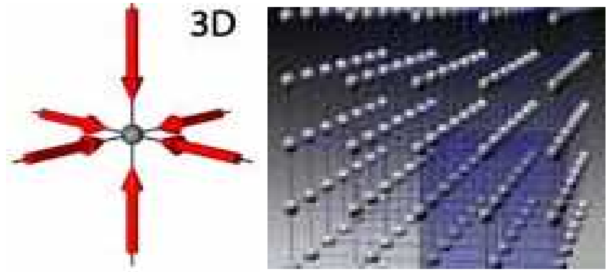
MOTIVATION:
3 IS MUCH MORE THAN 2

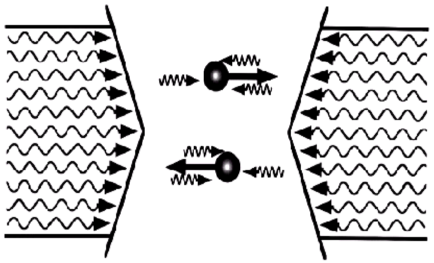
TWO-LEVEL ATOM:

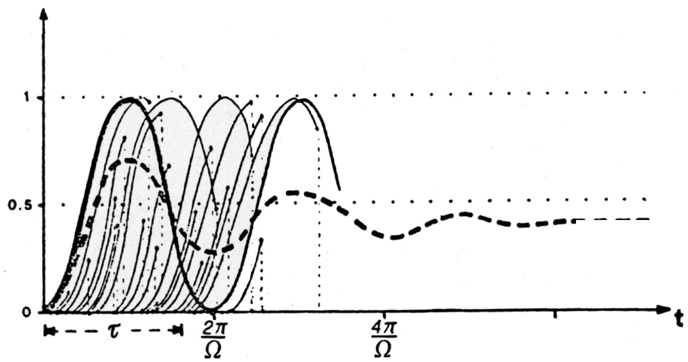


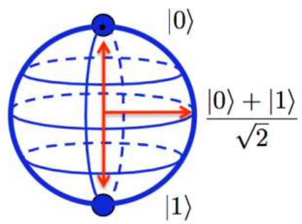
- AC-Stark splitting
- Dressed atom
- Light shifts
- Dipole force
- Rabi oscillations
- Qubit gates
- Optical nutation
- RAP
- Radiation pressure
- ....

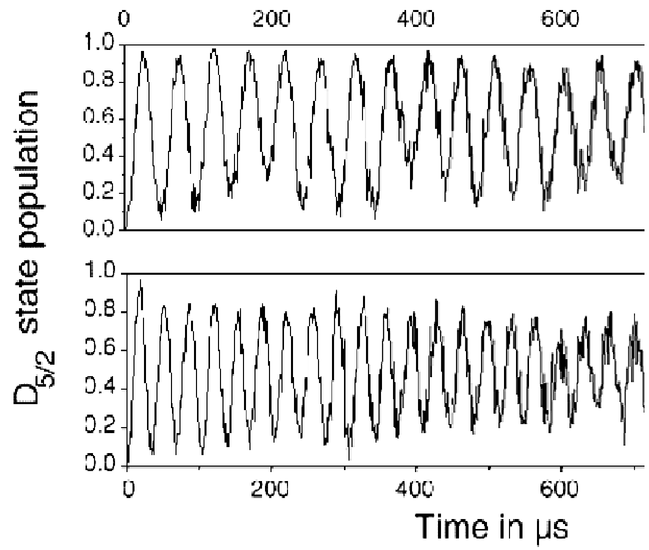






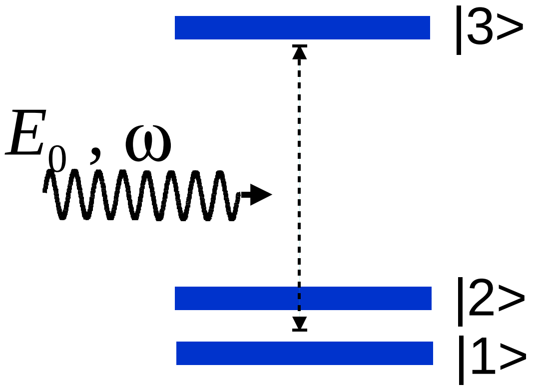






MOTIVATION: 3 IS MUCH MORE THAN 2 (4)

THREE-LEVEL ATOM:



Assume the atom initially in:

$$|\psi(t=0)\rangle = c_1|1\rangle + c_2|2\rangle$$

$$P_{abs} = ct\cdot E_0^2 |c_1 + c_2|^2 = ct\cdot E_0^2 (c_1c_1^* + c_2c_2^* + c_1c_2^* + c_2c_1^*)$$

$$= ct\cdot E_0^2 (\rho_{11} + \rho_{22} + \rho_{12} + \rho_{21})$$

$$= ct\cdot E_0^2 (\rho_{11} + \rho_{22} + 2\,\mathrm{Re}\,\rho_{12})$$

populations

atomic coherence
(quantum interference)

Example: for $|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle)$ $\Rightarrow$ $Pabs = ct\cdot E_0^2 (\rho_{11} + \rho_{22} + 2\,\mathrm{Re}\,\rho_{12}) = 0$

Dark state

ATOMIC COHERENCE EFFECTS

Three-level optical techniques:

- Coherent Population Trapping (CPT)
- Electromagnetically Induced Transparency (EIT)
- Stimulated Raman Adiabatic Passage (STIRAP)

} **DARK STATE PHYSICS**

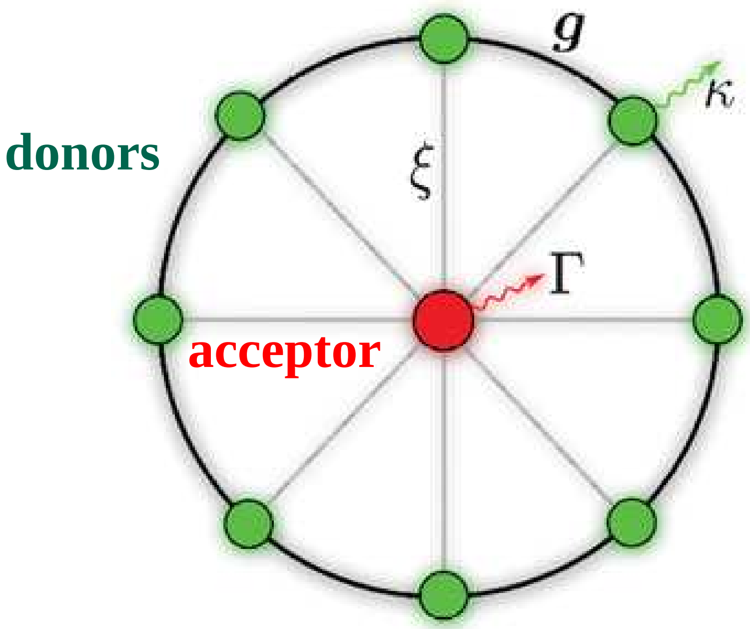
Some applications:

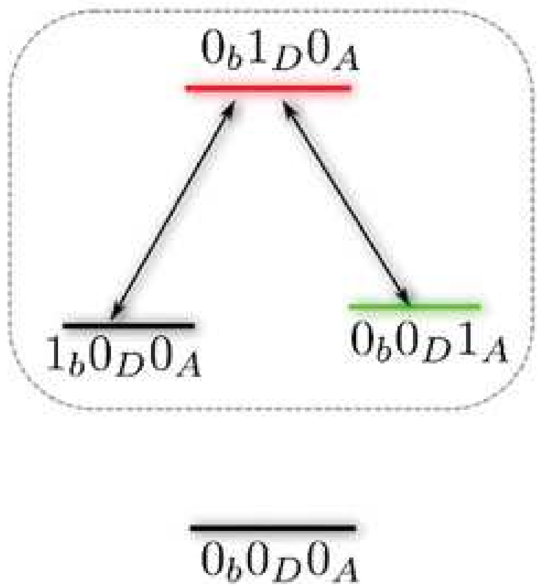
- **Laser cooling** based on CPT
- High precission **magnetometry** (atomic clocks) based on CPT
- **Lasing without inversion** based on EIT or CPT
- **Slow/super luminal** light based on EIT
- **Quantum memories** based on EIT
- Robust and efficient **population transfer** based on STIRAP
- **Subwavelength lithography/microscopy** based on STIRAP
- **Single photon gun** based on STIRAP
- …

