

### SPECIAL RELATIVITY HOMEWORK – WEEK 3

**Exercise 1.** Write the generator  $J_{uy}$  and group element  $e^{\theta J_{uv}}$  of lightlike boosts, as  $3 \times 3$  matrices in the  $(u, v, y)$  basis.

**Exercise 2.** Consider the intersection of the  $\mathbb{R}^{2,1}$  lightcone with the lightlike plane  $v = 1$ . Express the  $(u, v, y)$  coordinates of points on this intersection in terms of our projective lightcone coordinate  $\xi = \cot \frac{\phi}{2}$ .

**Exercise 3.** Recall the vector multiplication table for the unit basis vectors in  $\mathbb{R}^3$ :

$$\begin{aligned} \hat{x} \times \hat{x} &= 0 ; & \hat{x} \times \hat{y} &= \hat{z} ; & \hat{x} \times \hat{z} &= -\hat{y} ; \\ \hat{y} \times \hat{x} &= -\hat{z} ; & \hat{y} \times \hat{y} &= 0 ; & \hat{y} \times \hat{z} &= \hat{x} ; \\ \hat{z} \times \hat{x} &= \hat{y} ; & \hat{z} \times \hat{y} &= -\hat{x} ; & \hat{z} \times \hat{z} &= 0 . \end{aligned} \tag{1}$$

Write a similar table for a Lorentz-invariant vector product in  $\mathbb{R}^{2,1}$ , in terms of the unit vectors  $\hat{t}, \hat{x}, \hat{y}$ . Justify your signs carefully!

**Exercise 4.** Consider two  $\mathbb{R}^{2,1}$  vectors  $a^\mu, b^\mu$ , encoded as traceless matrices  $A^\alpha_\beta, B^\alpha_\beta$ . What is the geometric meaning of the matrix product  $(AB)^\alpha_\beta$ ? Back up your answer with examples.