

Liouville quantum gravity is not enough

Shota Komatsu



There is a non-negligible **probability** that  
Liouville quantum **gravity** coupled to  
**conformal field theory** is not enough

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Based on works in progress / to appear with **Pronobesh Maity** (ICTS, Bangalore)

# Introduction & Motivation

- **String theory** was first “discovered” in an attempt to describe strongly coupled dynamics of **four-dimensional gauge theory**.

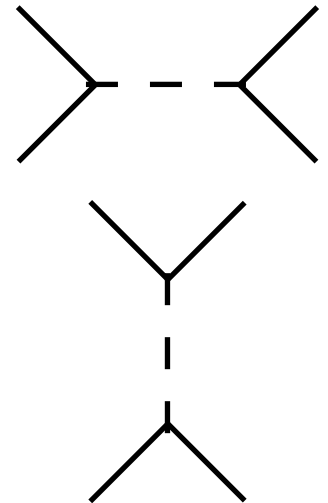


Gabriele Veneziano

- Veneziano amplitude (1968)

$$A(s, t) = \frac{\Gamma(-s - 1)\Gamma(-t - 1)}{\Gamma(-s - t - 2)}$$

$$\begin{aligned} &\nearrow \sum_n \frac{1}{s - n} \\ &\searrow \sum_n \frac{1}{t - n} \end{aligned}$$



- It was later understood (by Nambu, Nielsen, Susskind) that the formula arises from **two-dimensional string world sheet**.

# Introduction & Motivation

- **However**, it was soon realized that string theory can only be consistently defined in **26 or 10 dimensions**.
- On the other hand, gauge theories are defined (renormalizable) in **4 (or less)** dimensions.
- Nevertheless, there were various indications that  $SU(N_c)$  gauge theory in  $N_c \rightarrow \infty$  limit can be reformulated as string theory

't Hooft, Polyakov,.....

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- **Question (ver.1):**  
Can we define string theory in smaller dimensions?

- **Non-critical string theory** Das, Naik, Wadia, Polyakov, Distler, Kawai,...

“Replace 2d gravity with Liouville gravity”

$$S_{\text{Liouville}} \sim \int d^2z [\partial\phi\bar{\partial}\phi + e^\phi] \quad d + c_{\text{Liouville}} = 26 \text{ (or 10)}$$

# Introduction & Motivation

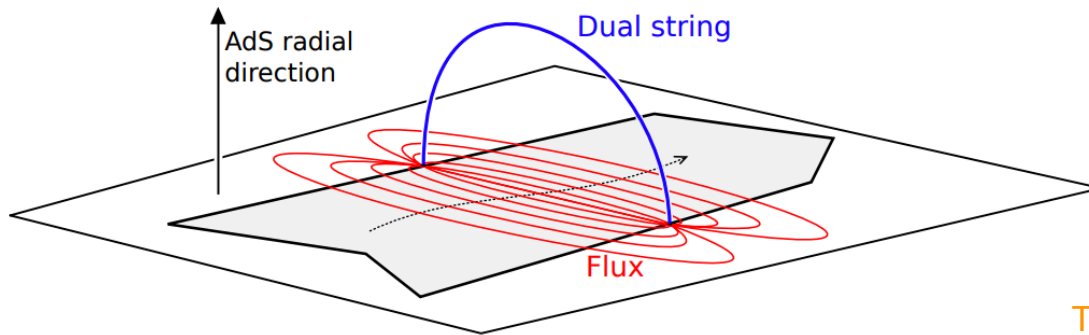
$$d + c_{\text{Liouville}} = 26 \text{ (or 10)}$$

$$S_{\text{Liouville}} \sim \int d^2z [\partial\phi\bar{\partial}\phi + e^\phi]$$

- Concrete realization: minimal strings,  $c=1$  string  
Large N matrix models  $\leftrightarrow$  Liouville gravity + “matter”
- And of course, we now have **AdS/CFT** Maldacena  
Large N SUSY gauge theories  $\leftrightarrow$  String in AdS

# Common feature

Existence of **emergent/Liouville/holographic** direction, which corresponds to **extra scalar mode** on the world sheet



Taken from [Basso, Sever, Vieira]

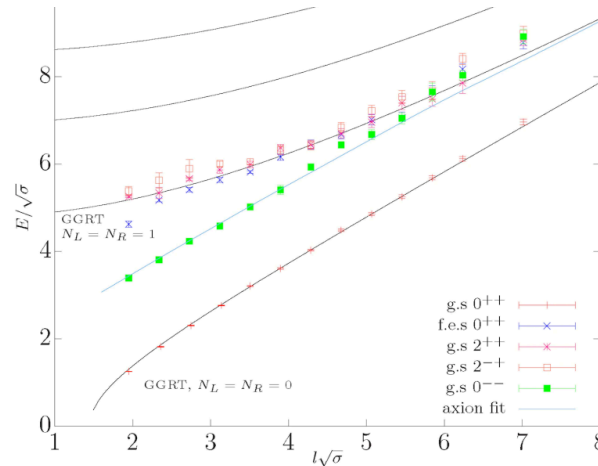
Q: Is this always true?