RECENT APPLICATION: LIGHT HARVESING

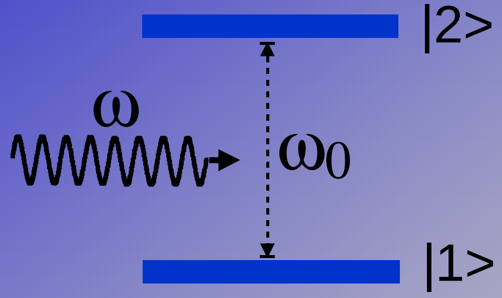
Coherent excitation transfer *via* the dark-state channel in a bionic system
Hui Dong, Da-Zhi Xu, Jin-Feng Huang and Chang-Pu Sun
Light: Science & Applications (2012) **1**, e2; doi:10.1038/lsa.2012.2

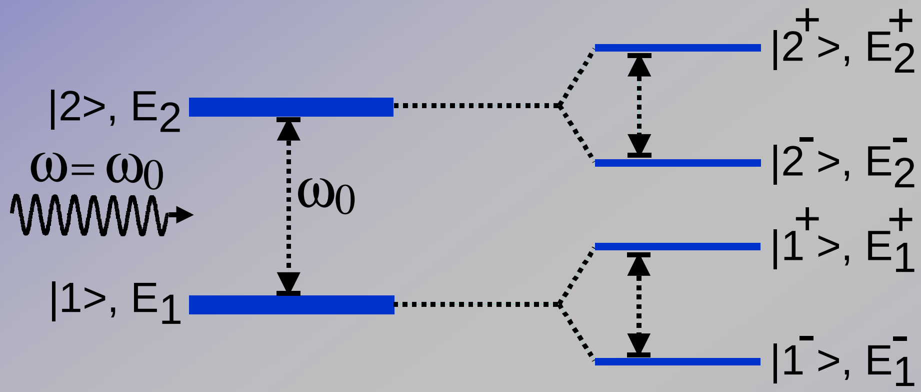
Light absorption and energy transfer were studied in a bionic system with donors and an acceptor. In the optimal case of uniform couplings, this seemingly complicated system was reduced to a three-level Λ-type system. With this observation, we showed that the efficiency of energy transfer through a **dark-state** channel, which is free of the spontaneous decay of the donors, was dramatically improved…

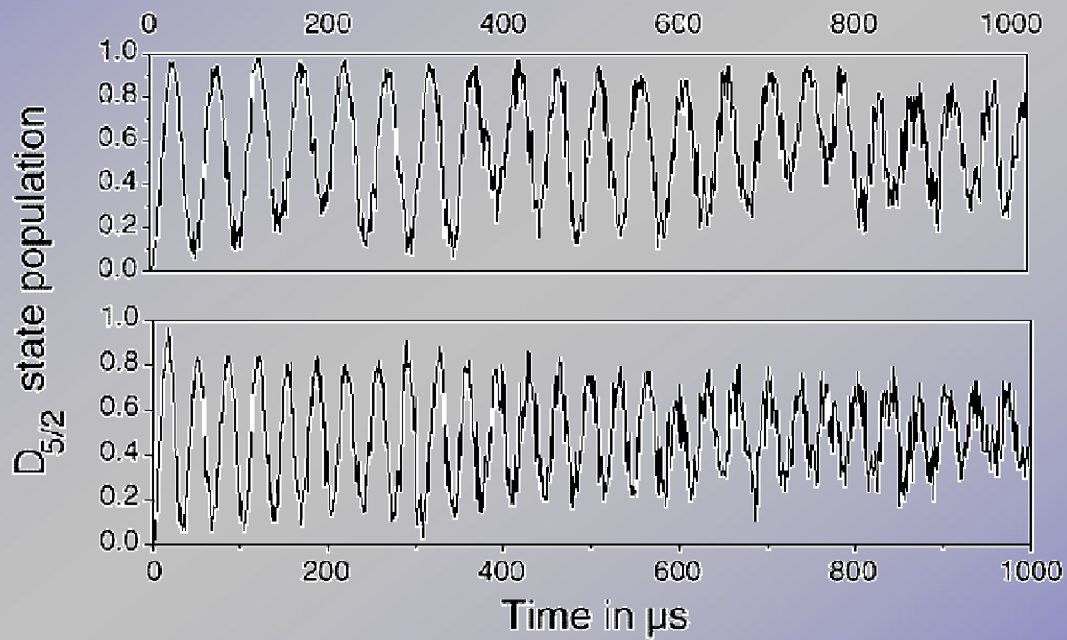




OUTLINE OF THE LECTURE







1 REVIEW OF TWO-LEVEL OPTICAL SYSTEMS

OUTLINE

1.1 TWO-LEVEL ATOM: SEMICLASSICAL THEORY

Introduction

Semiclassical Theory {
- *Light classically described*: Maxwell Eqs.
- *Quantum matter*: Schrödinger eq. + Bohr's atomic model

- *Some examples of phenomena described by the Semiclassical Theory:*

Radiation pressure	⟷	Laser cooling of atoms and ions
Dipole force	⟷	Optical traps for neutral atoms
Rabi oscillations	⟷	Qubit manipulation
Optical nutation	⟷	Decoherence

Perturbation of a quantum system: Temporal evolution of the probability amplitudes

- Evolution of a free atom with a Hamiltonian H_0

$$i\hbar \frac{\partial}{\partial t}\left|\psi(t)\right\rangle = H_0\left|\psi(t)\right\rangle$$

General solution:

$$\left|\psi(t)\right\rangle = \sum_{i=1}^{n} a_i e^{-i\omega_i t}\left|i\right\rangle \qquad \begin{cases} \sum_{i=1}^{n}\left|a_i\right|^2 = 1 \\ H_0\left|i\right\rangle = E_i\left|i\right\rangle \end{cases}$$

$$\left|\left\langle i\middle|\psi(t)\right\rangle\right|^2 = \left|a_i\right|^2$$

- Perturbation: $H \equiv H_0 + V \quad with \quad V << H_0$

$$a_i \rightarrow a_i(t)$$

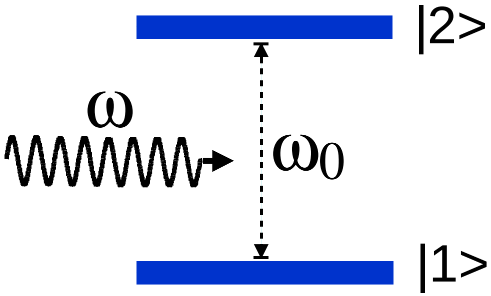
Then:

$$i\hbar \sum_{i=1}^{n}(\dot{a}_i - i\omega_i a_i)e^{-i\omega_i t}\left|i\right\rangle = \sum_{i=1}^{n}(\hbar\omega_i + V)a_i e^{-i\omega_i t}\left|i\right\rangle$$

$$\dot{a}_k = -\frac{i}{\hbar}\sum_{i=1}^{n}\left\langle k\right|V\left|i\right\rangle a_i e^{-i(\omega_i - \omega_k)t}$$

(Electric dipole) interaction of a plane wave light field with a two-level atom:

- Model:



$$\vec{E}(z,t) = \vec{E}_0 \cos(\omega t - kz) \longrightarrow \vec{E}(t) = \vec{E}_0 \cos(\omega t)$$

$$\lambda = \frac{2\pi}{k} >> a_0 = 0.53\,\mathring{A}$$

EDA (Electric Dipole approximation)

