SPECIAL RELATIVITY – FINAL EXAM

Exercise 1. *A particle at rest with mass M decays into three particles, each with mass m. What is the smallest possible velocity for one of the outgoing particles? What is the largest? What are the velocities of the other two particles in each of these two cases?*

Exercise 2. *Let's investigate Lorentz length contraction from the vantage point of a physical observer. Consider a square with sides of length a along the x and y axes (as measured in its rest frame). An observer at rest is situated at $\mathbf{x} = \vec{0}$. If the square is also at rest at $\mathbf{x} = (0, 0, L)$ with $L \gg a$, then the observer sees it centered along the z axis, with apparent angular size a/L along both x and y axes. Now, suppose the square moves at constant velocity v in the x direction, so that its center's trajectory is $\mathbf{x} = (vt, 0, L)$. At what time t will the observer see it centered along the z axis? At that time, what will be its apparent angular size along the x and y axes?*

Exercise 3. *In $\mathbb{R}^3$, we understood the geometric meaning of a spinor ψ^α in terms of the null complex vector $\psi_\alpha \boldsymbol{\sigma}^\alpha{}_\beta \psi^\beta$, its complex conjugate $-\bar{\psi}_\alpha \boldsymbol{\sigma}^\alpha{}_\beta \bar{\psi}^\beta$, and the real vector $\bar{\psi}_\alpha \boldsymbol{\sigma}^\alpha{}_\beta \psi^\beta$ orthogonal to both. What is the geometric meaning of a spinor sum $\psi^\alpha + \chi^\alpha$?*

Exercise 4. *In $\mathbb{R}^{3,1}$, write the formulas $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (for the EM field strength) and $\dot{p}_\mu = q F_{\mu\nu} \dot{x}^\nu$ (for the Lorenz force law) in spinor-index notation. Use $\partial_{\alpha\dot{\alpha}}, A_{\alpha\dot{\alpha}}, p_{\alpha\dot{\alpha}}, x^{\alpha\dot{\alpha}}$ to denote the corresponding vectors, and $f_{\alpha\beta}, \bar{f}_{\dot{\alpha}\dot{\beta}}$ to denote the EM field strength.*