SPECIAL RELATIVITY HOMEWORK – WEEK 4

Exercise 1. *In* $\mathbb{R}^{3,1}$*, we have represent spacetime vectors as* 2×2 *matrices via:*

$$x^\mu \sigma_\mu^{\alpha\dot{\alpha}} = \begin{pmatrix} t+z & x-iy \\ x+iy & t-z \end{pmatrix} \equiv X^{\alpha\dot{\alpha}} \ . \tag{1}$$

Within this setup, write all 6 Lorentz generators $J_{\mu\nu}$ *as* $SL(2,\mathbb{C})$ *generator matrices* $(J_{\mu\nu})^\alpha{}_\beta$*.*

Exercise 2. *A spacetime matrix* $A_{\mu\nu}$ *in* $\mathbb{R}^{3,1}$ *can be decomposed into three irreducible pieces – symmetric-traceless, antisymmetric, and trace:*

$$A_{\mu\nu} = \left(A_{(\mu\nu)} - \frac{1}{4} A^\rho{}_\rho \eta_{\mu\nu} \right) + A_{[\mu\nu]} + \frac{1}{4} A^\rho{}_\rho \eta_{\mu\nu} \ . \tag{2}$$

Write the corresponding decomposition for the spinor form $A_{\alpha\dot{\alpha}\beta\dot{\beta}}$ *of* $A_{\mu\nu}$*.*