- Electric dipole interaction: $V^{(ADE)} = -\vec{\mu}\cdot\vec{E}(t) = -(-e)\vec{r}\cdot\vec{E}(t)$

➡ Electric dipole moment operator:

$$\vec{\mu}_{ij} = -e\vec{r}_{ij} = -e\langle j|\vec{r}|i\rangle = -e\int \vec{u}_j^*(\vec{r})\vec{r}\vec{u}_i(\vec{r})d^3r$$

$\vec{u}_i(\vec{r})$ has, in general, a well defined parity

$|\vec{u}_i(\vec{r})|^2$ is a symmetric function in $\vec{r}$

➡ $\vec{\mu}_{ii} = -e\int \vec{r}|\vec{u}_i(\vec{r})|^2 d^3r = 0$

if $\vec{u}_i(\vec{r})$ and $\vec{u}_j(\vec{r})$ have same parity ➡ $\vec{\mu}_{ij} = 0$

if $\vec{u}_i(\vec{r})$ and $\vec{u}_j(\vec{r})$ have oposite parity ➡ $\vec{\mu}_{ij} = \vec{\mu}_{ji} \neq 0$

For the two-level atom, we end up with: $\vec{\mu} = \begin{pmatrix} 0 & \vec{\mu}_0 \\ \vec{\mu}_0 & 0 \end{pmatrix}$

➡ $V = \begin{pmatrix} 0 & \hbar\Omega\cos(\omega t) \\ \hbar\Omega\cos(\omega t) & 0 \end{pmatrix}$ where $\Omega \equiv -\frac{\vec{\mu}_0 \cdot \vec{E}_0}{\hbar}$ is the Rabi frequency

Atomic evolution

$$|\psi(t)\rangle = a_1(t)e^{-i\omega_1 t}|1\rangle + a_2(t)e^{-i\omega_2 t}|2\rangle$$

with:

$$\begin{cases} \dot{a}_1 = -\frac{i}{\hbar}\langle 1|V|2\rangle e^{-i\omega_0 t} a_2 = i\Omega\cos(\omega t)e^{-i\omega_0 t} a_2 \\ \dot{a}_2 = -\frac{i}{\hbar}\langle 2|V|1\rangle e^{i\omega_0 t} a_1 = i\Omega\cos(\omega t)e^{i\omega_0 t} a_1 \end{cases}$$

"Rotating Wave Aproximation" (RWA):

$$\begin{cases} \dot{a}_1 = i\frac{\Omega}{2}\left[e^{-i(\omega_0-\omega)t} + e^{-i(\omega_0+\omega)t}\right]a_2 \\ \dot{a}_2 = i\frac{\Omega}{2}\left[e^{i(\omega_0-\omega)t} + e^{i(\omega_0+\omega)t}\right]a_1 \end{cases}$$

RWA ⇒

$$\begin{cases} \dot{a}_1 = i\frac{\Omega}{2}e^{i\Delta t} a_2 \\ \dot{a}_2 = i\frac{\Omega}{2}e^{-i\Delta t} a_1 \end{cases}$$

with $\Delta \equiv \omega - \omega_0$ being the detuning

Exact solution within the RWA

Two-level atom in the electric dipole interaction with a plane wave (EDA+RWA):

$$\begin{cases} \dot{a}_1 = i\dfrac{\Omega}{2}e^{i\Delta t}\ a_2 \\ \dot{a}_2 = i\dfrac{\Omega}{2}e^{-i\Delta t}\ a_1 \end{cases} \Rightarrow \begin{aligned} &\ddot{a}_1 - i\Delta\dot{a}_1 + \left(\frac{\Omega}{2}\right)^2 a_1 = 0 \\ &\ddot{a}_2 + i\Delta\dot{a}_2 + \left(\frac{\Omega}{2}\right)^2 a_2 = 0 \end{aligned}$$

General solution:

$$a_1(t) = Ae^{i(\Delta-\Omega')t/2} + Be^{i(\Delta+\Omega')t/2}$$
$$a_2(t) = Ce^{-i(\Delta-\Omega')t/2} + De^{-i(\Delta+\Omega')t/2}$$

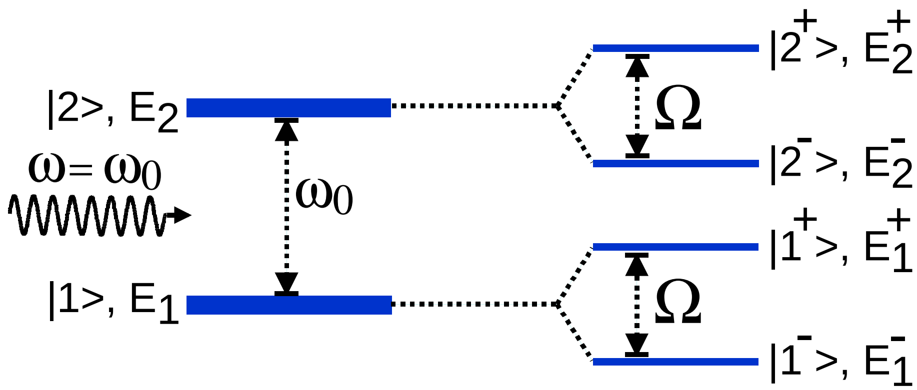
with $\Omega' \equiv \sqrt{\Omega^2+\Delta^2}$ being the **Generalized Rabi Frequency**

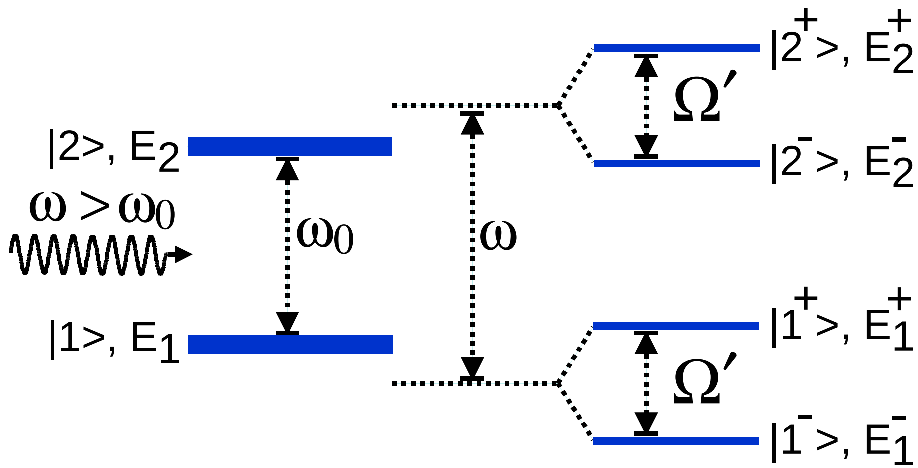
1.2 AC-STARK SPLITTING AND DRESSED ATOM

$$\left|\Psi(t)\right\rangle = \left(Ae^{i(\Delta-\Omega')t/2} + Be^{i(\Delta+\Omega')t/2}\right)e^{-iE_1 t/\hbar}\left|1\right\rangle + \left(Ce^{-i(\Delta-\Omega')t/2} + De^{-i(\Delta+\Omega')t/2}\right)e^{-iE_2 t/\hbar}\left|2\right\rangle$$

$$E_1^{\pm} = E_1 - \hbar\frac{\Delta}{2} \pm \hbar\frac{\Omega'}{2} \qquad E_2^{\pm} = E_2 + \hbar\frac{\Delta}{2} \pm \hbar\frac{\Omega'}{2}$$

On resonance light field	Blue detuned light field
$\Delta = \omega - \omega_0 = 0$	$\Delta = \omega - \omega_0 > 0$





"LIGHT SHIFTS"

Cohen-Tannoudji, Ann. Phys. (Paris) ***7*** *(1962) 423 and 469*

If $|\Delta| >> \Omega$ ⇒ $\Omega' = |\Delta|\sqrt{1+\left(\frac{\Omega}{\Delta}\right)^2} \approx |\Delta|\left[1+\frac{1}{2}\left(\frac{\Omega}{\Delta}\right)^2+\ldots\right]$