**An inconvenient truth**

# An inconvenient truth

- **Lattice** studies of confining string in **large N YM** in 3d and 4d



Teper et al, Dubovsky et al.

**3d**

No extra mode on the worldsheet

**4d**

A pseudo-scalar resonance on the worldsheet

Do not quite fit “emergent/Liouville/holographic” paradigm.

## Question (ver. 2)

Can we define **non-critical string** (i.e. string in smaller dimensions) **without introducing extra scalar mode**?

# Effective action for confining string

Polchinski, Strominger,..., Aharony, Komargodski,  
.....Hellerman, Maeda, Maltz, Swanson,.....

- Polchinski-Strominger: Effective action around long string

$$S_{\text{Nambu-Goto}} + (26 - D)S_{\text{PS}} + \dots$$

Needed for restoring Weyl inv.

- Reformulation by Hellerman et al: “Composite linear dilaton”

$$S_{\text{Polyakov}} + \frac{26 - D}{12\pi} \int d^2z \left( \partial\varphi\bar{\partial}\varphi + R\varphi/4 \right) + \dots$$

$$\varphi = \frac{1}{2} \log \partial X^\mu \bar{\partial} X_\mu$$

- Reminiscent of Polyakov’s Liouville action, but **the extra mode is composite**
- This is just an effective field theory, not UV complete theory.

## Question (final)

Can we write a string dual of confining gauge theory which

- does not involve an extra scalar
- is UV complete (i.e. path integral can be performed exactly)

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I will present one such realization for the simplest toy model, **Chiral 2d YM**

# Punchline / Plan

- String dual for **chiral 2d YM**
- String dual for **symmetric product orbifolds of arbitrary CFT**
- Relation to  $T\bar{T}$  deformation
- Potential relation to Eberhardt-Gaberdiel-Gopakumar
- Potential relation to matrix string



Speculative

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**Calculable, UV complete** worldsheet theory for which the extra mode is composite.

# Lightening review of 2d YM

- 2d YM = Yang-Mills theory in two dimensions



# Lightening review of 2d YM

- 2d YM: Simplest possible confining theory.  
Linear potential, area law, meson spectrum.....
- Completely solvable even at finite N:

$$Z_{\mathcal{M}} = \sum_{\text{rep of } U(N)} (\dim R)^{2-2G} e^{-g_{\text{YM}}^2 A C_2(R)}$$

- At large N, the result factorizes into “chiral” and “anti-chiral” parts

$$Z_{\mathcal{M}} \xrightarrow{\text{large N}} Z_{\mathcal{M}}^{\text{chiral}} Z_{\mathcal{M}}^{\text{anti}}$$

- At finite N, each partition function receives corrections. In addition there will be interactions between chiral and anti-chiral parts.

$$Z_{\mathcal{M}}^{\text{chiral}} = Z_0^{\text{chiral}} + \frac{Z_1^{\text{chiral}}}{N^2} + \dots$$

$$Z_{\mathcal{M}} = Z_{\mathcal{M}}^{\text{chiral}} Z_{\mathcal{M}}^{\text{anti}} + \frac{Z_{\text{int}}}{N^2} + \dots$$

## 2d YM as string theory

- **Gross Taylor:**  $1/N$  expansion can be interpreted as string genus expansion.
- Chiral and anti-chiral parts correspond to worldsheets wrapping the target space with different orientations.
- But the explicit worldsheet action was not written down.

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- **Horava:** Proposed a “rigid topological string”. Much more complicated than standard string theory.
- **Cordes, Moore, Rangoolam:** “zero area limit” of 2d YM can be interpreted as topological string.
- **Vafa: Aganagic, Ooguri, Saulina, Vafa:** Partition functions on Riemann surfaces coincide with topological string partition function.

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- **Vafa: Aganagic, Ooguri, Saulina, Vafa:** Partition functions on Riemann surfaces coincide with topological string partition function.
- Our worldsheet theory: standard bosonic string. In principle we can use it to compute more general observables. We may be able to add boundaries to study mesons.

