GENERAL RELATIVITY HOMEWORK – WEEK 2

Exercise 1. *Defend the honor of our course. Do the final exercise from the previous week properly. Collaborate with each other as necessary.*

Exercise 2. *Consider a cylindrically symmetric configuration of matter, with constant mass density per unit volume* ρ*. The matter is rotating around the axis of symmetry, with constant angular velocity* ω*. Express* ω *in terms of* ρ*, assuming that the only relevant force is Newtonian gravity. What would happen if we replaced gravity with Coulomb's electric force?*

Exercise 3. *In Newtonian gravity, consider a surface composed of probe particles. The surface initially encloses a volume* V*, within which there is a mass* M*. The probe particles' initial velocity is zero. Find the second time derivative* $\ddot{V}$ *in the first instant after the particles are released.*

Exercise 4. *Consider a small block of probe particles hanging above the Earth, at radius* R *from the Earth's center. The Earth's mass is* M*. The block's height is* h*, and its base area is* A*. The particles are initially at rest. Find the second time derivatives* $\ddot{h}$ *and* $\ddot{A}$ *in the first instant after the particles are released. Check for consistency with exercise 3.*

Exercise 5. *Consider the following non-orthonormal basis:*

$$\mathbf{e_1} = (0,1,1) \ ; \quad \mathbf{e_2} = (1,0,1) \ ; \quad \mathbf{e_3} = (1,1,0) \ . \tag{1}$$

Find the dual basis $\mathbf{e^1}, \mathbf{e^2}, \mathbf{e^2}$*, the metric* g_{ab} *and the inverse metric* g^{ab}*. What are the components* x^a *and* x_a *of the vector* $\mathbf{x} = (1,0,0)$ *in both bases? Verify the equalities:*

$$x_a = g_{ab}x^b \ ; \quad x^a = g^{ab}x_b \ ; \quad \mathbf{x}\cdot\mathbf{x} = x_a x^a \ . \tag{2}$$