If $\Delta >> 0$ ⇒
(blue detuned light)

$$\begin{cases} E_1^+ = E_1 - \hbar\frac{\Delta}{2} + \hbar\frac{\Omega'}{2} \approx E_1 + \hbar\frac{(\Omega/2)^2}{\Delta} \\ E_2^- = E_2 + \hbar\frac{\Delta}{2} - \hbar\frac{\Omega'}{2} \approx E_2 - \hbar\frac{(\Omega/2)^2}{\Delta} \end{cases}$$

If $\Delta << 0$ ⇒
(red detuned light)

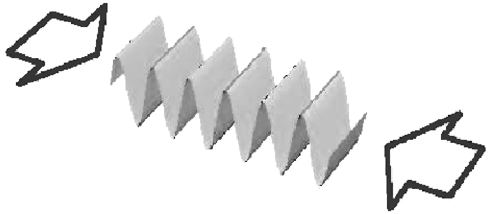
$$\begin{cases} E_1^- \approx E_1 + \hbar\frac{(\Omega/2)^2}{\Delta} \\ E_2^+ \approx E_2 - \hbar\frac{(\Omega/2)^2}{\Delta} \end{cases}$$

DIPOLE FORCE

$$\delta E_1 = \hbar\frac{(\Omega/2)^2}{\Delta} = -\delta E_2 \qquad \vec{F}(\vec{r}) = -\nabla E_1 = -\frac{\hbar}{4\Delta}\nabla\Omega^2(\vec{r}) \propto -\frac{1}{\Delta}\nabla I(\vec{r})$$

Dipole Force.
Example 1: Optical Lattices

- Stationary wave: interference of two counter-propagating lasers

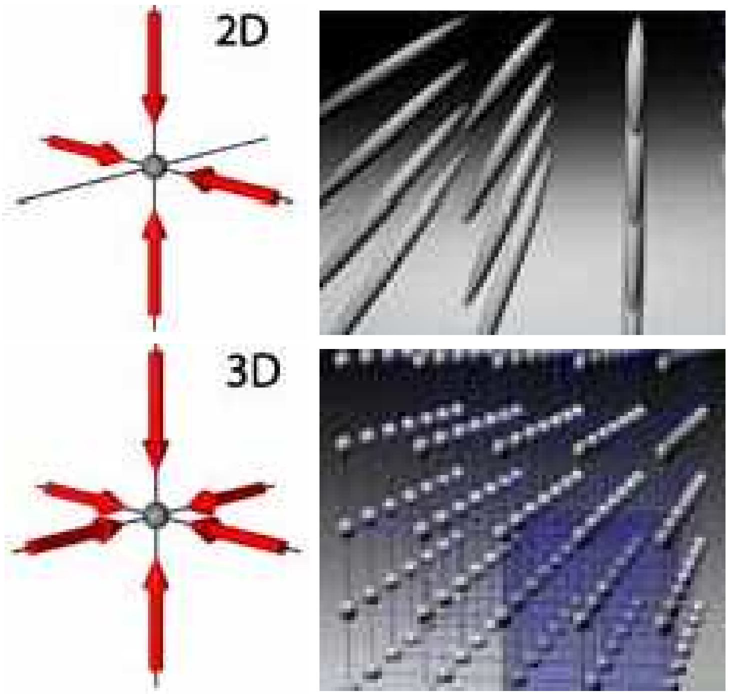


If $\Delta << 0$ ⇨ Atoms move towards the anti-nodes

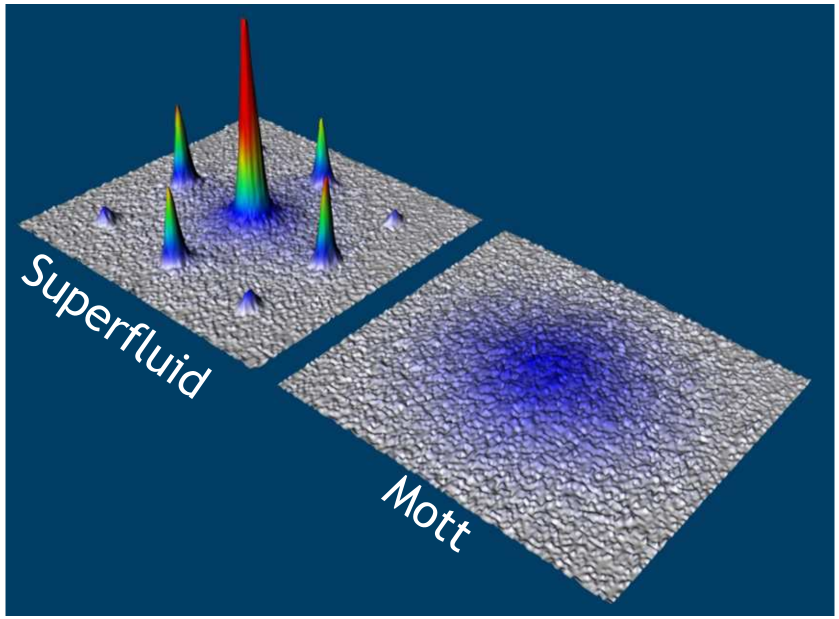
If $\Delta >> 0$ ⇨ Atoms move towards the nodes

Immanuel Bloch in Ted Hänsch's group at the LMU Munich and the MPQ in Garching

`http://www.mpq.mpg.de/~haensch/bec/experiments/lattice.html`



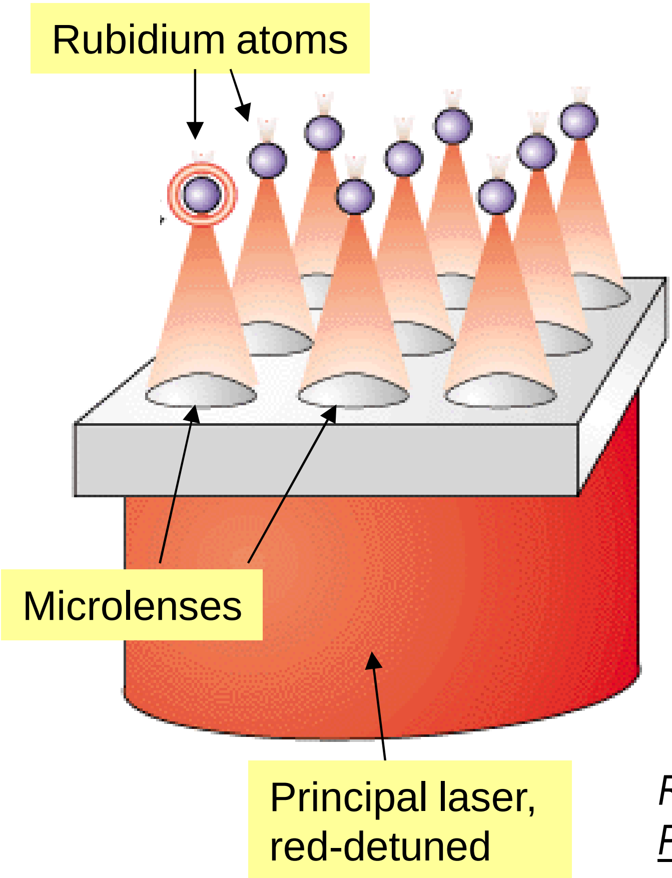
Wavelength: 850 nm
(approx. 60 nm detuning)
Lattice Spacing: 425 nm
Lattice type: simple cubic
Beam waist: 120 µm