# Worksheet actions

Inspired by the works by Ishibashi on noncritical SFT

- String dual for **chiral 2d YM** on a torus / cylinder

$$\int d^2z \underbrace{(\beta\bar{\partial}X + \bar{\beta}\partial\bar{X} + \lambda\partial X\bar{\partial}\bar{X})}_{\beta\gamma\text{-system}} - \frac{2}{\pi} \int d^2z \underbrace{(\partial\varphi\bar{\partial}\varphi + R\varphi/4)}_{\text{Chiral analog of PS term}}$$

$$\varphi = \frac{1}{2} \log \partial X \bar{\partial} \bar{X}$$

$$\lambda = g_{\text{YM}}^2 N$$

$$X \sim X + 2\pi R \text{ etc}$$

- String dual for symmetric product orbifolds for arbitrary CFT

$$\int d^2z (\beta\bar{\partial}X + \bar{\beta}\partial\bar{X}) - \frac{24 - c}{12\pi} \int d^2z (\partial\varphi\bar{\partial}\varphi + R\varphi/4) + S_{\text{seed CFT}}$$

- The action looks complicated.
- Integrating out  $\beta$ s forces  $X$  to be holomorphic  $\rightarrow$  path integral can be performed explicitly.
- Can be viewed as a non-critical version of **non-relativistic string theory** studied by Gomis and Ooguri.

# Nonrelativistic string

Gomis, Ooguri,,..., Bergshoff, Gomis, Yan,....

$$\int d^2z (\beta \bar{\partial} X + \bar{\beta} \partial \bar{X}) + (\text{Polyakov action for transverse modes})$$

- Worldsheet theory is **relativistic**, target space spectrum is **non-relativistic**.
- It can be obtained by a **double-scaling limit** of usual string theory in which one sends B-field large (critical) and  $\alpha' \rightarrow 0$ .  
(Open string sector becomes **noncommutative gauge theory**)
- Roughly speaking, the non-relativistic limit kills half of the spectrum.
- Here taking the limit makes the theory **chiral** and kill “**anti-strings**” which do not exist in the symmetric product orbifolds.

# Simple check: genus 1 partition function

$$\int d^2z (\beta \bar{\partial} X + \bar{\beta} \partial \bar{X}) + S_{\text{dilaton}} + \text{rest}$$

- Genus-1 partition function for the torus target space
- Integrating out  $\beta \rightarrow X$  : holomorphic

$$\int_{\text{fund}} \frac{d^2\tau}{\tau_2} \delta(\text{holo map}) \det(\text{rest}) e^{-S_{\text{dilaton}}}$$

- Holomorphic map from torus to torus: labelled by 4 integers.
- After doing all integration, we get for 2d YM

$$\sum_K \frac{1}{2K} e^{-\lambda K(\text{area})} \sum_{a \cdot d = K} (a + d)$$

- Reproduces the large N result by Gross and Taylor.
- We reproduced also scattering amplitudes, Wilson loop VEV.

# Simple check: genus 1 partition function

$$\int d^2z (\beta \bar{\partial} X + \bar{\beta} \partial \bar{X}) + S_{\text{dilaton}} + S_{\text{seed}}$$

- Doing the same, we can reproduce the (grand-canonical) partition function of symmetric product orbifolds.
- Result depends only on **complex structure moduli** (independent of Kahler moduli): special feature of nonrelativistic string.



# Plan

- String dual for **chiral 2d YM**
- String dual for **symmetric product orbifolds of arbitrary CFT**
- Relation to  $T\bar{T}$  deformation
- Potential relation to Eberhardt-Gaberdiel-Gopakumar
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Speculative

## Relation to $T\bar{T}$ deformation

$$\int d^2z (\beta \bar{\partial} X + \bar{\beta} \partial \bar{X}) + S_{\text{dilaton}} + S_{\text{seed}}$$

- Worksheet matter stress tensor

$$T_{\text{w.s.}}(z) = -\beta \partial X - \frac{24 - c}{12\pi} \{X, z\} + T_{\text{seed}} \stackrel{\text{Virasoro}}{=} 0$$

- Choose static gauge  $\partial X = \bar{\partial} \bar{X} = 1$ :

$$\beta \stackrel{\text{Virasoro}}{=} T_{\text{seed}}$$

- A naive guess for (symmetric orbifold of)  $T\bar{T}$ -deformation:

$$\int d^2z (\beta \bar{\partial} X + \bar{\beta} \partial \bar{X} + \beta \bar{\beta}) + S_{\text{dilaton}} + S_{\text{seed}}$$

- Integrating out  $\beta$ 's, we arrive at an action **almost identical** to the description of  $T\bar{T}$ -deformed CFT by **Callebaut, Kruthoff, Verlinde**

