GENERAL RELATIVITY HOMEWORK – WEEK 6

Exercise 1. *No, seriously, solve Question 1 from Week 4.*

Exercise 2. *Show that knowing the exterior and Lie derivatives is sufficient. Specifically, given a metric* $g_{\mu\nu}(x)$ *and a vector field* $v^\mu(x)$*, express the covariant derivative* $\nabla_\mu v^\nu$ *in terms of exterior and Lie derivatives. In other words, show that any non-tensorial partial derivatives involved in the definition of* $\nabla_\mu v^\nu$ *can be packaged into exterior and Lie derivatives.*

Exercise 3. *Calculate the Christoffel symbols* Γ^i_{jk} *for the 2d flat plane in polar coordinates* (r, ϕ)*, and for 3d flat space in spherical coordinates* (r, θ, ϕ)

Exercise 4. *Consider Newton's law in polar coordinates* $x^i = (r, \phi)$*:*

$$\frac{Dv^i}{dt} = F^i \ , \tag{1}$$

where $v^i = dx^i/dt$ *is the velocity,* F^i *is the force, and* $Dv^i \equiv dv^i + dx^j \Gamma^i_{jk} v^k$ *is the covariant differential. We normalized the mass to* $m = 1$*.*

1. *Consider general circular motion* $\phi(t)$ *at a constant radius* r*. Find the velocity and force components* $v^i = (v^r, v^\phi)$ *and* $F^i = (F^r, F^\phi)$*. Notice how, for* <u>*uniform*</u> *circular motion* $\phi = \omega t$*, we can have acceleration despite* $dv^i/dt = 0$*.*
2. *Consider now a general radial force* $F^r(r, \phi)$*, with* $F^\phi = 0$*, and general (not necessarily circular) motion. One of the components of* v^i *or* v_i *is conserved. Identify and name it.*