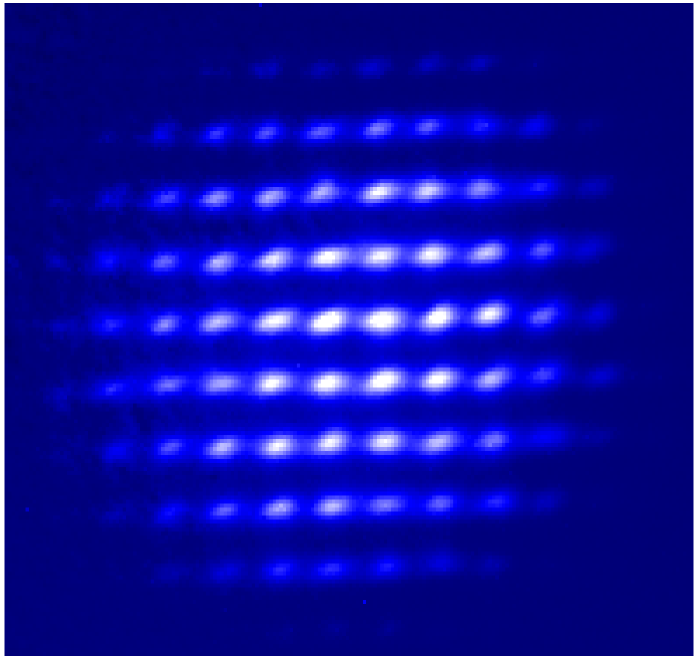
DIPOLE FORCE
EXAMPLE 2: OPTICAL MICROTRAPS

- Trapping at the focii of a microlenses array

G. Birkl and W. Ertmer, Universität Hannover



Fluorescence reported in the experiment



- ^{85}Rb
- Diode laser of 1-10 mW
- P = 1 mW per trap
- 100 μm lense separation
- Trap width = 7 μm (1μm)
- Trapping frequency:

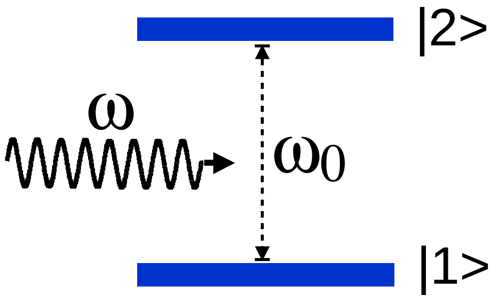
$$\omega_x = 10^4 - 10^5\ s^{-1}$$
$$\omega_\perp \approx 10\omega_x$$

- Trap depth = 1 mK
- Atoms per trap: 100 (1)

*R. Dumke, M. Volk, T. Müther, F.B.J. Buchkremer, G. Birkl, W. Ertmer, Phys. Rev. Lett. **89**, 097903 (2002)*

*G. Birkl, F.B.J. Buchkremer, R. Dumke, M. Volk, W. Ertmer, Optics Comm. **191**, 67 (2001)*

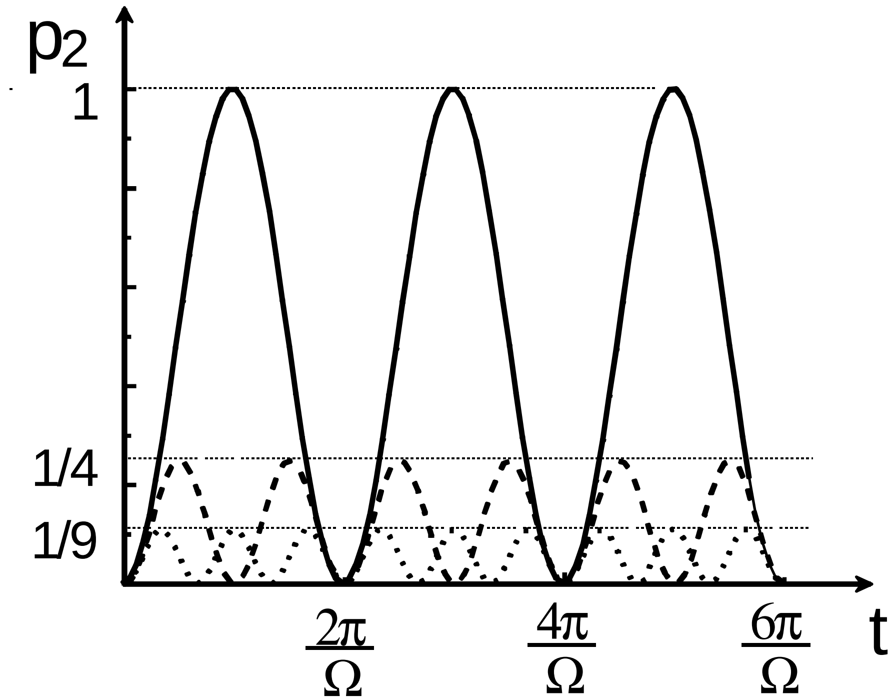
1.3 RABI OSCILLATIONS



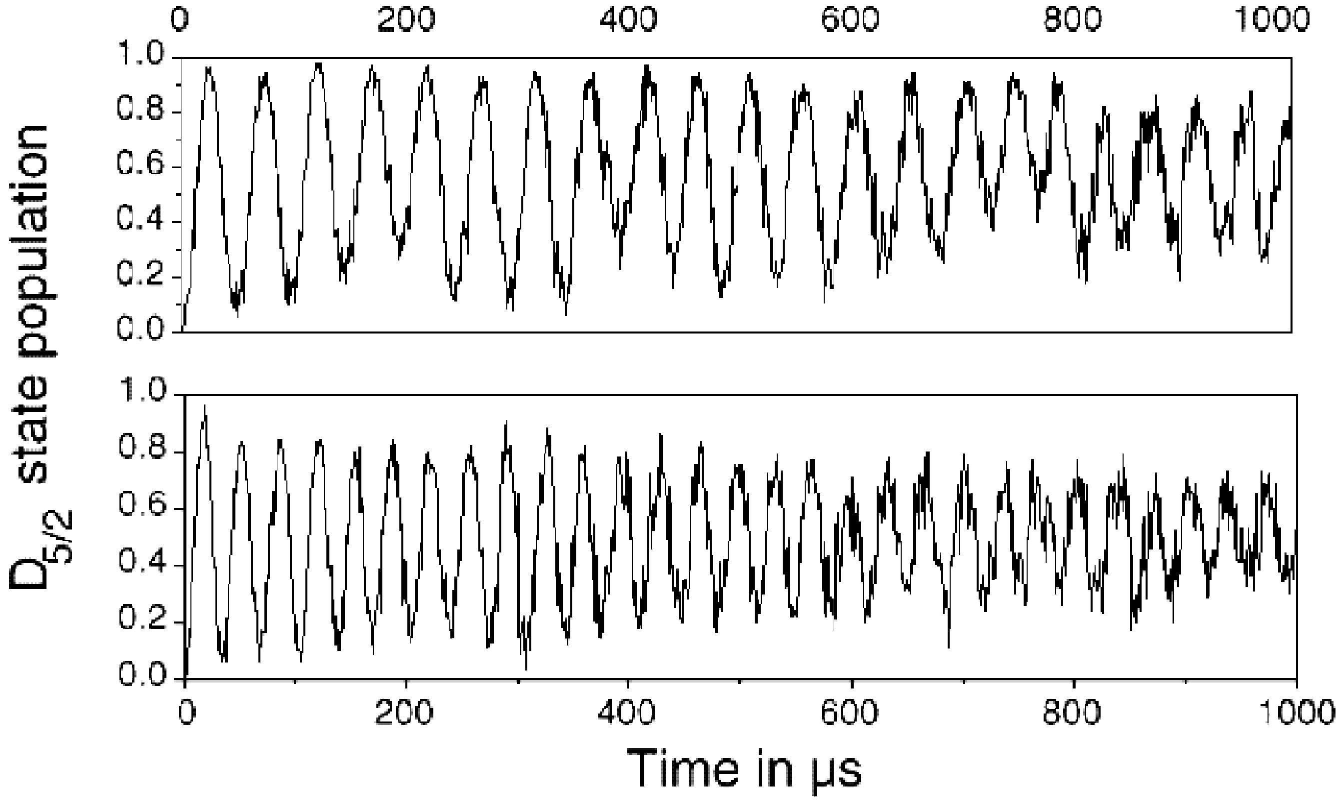
$$|\Psi(t)\rangle = \left(Ae^{i(\Delta-\Omega')t/2} + Be^{i(\Delta+\Omega')t/2}\right)e^{-iE_1t/\hbar}|1\rangle + \left(Ce^{-i(\Delta-\Omega')t/2} + De^{-i(\Delta+\Omega')t/2}\right)e^{-iE_2t/\hbar}|2\rangle$$

