SPECIAL RELATIVITY – MIDTERM EXAM

Exercise 1. *In $\mathbb{R}^{2,1}$, consider a spinor ψ^α and a unit vector n^μ, with corresponding spinor matrix $N^\alpha{}_\beta = n^\mu(\sigma_\mu)^\alpha{}_\beta$. What is the geometric meaning of the linear transformation $\psi^\alpha \to N^\alpha{}_\beta\psi^\beta$? Illustrate by examples.*

Exercise 2. *When we intersect a cone in $\mathbb{R}^3$ by various planes, we get Archimedes' conic sections: the circle, ellipse, parabola and hyperbola. Which surfaces do we get when we intersect the lightcone in $\mathbb{R}^{3,1}$ with a spacelike hypersurface? A timelike hypersurface? A lightlike hypersurface? Hint: in the latter case, use $x^a = (u,v,x,y)$ coordinates, and write the squared distance $(x_a - x'_a)(x^a - x'^a)$ between two points on the section.*

Exercise 3. *Let us parameterize the $\mathbb{R}^{3,1}$ lightcone as:*

$$x^\mu = (t, t\sin\theta\cos\phi, t\sin\theta\sin\phi, t\cos\theta) \ , \tag{1}$$

and introduce a complex coordinate $\xi = \cot\frac{\theta}{2}\, e^{i\phi}$ for the projective lightcone. Identify the $SO(3,1)$ generators which:

1. *Preserve the points $\xi = 0$ and $\xi = \infty$.*
2. *Preserve $\xi = \infty$ but not $\xi = 0$.*
3. *Preserve $\xi = 0$ but not $\xi = \infty$.*

What is the action of these generators on ξ?