# Relation to Eberhardt-Gaberdiel-Gopakumar

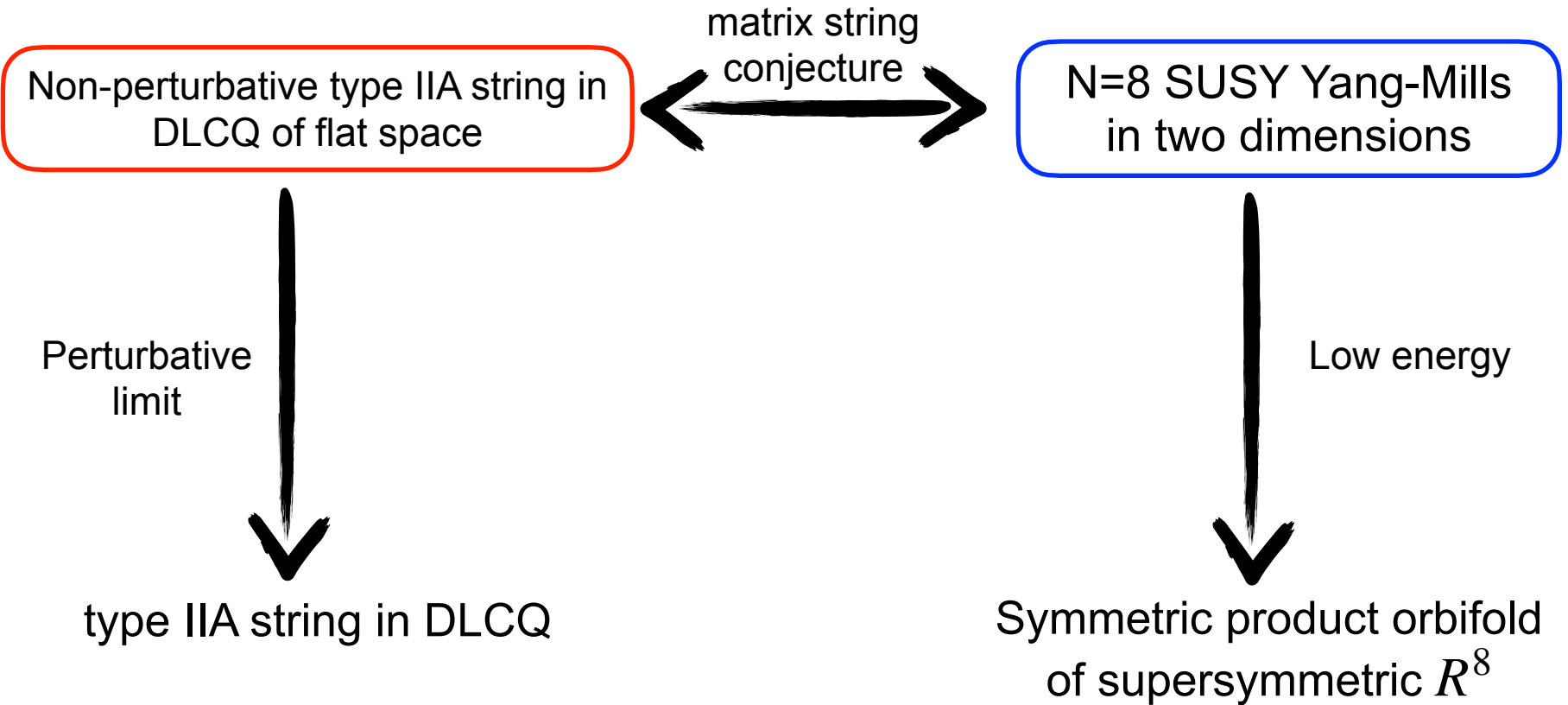
- Free field representation of  $SL(2,R)$  WZW:

$$\int d^2z (\beta\bar{\partial}X + \bar{\beta}\partial\bar{X} + 4\underline{\partial\Phi\bar{\partial}\Phi} - \underline{e^{-2\Phi}\beta\bar{\beta}} - R\Phi)$$

- Similar to our action, but  $\Phi$  is not composite.
  - Additional term that looks like  $T\bar{T}$ -deformation
- They argued that, for tensionless string ( $k=1$ ), the path integral localizes to
$$\Phi = -\log \epsilon + \log \partial X \bar{\partial} \bar{X}$$
  - After the replacement, the action has the same structure as ours.
  - **The relation is not rigorous**: their theory is supersymmetric, our theory is purely boson.