Si $\begin{cases} a_1(t=0)=1 \\ a_2(t=0)=0 \end{cases} \Rightarrow A=\dfrac{\Omega'+\Delta}{2\Omega'} \quad B=\dfrac{\Omega'-\Delta}{2\Omega'} \quad C=-\dfrac{\Omega}{2\Omega'} \quad D=\dfrac{\Omega}{2\Omega'}$

$$p_2(t) \equiv c_2c_2^* = \left(\frac{\Omega}{\Omega'}\right)^2 \sin^2\left(\frac{\Omega'}{2}t\right)$$



Rabi Oscillations
Example: Single $^{40}Ca^{+}$ Ion in a Paul Trap



F. Schmidt-Kaler et. al. arXiv:quant-ph/0003096 (21/March/2000)
JOURNAL OF MODERN OPTICS, 2000, VOL. 47, NO. 14/15, 2573 -2582

Light pulses of well defined area

Si $\Delta = 0$ ⇒ $|\psi(t)\rangle = \cos\left(\frac{\Omega}{2}t\right)e^{-iE_1 t/\hbar}|1\rangle + i\sin\left(\frac{\Omega}{2}t\right)e^{-iE_2 t/\hbar}|2\rangle$

$$= \cos\left(\frac{\Omega}{2}t\right)|\tilde{1}\rangle + i\sin\left(\frac{\Omega}{2}t\right)|\tilde{2}\rangle$$

Singel qubit operations

$\Omega t = 0 \Rightarrow |\Psi(t)\rangle = |\tilde{1}\rangle$

$\Omega t = \pi/2 \Rightarrow |\Psi(t)\rangle = (\sqrt{2}/2)(|\tilde{1}\rangle + i|\tilde{2}\rangle)$ ← Hadamard gate

$\Omega t = \pi \Rightarrow |\Psi(t)\rangle = i|2\rangle$ ← Bit flip

$\Omega t = 3\pi/2 \Rightarrow |\Psi(t)\rangle = (\sqrt{2}/2)(-|\tilde{1}\rangle + i|\tilde{2}\rangle)$

$\Omega t = 2\pi \Rightarrow |\Psi(t)\rangle = -|\tilde{1}\rangle$ ← Phase flip

$\Omega t = 5\pi/2 \Rightarrow |\Psi(t)\rangle = -(\sqrt{2}/2)(|\tilde{1}\rangle + i|\tilde{2}\rangle)$

$\Omega t = 3\pi \Rightarrow |\Psi(t)\rangle = -i|\tilde{2}\rangle$

$\Omega t = 7\pi/2 \Rightarrow |\Psi(t)\rangle = (\sqrt{2}/2)(|\tilde{1}\rangle - i|\tilde{2}\rangle)$

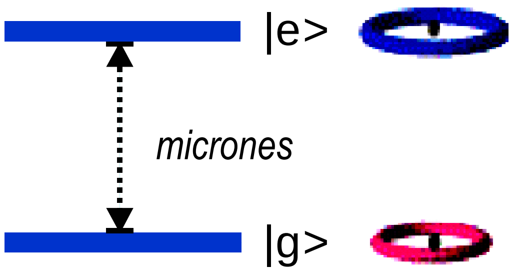
$\Omega t = 4\pi \Rightarrow |\Psi(t)\rangle = |\tilde{1}\rangle$

Light pulses of well defined area
Example: Cavity Quantum ElectroDynamics (CQED)

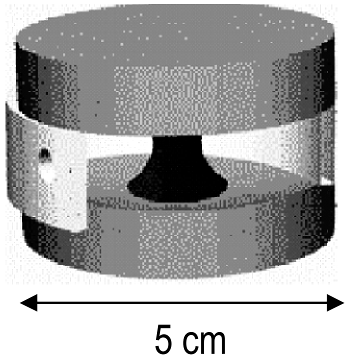
S. Haroche Group. ENS & College de France

Two-level atom

Rydberg states (n=50,51)

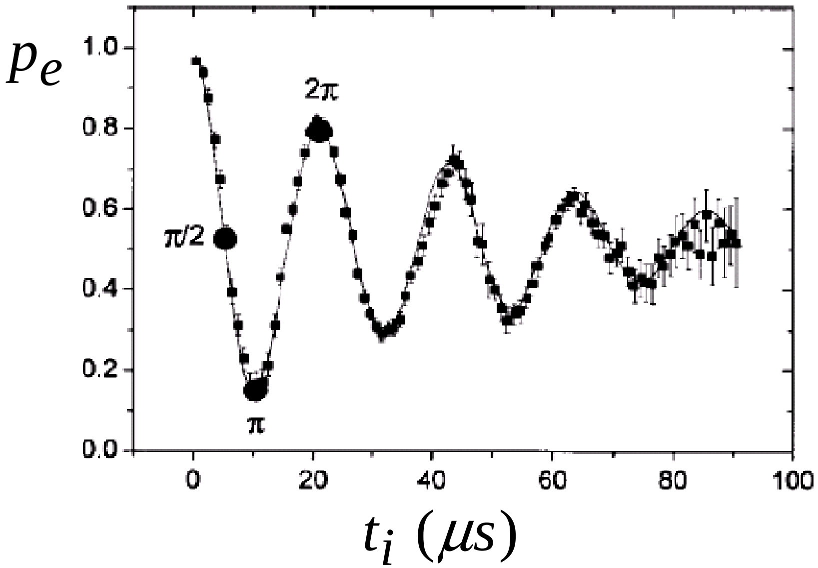


Superconducting Fabry-Perot resonator



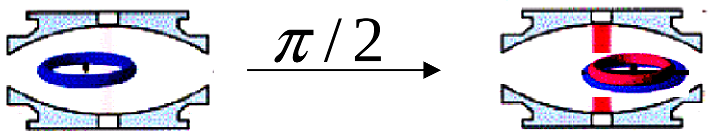
$$\Omega = g\sqrt{n+1}$$

Quantum Rabi oscillations



Atom-photon entanglement

$$|\psi\rangle_{in} = |e\rangle_{atom} \otimes |0\rangle_{camp}$$

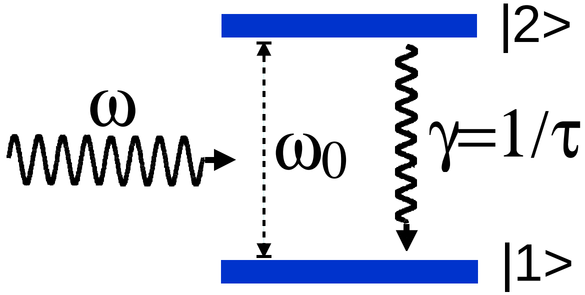


$$|\psi\rangle_{out} = \frac{1}{\sqrt{2}}\left(|e\rangle \otimes |0\rangle + i|g\rangle \otimes |1\rangle\right)$$

Hagley et al., Phys. Rev. Lett. ***79****, 1 (1997)*

Optical Nutation

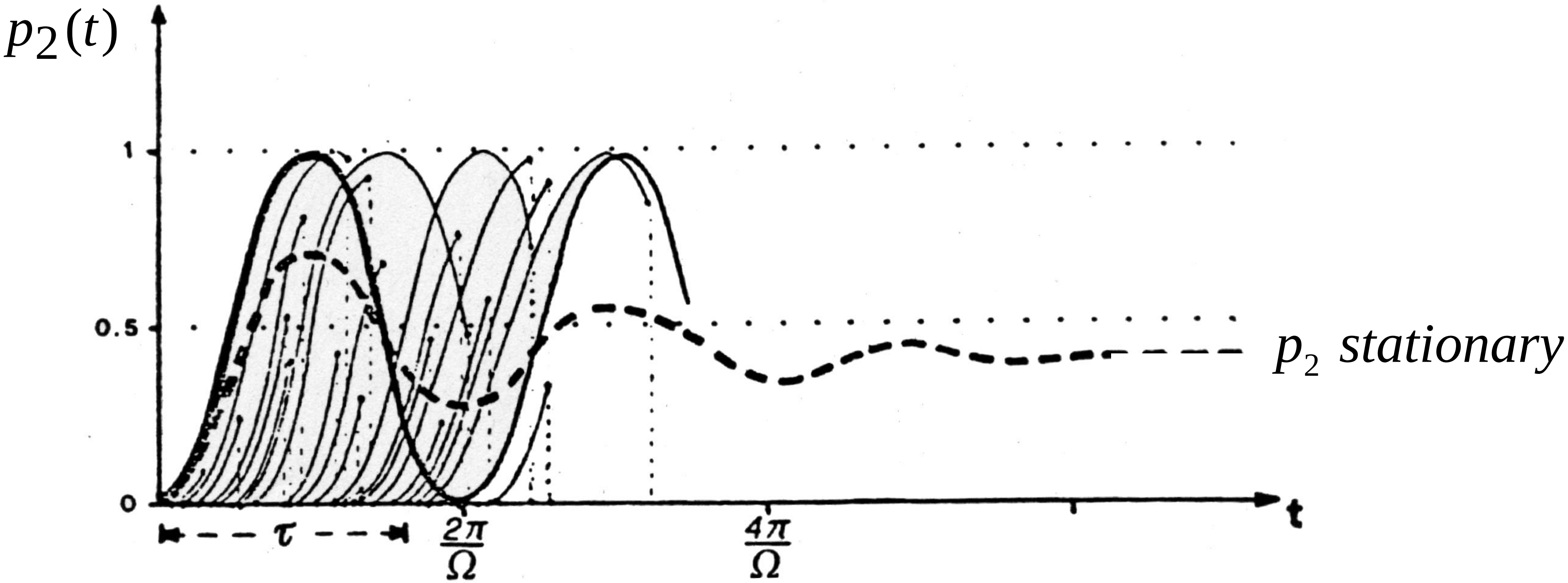
- *Two-level atom + spontaneous emission*



Which is the probability dp that an atom suffers a spontaneous emission in a dt?

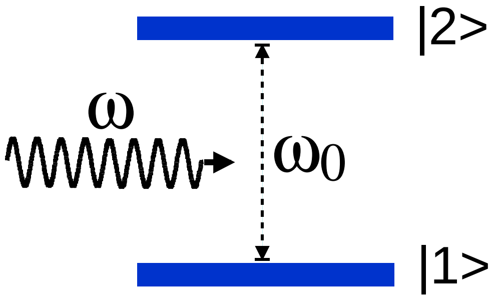
$$dp_{spon.\,emission} = \gamma \cdot p_2(t) \cdot dt$$

- *Collection of two-level atoms + spontaneous emission*



1.4 RAPID ADIABATIC PASSAGE

N. V. Vitanov, T. Halfmann, B. W. Shore, K. Bergmann, Annual Review of Physical Chemistry, **52**, 763 (2001)



$$\begin{cases} \dot{a}_1 = i\dfrac{\Omega}{2} e^{i\Delta t}\, a_2 \\ \dot{a}_2 = i\dfrac{\Omega}{2} e^{-i\Delta t}\, a_1 \end{cases}$$

$$\Omega \equiv -\frac{\vec{\mu}_0 \cdot \vec{E}_0}{\hbar}$$

$$\Delta \equiv \omega - \omega_0$$

- Change of variables:

$$\tilde{a}_1 \leftrightarrow a_1$$
$$\tilde{a}_2 \leftrightarrow a_2 e^{i\Delta t}$$

$$\Rightarrow \begin{cases} \dot{\tilde{a}}_1 = i\dfrac{\Omega}{2}\tilde{a}_2 \\ \dot{\tilde{a}}_2 = i\dfrac{\Omega}{2}\tilde{a}_1 + i\Delta\tilde{a}_2 \end{cases}$$

Taking out ~ for simplicity

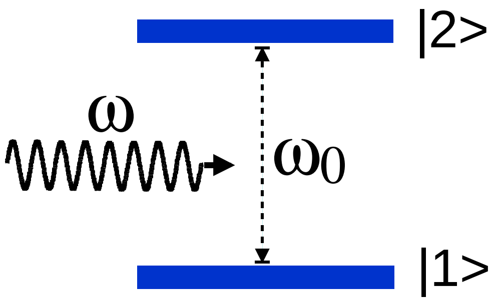
$$\Rightarrow i\begin{pmatrix} \dot{a}_1 \\ \dot{a}_2 \end{pmatrix} = \begin{pmatrix} 0 & -\Omega/2 \\ -\Omega/2 & -\Delta \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

↑ **Two state Hamiltonian**

- **Eigenstates and eigenvalues:**

$$|+\rangle = -\sin\vartheta|1\rangle - \cos\vartheta|2\rangle$$
$$|-\rangle = \cos\vartheta|1\rangle - \sin\vartheta|2\rangle$$

with $\tan(2\vartheta) = \dfrac{\Omega}{\Delta}$ and $E_{\pm} = \dfrac{1}{2}\left(-\Delta \pm \sqrt{\Delta^2 + \Omega^2}\right)$



$$\Omega \equiv -\frac{\vec{\mu}_0 \cdot \vec{E}_0}{\hbar}$$

$$\Delta \equiv \omega - \omega_0$$

Eigenstates and eigenvalues:

$$|+\rangle = -\sin\vartheta|1\rangle - \cos\vartheta|2\rangle$$

$$|-\rangle = \cos\vartheta|1\rangle - \sin\vartheta|2\rangle$$

with $\tan(2\vartheta) = \frac{\Omega}{\Delta}$ and $E_{\pm} = \frac{1}{2}\left(-\Delta \pm \sqrt{\Delta^2 + \Omega^2}\right)$

- Adiabatic theorem by Max Born and Vladimir Fock:
  M. Born and V. Fock. Zeitschrift für Physik, **51**, 165-180 (1928)

  "A physical system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough and if there is a gap between the eigenvalue and the rest of the Hamiltonian's spectrum"

- **Rapid Adiabatic Passage (RAP). Assume:** $\Delta = \Delta(t); \Omega = \Omega(t)$

  Consists in adiabatically following $|+\rangle$ **or** $|-\rangle$ **from** $|1\rangle$ **to** $|2\rangle$

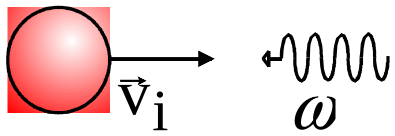
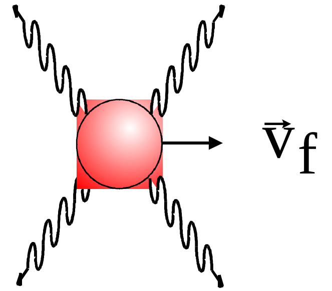
  ***Example*: the atom initially at** $|\Psi(t_0)\rangle = |1\rangle$

  At t_0 **:** $\Delta > 0$ **with** $\Delta >> \Omega \Rightarrow \vartheta(t_0) = 0 \Rightarrow |\Psi(t_0)\rangle = |-(t_0)\rangle = |1\rangle$