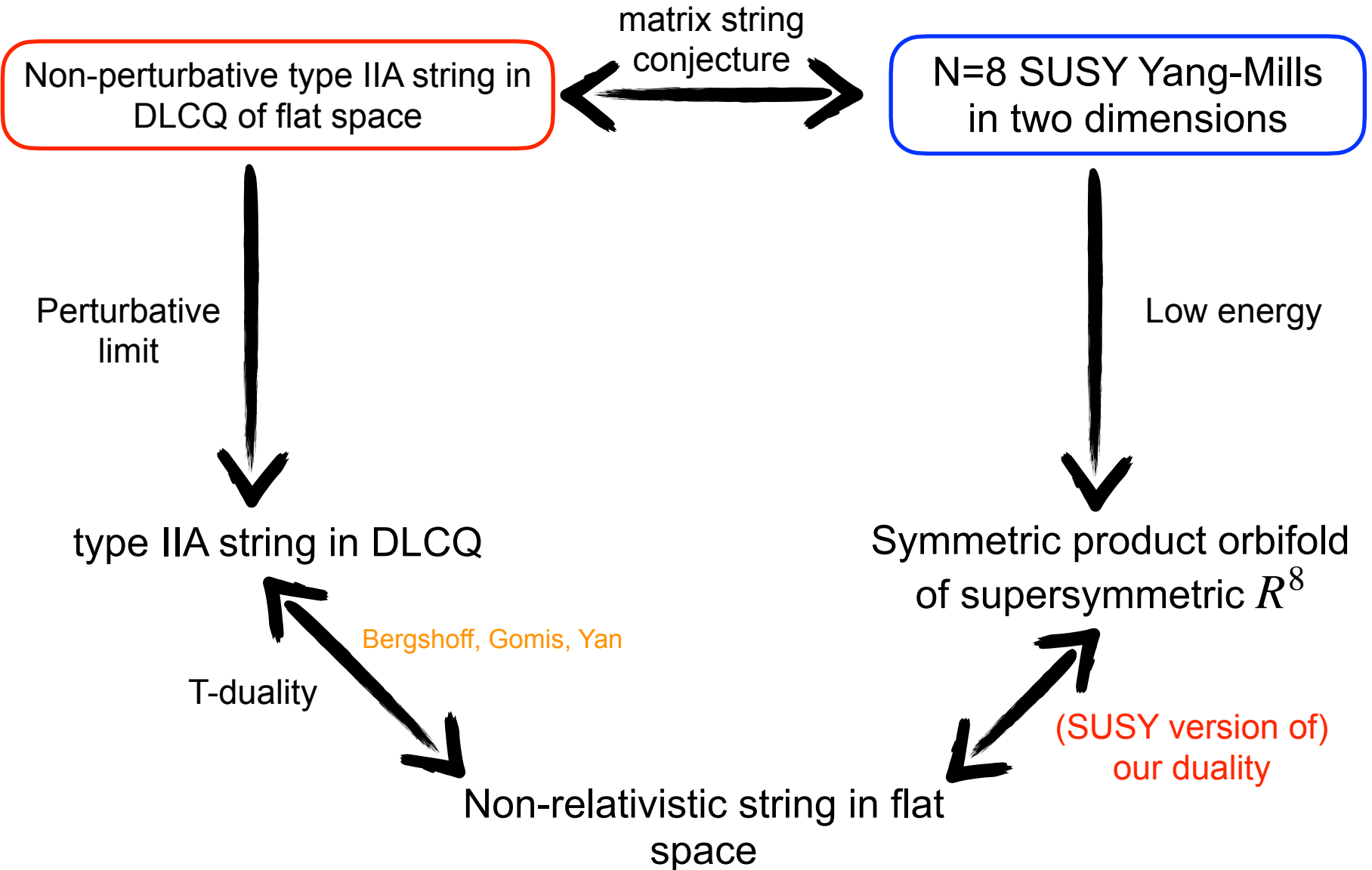
# Relation to matrix string

Motl, Dijkgraaf, Verlinde, Verlinde



# Relation to matrix string

Motl, Dijkgraaf, Verlinde, Verlinde



# Conclusion

- Proposals for string duals to chiral 2d YM and symmetric product orbifolds.
- Duality without holographic directions.
- Interesting connections to recent developments in  $AdS_3/CFT_2$ , nonrelativistic string, matrix string,.....

# Future

- Meson spectrum.
- Non-perturbative corrections and D-instantons.
- Understand the relation with topological string by Vafa et al.
  - “Instantons beyond topological theory” Frenkel, Losev, Nekrasov
- Compute other observables for symmetric product orbifolds, supersymmetrize.....
- Revisiting matrix string proposal?