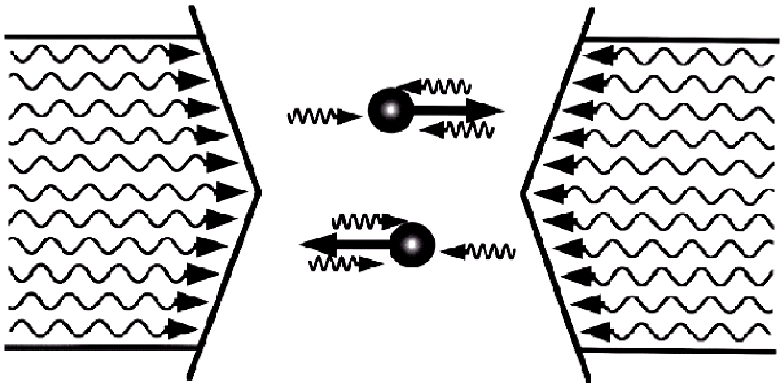
  At t_f **:** $\Delta < 0$ **with** $|\Delta| >> \Omega \Rightarrow \vartheta(t_f) = \frac{\pi}{2} \Rightarrow |\Psi(t_f)\rangle = |-(t_f)\rangle = -|2\rangle$

1.5 RADIATION PRESSURE

- On resonance force (nothing to do with the dipole force that works far from resonance):

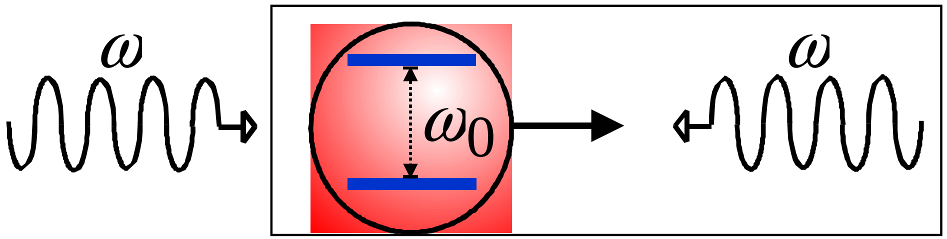
1. Light absorption	2. Spontaneous emission
	 $\langle \vec{v}_f \rangle = \vec{v}_i - \hbar\vec{k}$

- Laser cooling:



(preferential absorption)

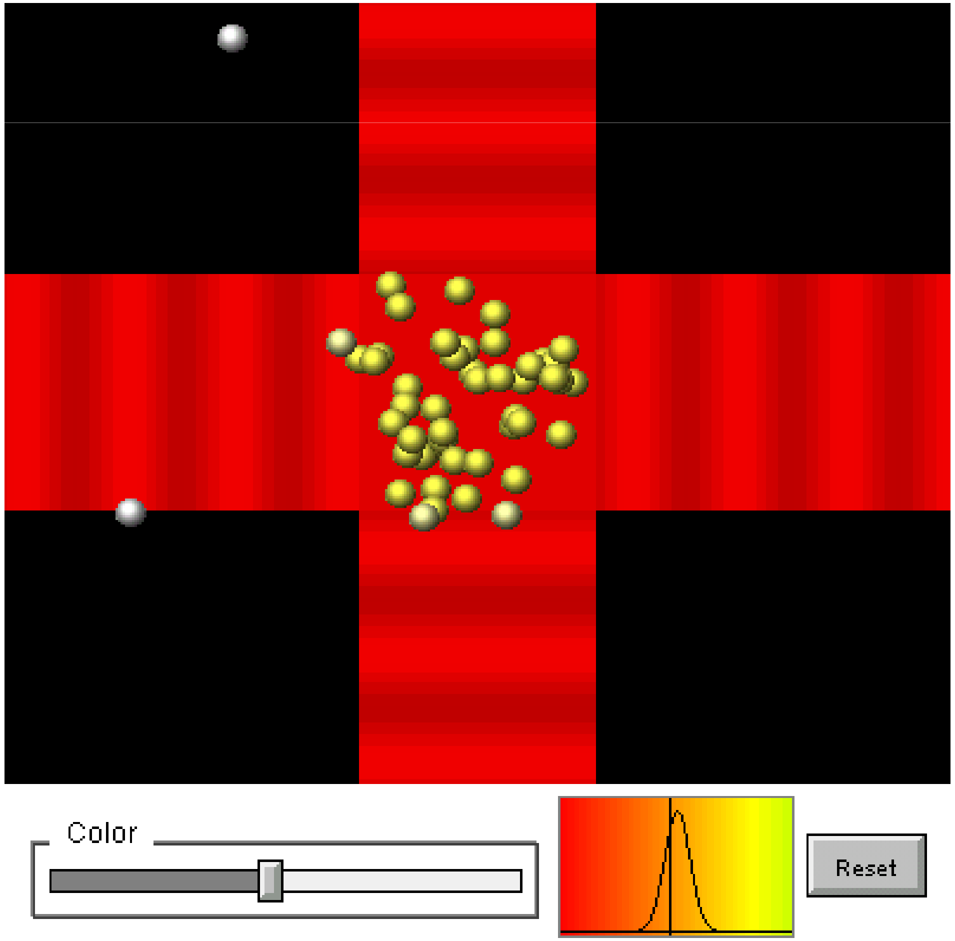
- Doppler effect: $\omega' = \omega - \vec{k}\cdot\vec{v}$



$(\omega_0 > \omega)$

“Optical Molasses”

T. Hänsch, A. Schawlow, Opt. Commun ***13*** *(1975) 68.*

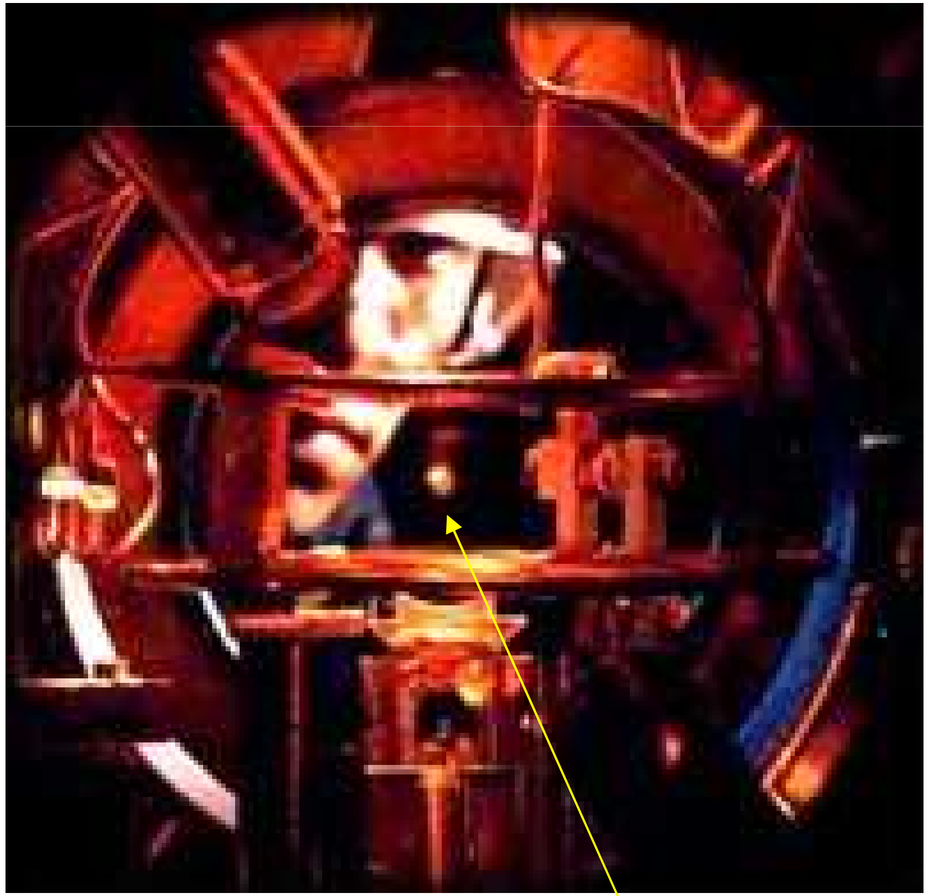


http://www.colorado.edu/physics/2000/index.pl
(The Atomic Lab / Bose Einstein Condensate)

Example:

Magneto Optical Trap (MOT)

W. B. Phillips Group, “Laser Cooling and Trapping”, NIST



http://physics.nist.gov/Divisions/Div842/Gp4/group4.html

Na atoms
$T < 1$ mK